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General Certificate of Education

Statistics 6380

SS05 Statistics 5

Mark Scheme

2009 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

SS05

Q	Solution	Marks	Total	Comments
1(a)	mean 1/0.05 = 20 s.d. 1/0.05 = 20	M1		Method for both
		A1	2	20 both, CAO
(b)	$1 - e^{-0.05 \times 20}$	B1		0.05 × 20
	$= 1 - e^{-1}$	M1		Method - allow wrong tail
	= 0.632	A1	3	0.6315 ~ 0.6325
(c)	$e^{-0.05 \times 10}$	M1		Attempt to find > or < 10 from exponential parameter 0.05 or equivalent
	$= e^{-0.5}$	m1		Method - allow wrong tail
	= 0.607	A1	3	0.606 ~ 0.607
	Total		**8**	

SS05 (cont)

Q	Solution	Marks	Total	Comments
2(a)	Range 15g > 10 × 1.4 or 6 × 1.4 = 8.4 < 15	E1		Comparison of range and s.d.
	Range is too large if σ = 1.4	E1	2	Full explanation
(b)	$s = 4.8033$	B1		4.8033 (4.80 ~ 4.81) or 23.07 (23 ~ 23.1) or 161.5 (161 ~ 162)
	$2.167 < 7 \times 4.8033^2/\sigma^2 < 14.067$	M1		Any correct expression; allow small slip, incorrect χ^2
		m1		Correct expression, allow incorrect χ^2
		B1		7 df
		B1		14.067 (14 ~14.1) and 2.167 (2.16 ~ 2.17)
	$161.5/14.067 < \sigma^2 < 161.5/2.167$	m1		Correct method for interval for σ, or σ^2 provided it is clearly called σ^2 or variance
	$11.481 < \sigma^2 < 74.527$			
	$3.39 < \sigma < 8.63$	A1	7	3.39 (3.385 ~3.395) and 8.63 (8.63 ~8.64)
	or using F $4.8033^2/\sigma^2 < 2.010$ *and* $\sigma^2/4.8033^2 < 3.230$			
(c)	Statement supported since 1.4 is below lower bound of confidence interval	B1√ E1	2	Statement supported - their c.i. Explanation
(d)	$\bar{x} = 218.75$ 95% confidence interval			
	$218.75 \pm 2.365 \times 4.8033 / \sqrt{8}$	M1		Use of their s.d. / $\sqrt{8}$
		M1		Attempt at c.i. using t
		m1		Method - allow incorrect t-value
		B1		2.365 (2.36 ~ 2.37)
	218.75 ± 4.02			
	214.7 ~ 222.8	A1	5	214.7 (214.7 ~ 214.8) and 222.8 (222.7 ~ 222.8); allow 215 and 223
(e)	Confidence interval indicates that mean is above 212. Hence mean could be reduced and the mean could still be greater than 212. However, the fact that one member of the sample only contains 212g indicates that if the mean were reduced some individual jars would contain less than 212g.	E1 E1 E1	3	Statement incorrect Mean could be reduced and still be greater than 212 If mean were reduced some individual jars would contain less than 212g
	Total		**19**	

SS05 (cont)

<table>
<tr><th>Q</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>3(a)</td><td><table><tr><th>Size</th><th>Frequency</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>10</td></tr><tr><td>4</td><td>12</td></tr><tr><td>5</td><td>11</td></tr></table></td><td>M1

A1</td><td>

2</td><td>Method for frequency distribution

Frequencies CAO</td></tr>
<tr><td>(b)</td><td><table><tr><th>Size</th><th>O</th><th>E</th></tr><tr><td>1</td><td>2</td><td>6</td></tr><tr><td>2</td><td>5</td><td>8</td></tr><tr><td>3</td><td>10</td><td>12</td></tr><tr><td>4</td><td>12</td><td>8</td></tr><tr><td>5</td><td>11</td><td>6</td></tr></table></td><td>B1</td><td></td><td>Correct values for E</td></tr>
<tr><td></td><td>H_0: Probability distribution is adequate model
H_1: Probability distribution is not adequate model</td><td>B1</td><td></td><td>Hypotheses - may be earned in conclusion</td></tr>
<tr><td></td><td>$\Sigma(O - E)^2/E$
$= 4^2/6 + 3^2/8 + 2^2/12 + 4^2/8 + 5^2/6$</td><td>M1</td><td></td><td>Attempt at $\Sigma(O - E)^2/E$ - their Es and Os</td></tr>
<tr><td></td><td>$= 10.3$</td><td>A1</td><td></td><td>10.25 ~ 10.35</td></tr>
<tr><td></td><td>c.v. χ_4^2 is 9.488</td><td>B1
B1√</td><td></td><td>4 df
9.488 - their df</td></tr>
<tr><td></td><td>Significant evidence that the probability distribution does not adequately model the distribution of required helmet sizes</td><td>A1√</td><td>7</td><td>Conclusion - needs correct method for Es and Os and comparison with upper tail of χ^2</td></tr>
<tr><td>(c)</td><td>Modify order - more large helmets, less small helmets than suggested by probability distribution</td><td>E1√

