



General Certificate of Education  
Advanced Subsidiary Examination  
January 2010

## Mathematics

## MPC2

### Unit Pure Core 2

Monday 11 January 2010 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

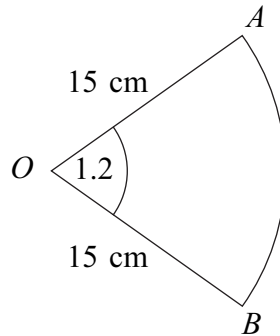
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

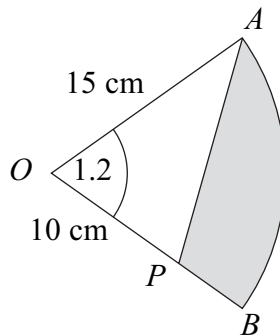
Answer **all** questions.

- 1 The diagram shows a sector  $OAB$  of a circle with centre  $O$ .



The radius of the circle is 15 cm and angle  $AOB = 1.2$  radians.

- (a) (i) Show that the area of the sector is  $135 \text{ cm}^2$ . (2 marks)
- (ii) Calculate the length of the arc  $AB$ . (2 marks)
- (b) The point  $P$  lies on the radius  $OB$  such that  $OP = 10$  cm, as shown in the diagram below.



Calculate the perimeter of the shaded region bounded by  $AP$ ,  $PB$  and the arc  $AB$ , giving your answer to three significant figures. (5 marks)

2 At the point  $(x, y)$  on a curve, where  $x > 0$ , the gradient is given by

$$\frac{dy}{dx} = 7\sqrt{x^5} - 4$$

(a) Write  $\sqrt{x^5}$  in the form  $x^k$ , where  $k$  is a fraction. (1 mark)

(b) Find  $\int (7\sqrt{x^5} - 4) dx$ . (3 marks)

(c) Hence find the equation of the curve, given that the curve passes through the point  $(1, 3)$ . (3 marks)

3 (a) Find the value of  $x$  in each of the following:

(i)  $\log_9 x = 0$ ; (1 mark)

(ii)  $\log_9 x = \frac{1}{2}$ . (1 mark)

(b) Given that

$$2 \log_a n = \log_a 18 + \log_a (n - 4)$$

find the possible values of  $n$ . (5 marks)

4 An arithmetic series has first term  $a$  and common difference  $d$ .

The sum of the first 31 terms of the series is 310.

(a) Show that  $a + 15d = 10$ . (3 marks)

(b) Given also that the 21st term is twice the 16th term, find the value of  $d$ . (3 marks)

(c) The  $n$ th term of the series is  $u_n$ . Given that  $\sum_{n=1}^k u_n = 0$ , find the value of  $k$ . (4 marks)

Turn over ►

5 A curve has equation  $y = \frac{1}{x^3} + 48x$ .

(a) Find  $\frac{dy}{dx}$ . (3 marks)

(b) Hence find the equation of each of the two tangents to the curve that are parallel to the  $x$ -axis. (4 marks)

(c) Find an equation of the normal to the curve at the point  $(1, 49)$ . (3 marks)

6 (a) Sketch the curve with equation  $y = 2^x$ , indicating the coordinates of any point where the curve intersects the coordinate axes. (2 marks)

(b) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for  $\int_0^2 2^x dx$ , giving your answer to three significant figures. (4 marks)

(ii) State how you could obtain a better approximation to the value of the integral using the trapezium rule. (1 mark)

(c) Describe a geometrical transformation that maps the graph of  $y = 2^x$  onto the graph of  $y = 2^{x+7} + 3$ . (3 marks)

(d) The curve  $y = 2^{x+k} + 3$  intersects the  $y$ -axis at the point  $A(0, 8)$ .

Show that  $k = \log_m n$ , where  $m$  and  $n$  are integers. (2 marks)

7 (a) The first four terms of the binomial expansion of  $(1 + 2x)^7$  in ascending powers of  $x$  are  $1 + ax + bx^2 + cx^3$ . Find the values of the integers  $a$ ,  $b$  and  $c$ . (4 marks)

(b) Hence find the coefficient of  $x^3$  in the expansion of  $\left(1 - \frac{1}{2}x\right)^2 (1 + 2x)^7$ . (4 marks)

- 8 (a) Solve the equation  $\tan(x + 52^\circ) = \tan 22^\circ$ , giving the values of  $x$  in the interval  $0^\circ \leq x \leq 360^\circ$ . (3 marks)

- (b) (i) Show that the equation

$$3 \tan \theta = \frac{8}{\sin \theta}$$

can be written as

$$3 \cos^2 \theta + 8 \cos \theta - 3 = 0 \quad (3 \text{ marks})$$

- (ii) Find the value of  $\cos \theta$  that satisfies the equation

$$3 \cos^2 \theta + 8 \cos \theta - 3 = 0 \quad (2 \text{ marks})$$

- (iii) Hence solve the equation

$$3 \tan 2x = \frac{8}{\sin 2x}$$

giving all values of  $x$  to the nearest degree in the interval  $0^\circ \leq x \leq 180^\circ$ . (4 marks)

**END OF QUESTIONS**

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