Version 1.0

General Certificate of Education
June 2010

Mathematics MM04

Mechanics 4

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

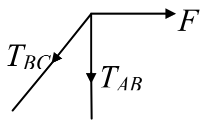
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

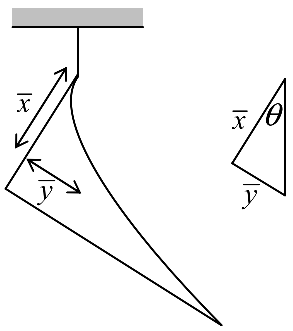
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM04

Q	Solution	Marks	Total	Comments
1(a)	Moments at A:			
	$50(2\cos 30°) = F(4\cos 30°)$	M1		One side correct. Use of ratios ok
	$\therefore F = 25\text{N}$	A1	2	
(b)	Magnitude 25 N, to the right $(\rightarrow)$	B1,B1	2	B1 each part
(c)(i)				
	Resolving horizontally at B:			
	$T_{BC}\ \sin 30° = F$	M1		Attempt at an equation to find T_{BC}
	$T_{BC} = 50\,\text{N}$	A1F	2	ft part (a)
(ii)	Resolving vertically at B:			
	$T_{AB} + T_{BC}\ \cos 30° = 0$	M1		Attempt at an equation to find T_{AB}
	$\lvert T_{AB}\rvert = 25\sqrt{3}$ or $43.3\,\text{N}$	A1F	2	ft part (a); must be positive for A1
	Total		**8**	
2(a)	Momentum = $I\omega$			
	$= 0.6 \times 3$	M1		Evidence of $I\omega$
	$= 1.8\ \text{kg m}^2\,\text{s}^{-1}$	A1	2	Units required
(b)	$0.45\omega_1 = 1.8$	M1		Forming equation – conservation of angular momentum
	$\omega_1 = 4\,\text{rad}\,\text{s}^{-1}$	A1F	2	ft their part (a); units required
	Total		**4**	

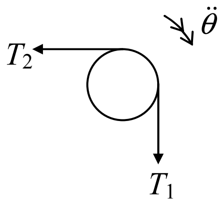
MM04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Area = $\int_0^2 kx^3\,dx = \left[\frac{kx^4}{4}\right]_0^2$	M1		Attempt to integrate
	$= 4k$	A1	2	
(b)	$\int xy\,dx = \int_0^2 kx^4\,dx = \left[\frac{kx^5}{5}\right]_0^2$	M1		Attempt to use $\int xy\,dx$
	$= \frac{32k}{5}$	A1		
	$\bar{x} = \frac{\int xy\,dx}{\int y\,dx} = \frac{\frac{32k}{5}}{4k}$	M1		Forming equation to find $\bar{x}$
	$= 1.6$	A1F	4	ft 'their' part (a); must not contain k
(c)(i)	$\frac{1}{2}\int y^2\,dx = \frac{1}{2}\int_0^2 k^2x^6\,dx = \left[\frac{k^2x^7}{14}\right]_0^2$	M1		Attempt to use $\frac{1}{2}\int y^2\,dx$
	$= \frac{64}{7}k$	A1		
	$\bar{y} = \frac{\frac{1}{2}\int y^2\,dx}{\int y\,dx} = \frac{\frac{64k^2}{7}}{4k} = \frac{16k}{7}$	M1		Finding $\bar{y}$ in terms of k
	$\therefore \frac{16k}{7} = 8$			
	$\therefore k = 3.5$	A1F	4	ft 'their' part (a)
(ii)				
	$\tan\theta = \frac{\bar{y}}{\bar{x}}$	M1		Use of $\tan\theta$
	$= \frac{8}{1.6} = 5$	A1F		ft 'their' part (b); $\frac{\bar{y}}{\bar{x}}$ structure
	$\theta = 78.7°$	A1F	3	
	Total		**13**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Moments about C:	B1		$P\cos\theta$ seen
	$mga = P\cos\theta a$	M1		Forming moments equation – 2 terms
	$P = \dfrac{mg}{\cos\theta}$	A1	3	
(b)	Resolve ↔ $F = P\cos\theta$ 1	M1		Resolve in two directions
	Resolve ↕ $mg = R + P\sin\theta$ 2	A1		Both equations correct
	Friction law (sliding) $F = \mu R$ 3			
	Substituting 1 and 2 in 3:			
	$P\cos\theta = \mu(mg - P\sin\theta)$	m1		Substituting in $F = \mu R$ – dep on first M1
	$P\cos\theta + P\mu\sin\theta = \mu mg$			
	$P = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$	A1	4	AG
(c)	Slides first ⇒			
	$\dfrac{\mu mg}{\cos\theta + \mu\sin\theta} < \dfrac{mg}{\cos\theta}$	M1		Set up inequality – expression in (b) < expression in (a)(ii)
	$\mu\cos\theta < \cos\theta + \mu\sin\theta$	A1F		Correct simplification – remove fractions ft parts (a) and (b)
	$\mu(\cos\theta - \sin\theta) < \cos\theta$			
	$\mu < \dfrac{\cos\theta}{\cos\theta - \sin\theta}$	A1	3	CAO ; Alternative: $\mu < \dfrac{1}{1-\tan\theta}$
(d)	Inequality independent of mass, so no change	E2,1F	2	**No change** (E1) and **reason** (E1) ft error in (c); must give consistent reason If no reason, E0
	Total		**12**	

