General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MM05**
Unit Mechanics 5

Tuesday 23 June 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

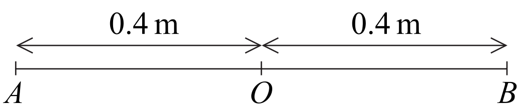
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves with simple harmonic motion along a straight line AOB, where $AO = OB = 0.4$ metres, as shown in the diagram.



The maximum speed of the particle is $1.2\,\text{m}\,\text{s}^{-1}$.

(a) Find the period of the motion. *(4 marks)*

(b) Find the magnitude of the maximum acceleration of the particle. *(2 marks)*

(c) The point C lies on OB. The speed of the particle as it passes through C is $0.9\,\text{m}\,\text{s}^{-1}$. Find the distance OC. *(3 marks)*

2 A simple pendulum consists of a particle, of mass m, attached to one end of a light inextensible string, of length l. The other end of the string is attached to a fixed point. The pendulum is set into motion in a vertical plane. At time t, the angle between the string and the downward vertical is θ.

(a) Using a small angle approximation, show that the motion of the pendulum can be modelled by the differential equation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g\theta}{l}$$

(4 marks)

(b) The period of the motion is 2.4 seconds. Find the value of l. *(2 marks)*

(c) During this motion, the pendulum is instantaneously at rest when the string is at an angle of 0.15 radians to the vertical. Find the maximum speed of the particle during the motion. *(3 marks)*

3 An astronaut on a spacewalk is travelling in a straight line. In order to increase his speed, he fires his rocket pack, which ejects burnt fuel backwards at a constant rate of $10\,\text{kg}\,\text{s}^{-1}$ and at a constant speed of $30\,\text{m}\,\text{s}^{-1}$ relative to the astronaut. Initially, the total mass of the astronaut with the rocket pack and fuel is 200 kg.

You should assume that gravitational forces can be ignored.

When the rocket pack has been fired for t seconds, the speed of the astronaut is $v\,\text{m}\,\text{s}^{-1}$.

(a) Show that, while the rocket pack is being fired,

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{30}{20 - t}$$ *(6 marks)*

(b) The initial speed of the astronaut is $2\,\text{m}\,\text{s}^{-1}$. Show that the speed of the astronaut at time t is given by

$$v = 30\ln\left(\frac{20}{20 - t}\right) + 2$$ *(4 marks)*

(c) The astronaut fires the rocket pack until his speed is $6\,\text{m}\,\text{s}^{-1}$. Calculate the time for which the rocket pack is fired. *(3 marks)*

4 A particle P, of mass m, is suspended from a fixed point O by a light elastic string, of natural length a and modulus of elasticity $5mn^2a$, where n is a positive constant. When the particle hangs in equilibrium vertically below O, the extension of the string is $\frac{g}{5n^2}$. At time $t = 0$, the particle is projected vertically downwards from this equilibrium position with speed U.

During its subsequent motion, when P is moving with speed v, it experiences a resistance force of magnitude $2mnv$. The displacement of P below its equilibrium position at time t is x.

(a) Show that, whilst the string is taut, x satisfies the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2n\frac{\mathrm{d}x}{\mathrm{d}t} + 5n^2x = 0$$ *(5 marks)*

(b) Find x in terms of U and t. *(7 marks)*

(c) State whether the nature of the damping caused by the resistance force is light, critical or heavy. *(1 mark)*

(d) Find, in terms of U and π, the speed of P when it first returns to its equilibrium position. *(4 marks)*

5 A particle P moves so that, at time t, its polar coordinates (r, θ) with respect to a fixed origin O are such that

$$r = \frac{2}{2 + \cos\theta}\ , \quad 0 \leqslant \theta < 2\pi$$

At all times during the motion the value of $r^2\dot{\theta} = 2$.

(a) Write down the value of $r\dot{\theta}$ in terms of θ. *(1 mark)*

(b) (i) Given that $\dot{r} = \sin\theta$, show that the speed of P is $\sqrt{5 + 4\cos\theta}$. *(3 marks)*

(ii) State the range of values of the speed of P. *(1 mark)*

(c) (i) Show that the transverse component of the acceleration of P is zero. *(2 marks)*

(ii) Hence find an expression for the acceleration of P in terms of r. *(4 marks)*

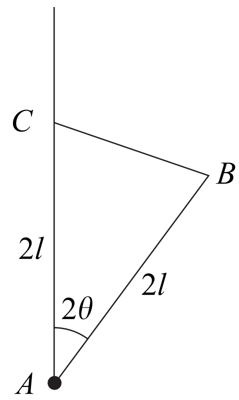
(iii) Deduce that the force acting on P is directed towards O at all times during the motion. *(2 marks)*

6 A uniform rod AB is smoothly hinged at A, and is free to move in a vertical plane. A light spring connects B to a point C, vertically above A.

The rod is of length $2l$ and of mass $2m$.

The spring is of natural length l and modulus $2mg$, and the distance AC is $2l$.

The angle BAC is 2θ, as shown in the diagram, where $0 < \theta \leqslant \frac{\pi}{2}$.



(a) The gravitational potential energy is taken to be zero at the level of A. Show that V, the total potential energy of the system, is given by

$$V = mgl(12\sin^2\theta - 8\sin\theta + 3)$$ *(6 marks)*

(b) Find the two values of θ for which the rod is in equilibrium. *(4 marks)*

(c) Determine, for each of these values, whether the rod is in stable or unstable equilibrium. *(4 marks)*

END OF QUESTIONS

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