

General Certificate of Education  
January 2008  
Advanced Level Examination



**MATHEMATICS**  
**Unit Further Pure 4**

**MFP4**

Wednesday 30 January 2008 9.00 am to 10.30 am

**For this paper you must have:**

- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

---

Answer **all** questions.

---

- 1 Give a full geometrical description of the transformation represented by each of the following matrices:

(a)  $\begin{bmatrix} 0.8 & 0 & -0.6 \\ 0 & 1 & 0 \\ 0.6 & 0 & 0.8 \end{bmatrix}$ ; (3 marks)

(b)  $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ . (2 marks)

- 2 It is given that  $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ ,  $\mathbf{b} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$  and  $\mathbf{c} = \mathbf{i} + 4\mathbf{j} + 28\mathbf{k}$ .

- (a) Determine:

(i)  $\mathbf{a} \cdot \mathbf{b}$ ; (1 mark)

(ii)  $\mathbf{a} \times \mathbf{b}$ ; (2 marks)

(iii)  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ . (2 marks)

- (b) Describe the geometrical relationship between the vectors:

(i)  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{a} \times \mathbf{b}$ ; (1 mark)

(ii)  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ . (1 mark)

- 3 A shear  $S$  is represented by the matrix  $\mathbf{A} = \begin{bmatrix} p & q \\ -q & r \end{bmatrix}$ , where  $p$ ,  $q$  and  $r$  are constants.

(a) By considering one of the geometrical properties of a shear, explain why  $pr + q^2 = 1$ . (2 marks)

- (b) Given that  $p = 4$  and that the image of the point  $(-1, 2)$  under  $S$  is  $(2, -1)$ , find:

(i) the value of  $q$  and the value of  $r$ ; (3 marks)

(ii) the equation of the line of invariant points of  $S$ . (3 marks)

4 The matrix  $\mathbf{T}$  has eigenvalues 2 and  $-2$ , with corresponding eigenvectors  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$  respectively.

- (a) Given that  $\mathbf{T} = \mathbf{U}\mathbf{D}\mathbf{U}^{-1}$ , where  $\mathbf{D}$  is a diagonal matrix, write down suitable matrices  $\mathbf{U}$ ,  $\mathbf{D}$  and  $\mathbf{U}^{-1}$ . (3 marks)
- (b) Hence prove that, for all **even** positive integers  $n$ ,

$$\mathbf{T}^n = f(n) \mathbf{I}$$

where  $f(n)$  is a function of  $n$ , and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix. (5 marks)

5 A system of equations is given by

$$\begin{aligned} x + 3y + 5z &= -2 \\ 3x - 4y + 2z &= 7 \\ ax + 11y + 13z &= b \end{aligned}$$

where  $a$  and  $b$  are constants.

- (a) Find the unique solution of the system in the case when  $a = 3$  and  $b = 2$ . (5 marks)
- (b) (i) Determine the value of  $a$  for which the system does not have a unique solution. (3 marks)
- (ii) For this value of  $a$ , find the value of  $b$  such that the system of equations is consistent. (4 marks)

**Turn over for the next question**

**Turn over ►**

6 (a) The line  $l$  has equation  $\mathbf{r} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}$ .

- (i) Write down a vector equation for  $l$  in the form  $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$ . (1 mark)
- (ii) Write down cartesian equations for  $l$ . (2 marks)
- (iii) Find the direction cosines of  $l$  and explain, geometrically, what these represent. (3 marks)

(b) The plane  $\Pi$  has equation  $\mathbf{r} = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ .

- (i) Find an equation for  $\Pi$  in the form  $\mathbf{r} \cdot \mathbf{n} = d$ . (4 marks)
- (ii) State the geometrical significance of the value of  $d$  in this case. (1 mark)
- (c) Determine, to the nearest  $0.1^\circ$ , the angle between  $l$  and  $\Pi$ . (4 marks)

7 The non-singular matrix  $\mathbf{M} = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ .

- (a) (i) Show that

$$\mathbf{M}^2 + 2\mathbf{I} = k\mathbf{M}$$

for some integer  $k$  to be determined. (3 marks)

- (ii) By multiplying the equation in part (a)(i) by  $\mathbf{M}^{-1}$ , show that

$$\mathbf{M}^{-1} = a\mathbf{M} + b\mathbf{I}$$

for constants  $a$  and  $b$  to be found. (3 marks)

- (b) (i) Determine the characteristic equation of  $\mathbf{M}$  and show that  $\mathbf{M}$  has a repeated eigenvalue, 1, and another eigenvalue, 2. (6 marks)
- (ii) Give a full set of eigenvectors for each of these eigenvalues. (5 marks)
- (iii) State the geometrical significance of each set of eigenvectors in relation to the transformation with matrix  $\mathbf{M}$ . (3 marks)

**END OF QUESTIONS**