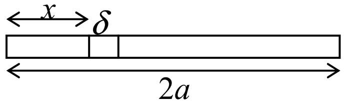
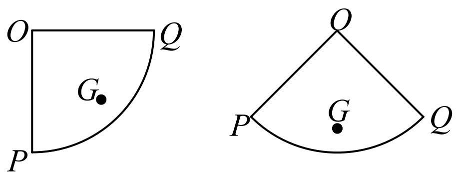
MM04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\text{MI}_{\text{disc}} = \frac{1}{2}Mr^2 = 6mr^2$	B1	1	
(b)(i)				
	Using $C = I\ddot{\theta}$:			
	$T_1r - T_2r$ or $(T_1 - T_2)r$	M1		Moments of both tensions seen
	$T_1r - T_2r = 6mr^2\ddot{\theta}$	m1		Equation formed using $C = I\ddot{\theta}$
	$T_1 - T_2 = 6mr\ddot{\theta}$ 1	A1	3	AG
(ii)	Equation of motion of R:			
	$T_2 = mr\ddot{\theta}$ 2	M1 A1		Evidence of $r\ddot{\theta}$ anywhere Correct equation
	Equation of motion of P:			
	$3mg - T_1 = 3mr\ddot{\theta}$ 3	M1 A1		Attempt at F = ma – three terms Correct equation
	Substituting 2 and 3 in 1: $(3mg - 3mr\ddot{\theta}) - mr\ddot{\theta} = 6mr\ddot{\theta}$	m1		Dep on previous M1 – solving three equations
	$3mg = 10mr\ddot{\theta}$			
	$\ddot{\theta} = \frac{3g}{10r}$	A1		AG
	Alternative to (b)(ii)			
	$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}mv^2 + \frac{3}{2}mv^2 = 3mgh$	(M1)		Attempt at Conservation of Energy – three 'types' of term
		(A1)		Fully current equation
	$\frac{1}{2}(6mr^2)\dot{\theta}^2 + \frac{1}{2}m(r\dot{\theta})^2 + \frac{3}{2}m(r\dot{\theta})^2 = 3mg(r\theta)$	(M1)		$v = r\ddot{\theta}$ and $h = r\ddot{\theta}$ used
	$5r\dot{\theta}^2 = 3g\theta$	(A1)		Simplified to two terms $a\dot{\theta}^2 = b\theta$
	Differentiate with respect to t			
	$10r\dot{\theta}\ddot{\theta} = 3g\dot{\theta}$	(m1)		Attempt to differentiate, dependent on first M1
	$\ddot{\theta} = \frac{3g}{10r}$	(A1)	6	
(iii)	$T_2 = mr\ddot{\theta} = \frac{3mg}{10r}$	B1F		ft 'their' equation for T_2
	$T_1 = 3mg - 3mr\ddot{\theta} = 3mg - \frac{9mg}{10}$	M1		Substituting in their equation for T_1
	$= \frac{21mg}{10}$	A1F	3	ft 'their' equation for T_1
	Total		**13**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\mathbf{r}_1 \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & 1 & 2 \\ \mathbf{j} & 0 & 0 \\ \mathbf{k} & 3 & a \end{vmatrix} = \begin{pmatrix} 0 \\ 6-a \\ 0 \end{pmatrix}$	M1 A1		Attempt at $\mathbf{r}_1 \times \mathbf{F}_1$ – one comp correct Fully correct
	$\mathbf{r}_2 \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & -1 & -2 \\ \mathbf{j} & 2 & 1 \\ \mathbf{k} & 0 & 3 \end{vmatrix} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$	M1 A1		Attempt at $\mathbf{r}_2 \times \mathbf{F}_2$ – one comp correct
	$\begin{pmatrix} 0 \\ 6-a \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 9-a \\ 3 \end{pmatrix}$	A1	5	Totalling $\mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$; AG Note $\mathbf{F} \times \mathbf{r}$ scores M1A0M1A1A0
(b)(i)	Magnitude of couple = 7 $\Rightarrow$			
	$6^2 + (9-a)^2 + 3^2 = 7^2$	M1		Forming equation – using part (a)
	$(9-a)^2 = 4$	M1		Solving – to obtain two values
	$\therefore\ 9-a = 2$ or $9-a = -2$ $a = 7$ or $a = 11$	A1,A1	4	AG (7)
	Special case (max 2): If $a = 7$, then resultant moment $= \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$			
	Magnitude $= \sqrt{6^2 + 2^2 + 3^2} = 7$	(M1) (A1)		
(ii)	$a = 7 \Rightarrow \mathbf{F}_1 + \mathbf{F}_2 = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$	M1		Attempt at $\mathbf{F}_1 + \mathbf{F}_2$
	$= \begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}$	A1	2	$\mathbf{F}_1 + \mathbf{F}_2$ correct
	Total		**11**	

