Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
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7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MPC2

Unit Pure Core 2

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

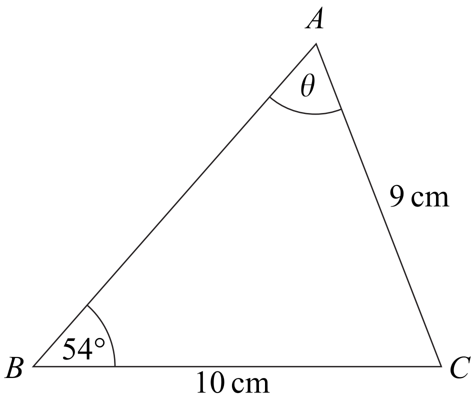
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The triangle ABC, shown in the diagram, is such that $AC = 9\text{ cm}$, $BC = 10\text{ cm}$, angle $ABC = 54°$ and the acute angle $BAC = \theta$.



(a) Show that $\theta = 64°$, correct to the nearest degree. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer to the nearest square centimetre. *(3 marks)*

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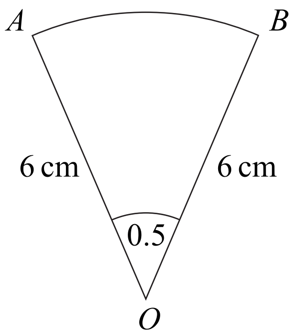
2 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle $AOB = 0.5$ radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) (i) Find the length of the arc AB. *(2 marks)*

(ii) Hence show that

the perimeter of the sector $OAB = k \times$ the length of the arc AB

where k is an integer. *(2 marks)*

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3 (a) The expression $(2+x^2)^3$ can be written in the form

$$8 + px^2 + qx^4 + x^6$$

Show that $p = 12$ and find the value of the integer q. *(3 marks)*

(b) (i) Hence find $\int \frac{(2+x^2)^3}{x^4}\,\mathrm{d}x$. *(5 marks)*

(ii) Hence find the exact value of $\int_1^2 \frac{(2+x^2)^3}{x^4}\,\mathrm{d}x$. *(2 marks)*

QUESTION PART REFERENCE

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4 (a) Sketch the curve with equation $y = 4^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) Describe the geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 4^x - 5$. *(2 marks)*

(c) (i) Use the substitution $Y = 2^x$ to show that the equation $4^x - 2^{x+2} - 5 = 0$ can be written as $Y^2 - 4Y - 5 = 0$. *(2 marks)*

(ii) Hence show that the equation $4^x - 2^{x+2} - 5 = 0$ has only one real solution. Use logarithms to find this solution, giving your answer to three decimal places. *(4 marks)*

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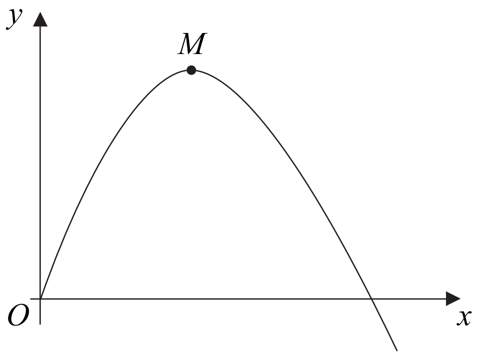
5 The diagram shows part of a curve with a maximum point M.



The curve is defined for $x \geqslant 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) (i) Hence find the coordinates of the maximum point M. *(3 marks)*

(ii) Write down the equation of the normal to the curve at M. *(1 mark)*

(c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.

(i) Find an equation of the normal to the curve at the point P, giving your answer in the form $ax + by = c$, where a, b and c are positive integers. *(4 marks)*

(ii) The normals to the curve at the points M and P intersect at the point R. Find the coordinates of R. *(2 marks)*

QUESTION PART REFERENCE

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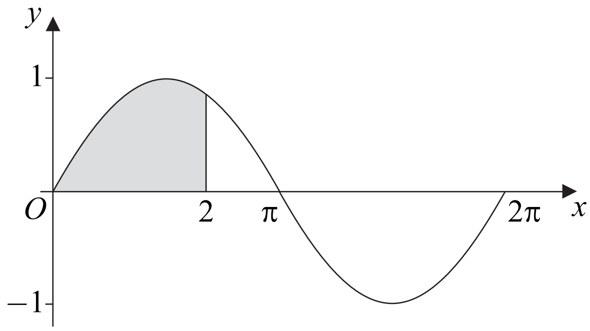
6 A curve C, defined for $0 \leqslant x \leqslant 2\pi$ by the equation $y = \sin x$, where x is in radians, is sketched below. The region bounded by the curve C, the x-axis from 0 to 2 and the line $x = 2$ is shaded.



(a) The area of the shaded region is given by $\int_0^2 \sin x \, dx$, where x is in radians.

Use the trapezium rule with five ordinates (four strips) to find an approximate value for the area of the shaded region, giving your answer to three significant figures.
(4 marks)

(b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = 2\sin x$.
(2 marks)

(c) Use a trigonometrical identity to solve the equation

$$2\sin x = \cos x$$

in the interval $0 \leqslant x \leqslant 2\pi$, giving your solutions in radians to three significant figures.
(4 marks)

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7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 60$ and $u_2 = 48$.

The limit of u_n as n tends to infinity is 12.

(a) Show that $p = \frac{3}{4}$ and find the value of q. *(5 marks)*

(b) Find the value of u_3. *(1 mark)*

QUESTION PART REFERENCE

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8 Prove that, for all values of x, the value of the expression

$$(3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$$

is an integer and state its value. *(4 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

9 The first term of a geometric series is 12 and the common ratio of the series is $\frac{3}{8}$.

(a) Find the sum to infinity of the series. *(2 marks)*

(b) Show that the sixth term of the series can be written in the form $\frac{3^6}{2^{13}}$. *(3 marks)*

(c) The nth term of the series is u_n.

(i) Write down an expression for u_n in terms of n. *(1 mark)*

(ii) Hence show that

$$\log_a u_n = n \log_a 3 - (3n - 5) \log_a 2$$

(4 marks)

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END OF QUESTIONS