E1</td><td>

2</td><td>Modify order

More large, less small</td></tr>
<tr><td></td><td></td><td></td><td>**11**</td><td></td></tr>
</table>

SS05 (cont)

Q	Solution	Marks	Total	Comments
4(a)	0.2/6 = 0.0333	M1 A1	 2	Method 0.0333 (0.033 ~ 0.034) or 1/30
(b)	mean 3	B1		CAO
	s.d. $6/\sqrt{12}$ = 1.73	M1 A1	 3	Correct method 1.73 (1.73 ~ 1.735) or $\sqrt{3}$ **SC** allow B1 instead of M1A1 for variance = 3
(c)	$z_1 = (3.1-3)/(1.732/\sqrt{46}) = 0.392$ $z_2 = (2.9-3)/(1.732/\sqrt{46}) = -0.392$	M1		Use of their s.d./$\sqrt{46}$
		m1 m1		z-values - their mean and s.d. method for z-values - requires correct method for mean and s.d.
	Probability between 2.9 and 3.1 = 0.6525 – (1 – 0.6525)	m1		Method
	= 0.305	A1	5	0.3 ~ 0.31
			10	

SS05 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	H$_0$: $\sigma_s = 0.65$ H$_1$: $\sigma_s \neq 0.65$	B1		Both hypotheses
	$s_s = 0.710466$ ($\bar{x}_s = 15.2143$)			
	$\sum(x-\bar{x})^2/\sigma^2 = 6 \times 0.710466^2/0.65^2$	M1		Method for test statistic - allow small slip, eg 7×0.710466^2
		m1		Correct method for test statistic
	$= 7.17$	A1		7.165 ~ 7.175
	c.v. χ_6^2 are 1.237 and 14.449	B1		6 df
		B1		1.237 and 14.449 - allow 1.24 and 14.4
	Accept H$_0$: $\sigma_s = 0.65$, ie accept standard deviation of breaking strength of standard line is 0.65kg	A1√		Conclusion - must be compared with least one χ^2 value
	Using F, compare 1.19 (1.19 ~ 1.2) with 2.408 (or reciprocals)			
(ii)	H$_0$: $\sigma_p = 0.95$ H$_1$: $\sigma_p \neq 0.95$	B1		Both hypotheses
	$s_p = 1.30945$ ($\bar{x}_p = 19.7333$)			
	$\sum(x-\bar{x})^2/\sigma^2 = 5 \times 1.30945^2/0.95^2$			
	$= 9.50$	A1		9.49 ~ 9.505
	c.v. χ_5^2 are 0.831 and 12.833	B1		Allow 0.83 and 12.8
	Accept H$_0$: $\sigma_p = 0.95$, ie accept standard deviation of breaking strength of premium line is 0.95kg	A1	11	Conclusion in context. Needs both c.v. – must mention standard deviation / variance and fishing line or breaking strength
	Using F, compare 1.90 (1.895 ~ 1.905) with 6.02 (or reciprocals)			*Use mark scheme for (i) in (ii) and for (ii) in (i) if more favourable to candidate*

SS05 (cont)

Q	Solution	Marks	Total	Comments
5(cont) (b)	H_0: $\mu_p = \mu_s + 5$ H_1: $\mu_p < \mu_s + 5$	B1		Hypotheses
	$z = \dfrac{19.7333 - 15.2143 - 5}{\sqrt{(0.95^2/6 + 0.65^2/7)}}$	M1 M1		Method for variance Method for z - their variance
	$= -1.05$	A1		–1.04 ~ –1.06 - ignore sign
	c.v. –1.2816	B1		–1.28 ~ –1.282 - ignore sign
	Accept H_0 , ie accept mean breaking strength of premium line is at least 5kg greater than that of standard line	A1√ A1√	7	Accept H_0 - must be compared with correct tail of z – needs both M marks Conclusion in context – needs previous A1√
	p-value 0.148 (0.146 ~ 0.149) compare with 0.1			*If t used, maximum B1M0M1A0B1(for 1.363)A0A0*
(c)	H_0: $\sigma_p = \sigma_s$ H_1: $\sigma_p \neq \sigma_s$	B1		Hypotheses
	$F = 1.30945^2 / 0.710466^2 = 3.40$	M1 A1		Method for F 3.40 (3.39 ~ 3.4)
	c.v. $F_{[5,6]}$ is 5.988 (or compare 0.294 with 0.167)	B1 B1		5,6 df (or 6,5 if $0.710^2/1.81^2$ calculated) 5.988 (5.98 ~ 6)
	Accept H_0: accept standard deviations of breaking strengths of two types of line are equal	A1√	6	Conclusion in context - must be compared with correct tail of F
(d)	Not all null hypotheses can be true. At least one Type II error (accepting a false null hypothesis must have been made). Accepting the null hypothesis only shows that any evidence against it is not significant, not that it is true. In this case the samples are small, so accepting the null hypothesis is quite a weak result.	E1 E1 E1	3	Not all null hypotheses can be true Type II error must have been made Accepting null hypothesis does not prove it is true Allow any other sensible comment, eg small samples (max 3)
	Total		**27**	
	TOTAL		**75**	