MM04 (cont)

Q	Solution	Marks	Total	Comments
7(a)				
	$\rho = \dfrac{m}{2a}$	B1		ρ seen anywhere
	MI for element $= (\rho\,\delta x)x^2$	M1		Attempt at mx^2
	MI rod $= \int_0^{2a} x^2 \rho\,\delta x = \int_0^{2a} \dfrac{mx^2}{2a}\,dx$			
	$= \left[\dfrac{mx^3}{6a}\right]_0^{2a}$	A1		Correct integration
	$= \dfrac{4}{3}ma^2$	A1	4	Correct use of units; AG
(b)(i)	$I_{OP} = I_{OQ} = \dfrac{4}{3}ma^2$			
	$I_{\text{seat}} = 4m(2a)^2 = 16ma^2$	M1A1		MI for seat – M1 for mx^2 form
	$\text{MI}_{\text{model}} = \dfrac{4}{3}ma^2 + \dfrac{4}{3}ma^2 + 16ma^2$	M1		Sum of three MIs
	$= \dfrac{56ma^2}{3}$	A1	4	AG
(ii)				
				Note $a = 1.5$ can be substituted anywhere
	KE gained $= \dfrac{1}{2}I\omega^2$	M1		Use of $\dfrac{1}{2}I\omega^2$
	$= \dfrac{28}{3}ma^2\omega^2$	A1		
	PE lost $= 6mg(1.44a) - 6mg(1.44a\sin 45°)$	M1		*mgh* used
	$= 2.53mga$	A1		Correct difference
	Conservation of energy $\Rightarrow$			
	$\dfrac{28}{3}ma^2\omega^2 = 2.53mga$	M1		Forming equation for C of E
	$a = 1.5 \Rightarrow \omega_{\max} = 1.33 \text{ rad s}^{-1}$	A1	6	
	Total		**14**	
	TOTAL		**75**	