

General Certificate of Education
June 2009
Advanced Level Examination



MATHEMATICS
Unit Statistics 2B

MS2B

Monday 15 June 2009 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A machine fills bottles with bleach. The volume, in millilitres, of bleach dispensed by the machine into a bottle may be modelled by a normal distribution with mean μ and standard deviation 8.

A recent inspection indicated that the value of μ was 768. Yvonne, the machine's operator, claims that this value has not subsequently changed.

Zara, the quality control supervisor, records the volume of bleach in each of a random sample of 18 bottles filled by the machine and calculates their mean to be 764.8 ml.

Test, at the 5% level of significance, Yvonne's claim that the mean volume of bleach dispensed by the machine has not changed from 768 ml. (6 marks)

- 2 John works from home. The number of business letters, X , that he receives on a weekday may be modelled by a Poisson distribution with mean 5.0.

The number of private letters, Y , that he receives on a weekday may be modelled by a Poisson distribution with mean 1.5.

(a) Find, for a given weekday:

(i) $P(X < 4)$; (2 marks)

(ii) $P(Y = 4)$. (2 marks)

(b) (i) Assuming that X and Y are independent random variables, determine the probability that, on a given weekday, John receives a **total** of more than 5 business and private letters. (3 marks)

(ii) Hence calculate the probability that John receives a **total** of more than 5 business and private letters on at least 7 out of 8 given weekdays. (3 marks)

(c) The numbers of letters received by John's neighbour, Brenda, on 10 consecutive weekdays are

15 8 14 7 6 8 2 8 9 3

(i) Calculate the mean and the variance of these data. (2 marks)

(ii) State, giving a reason based on your answers to part (c)(i), whether or not a Poisson distribution might provide a suitable model for the number of letters received by Brenda on a weekday. (2 marks)

- 3 A sample survey, conducted to determine the attitudes of residents to a proposed reorganisation of local schools, gave the following results.

| | | Against reorganisation | Not against reorganisation |
|-----------------|---------|------------------------|----------------------------|
| Age of resident | 16–17 | 9 | 2 |
| | 18–21 | 17 | 10 |
| | 22–49 | 115 | 90 |
| | 50–65 | 41 | 34 |
| | Over 65 | 3 | 4 |

Use a χ^2 test, at the 5% level of significance, to determine whether there is an association between the ages of residents and their attitudes to the proposed reorganisation of local schools. (12 marks)

- 4 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leq x \leq 1 \\ \frac{3-x}{4} & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the graph of f . (3 marks)
- (b) Explain why the value of η , the median of X , is 1. (2 marks)
- (c) Show that the value of μ , the mean of X , is $\frac{13}{12}$. (4 marks)
- (d) Find $P(X < 3\mu - \eta)$. (3 marks)

Turn over for the next question

Turn over ►

5 Joanne has 10 identically-shaped discs, of which 1 is blue, 2 are green, 3 are yellow and 4 are red. She places the 10 discs in a bag and asks her friend David to play a game by selecting, at random and without replacement, two discs from the bag.

(a) Show that:

(i) the probability that the two discs selected are the same colour is $\frac{2}{9}$; (2 marks)

(ii) the probability that exactly one of the two discs selected is blue is $\frac{1}{5}$. (2 marks)

(b) Using the discs, Joanne plays the game with David, under the following conditions:

If the two discs selected by David are the same colour, she will pay him 135p.

If exactly one of the two discs selected by David is blue, she will pay him 145p.

Otherwise David will pay Joanne 45p.

(i) When a game is played, X is the amount, in pence, **won by David**. Construct the probability distribution for X , in the form of a table. (2 marks)

(ii) Show that $E(X) = 33$. (2 marks)

(c) Joanne modifies the game so that the amount per game, Y pence, that **she wins** may be modelled by

$$Y = 104 - 3X$$

(i) Determine how much Joanne would expect to win if the game is played 100 times. (3 marks)

(ii) Calculate the standard deviation of Y , giving your answer to the nearest 1p. (4 marks)

6 Bishen believes that the mean weight of boxes of black peppercorns is 45 grams. Abi, thinking that this is not the case, weighs, in grams, a random sample of 8 boxes of black peppercorns, with the following results.

44 44 43 46 42 40 43 46

(a) (i) Construct a 95% confidence interval for the mean weight of boxes of black peppercorns, stating any assumption that you make. (6 marks)

(ii) Comment on Bishen's belief. (2 marks)

(b) (i) Abi claims that the mean weight of boxes of black peppercorns is less than 45 grams. Test this claim at the 5% level of significance. (6 marks)

(ii) If Bishen's belief is true, state, with a reason, what type of error, if any, may have occurred when conclusions to the test in part (b)(i) were drawn. (2 marks)

END OF QUESTIONS