Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MPC3

Unit Pure Core 3

Wednesday 19 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 (a) Find $\frac{dy}{dx}$ when $y = (x^3 - 1)^6$. *(2 marks)*

(b) A curve has equation $y = x \ln x$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the tangent to the curve $y = x \ln x$ at the point on the curve where $x = e$. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

2 A curve is defined by the equation $y = (x^2 - 4)\ln(x+2)$ for $x \geqslant 3$.

The curve intersects the line $y = 15$ at a single point, where $x = \alpha$.

(a) Show that α lies between 3.5 and 3.6. *(2 marks)*

(b) Show that the equation $(x^2 - 4)\ln(x+2) = 15$ can be arranged into the form

$$x = \pm\sqrt{4 + \frac{15}{\ln(x+2)}}$$ *(2 marks)*

(c) Use the iteration

$$x_{n+1} = \sqrt{4 + \frac{15}{\ln(x_n+2)}}$$

with $x_1 = 3.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

3 (a) Given that $x = \tan(3y + 1)$:

(i) find $\dfrac{dx}{dy}$ in terms of y; *(2 marks)*

(ii) find the value of $\dfrac{dy}{dx}$ when $y = -\dfrac{1}{3}$. *(2 marks)*

(b) Sketch the graph of $y = \tan^{-1} x$. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = 3\cos\tfrac{1}{2}x, \qquad \text{for } 0 \leqslant x \leqslant 2\pi$$

$$\mathrm{g}(x) = |x|, \qquad \text{for all real values of } x$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 1$, giving your answer in an exact form. *(2 marks)*

(c) (i) Write down an expression for $\mathrm{gf}(x)$. *(1 mark)*

(ii) Sketch the graph of $y = \mathrm{gf}(x)$ for $0 \leqslant x \leqslant 2\pi$. *(3 marks)*

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos x$ onto the graph of $y = 3\cos\frac{1}{2}x$. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

5 (a) Find $\int \frac{1}{3+2x}\,dx$. *(2 marks)*

(b) By using integration by parts, find $\int x \sin\frac{x}{2}\,dx$. *(4 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Do not write outside the box

6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^{0.4} \cos\sqrt{3x+1}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) Use the substitution $u = 3x + 1$ to find the exact value of $\int_0^1 x\sqrt{3x+1}\,dx$. *(6 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

7 (a) Solve the equation $\sec x = -5$, giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. *(3 marks)*

(b) Show that the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

can be written in the form

$$\sec^2 x = 25$$ *(4 marks)*

(c) Hence, or otherwise, solve the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. *(3 marks)*

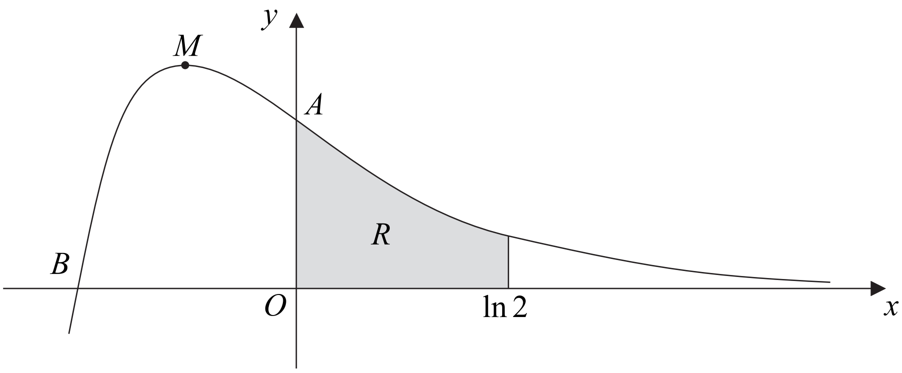
QUESTION PART REFERENCE

QUESTION PART REFERENCE

Do not write outside the box

8 (a) Given that $e^{-2x} = 4$, find the exact value of x. *(2 marks)*

(b) The diagram shows the curve $y = 4e^{-2x} - e^{-4x}$.



The curve crosses the y-axis at the point A, the x-axis at the point B, and has a stationary point at M.

(i) State the y-coordinate of A. *(1 mark)*

(ii) Find the x-coordinate of B, giving your answer in an exact form. *(3 marks)*

(iii) Find the x-coordinate of the stationary point, M, giving your answer in an exact form. *(3 marks)*

(iv) The shaded region R is bounded by the curve $y = 4e^{-2x} - e^{-4x}$, the lines $x = 0$ and $x = \ln 2$ and the x-axis.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $\frac{p}{q}\pi$, where p and q are integers. *(7 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

END OF QUESTIONS