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General Certificate of Education
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Mathematics MFP4

Further Pure 4

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1(a)	$\begin{vmatrix} 3 & 4 & -1 \\ -1 & 2 & 2 \\ 1 & 4 & 1 \end{vmatrix} = 6 + 8 + 4 + 2 - 24 + 4$	M1		Good attempt at det M0 for $\lvert .. \rvert = 0$ and no working
	or $3(2-8) - 4(-1-2) - 1(-4-2)$ etc or $3(2-8) + 1(4+4) + 1(8+2)$ etc			
	Correctly shown = 0	A1		
	Or $3\mathbf{p} + 4\mathbf{q} = 5\mathbf{r}$	(M1) (A1)	2	
(b)	For attempt at 2 of (±) $\overrightarrow{PQ}$, $\overrightarrow{PR}$, $\overrightarrow{QR}$	M1		
	Area $\Delta PQR = \frac{1}{2}\left\lvert \overrightarrow{QP} \times \overrightarrow{QR} \right\rvert$ e.g. $= \frac{1}{2}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 2 & -3 \\ 2 & 0 & -2 \end{vmatrix} = \frac{1}{2}\lvert \pm(4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) \rvert$	M1		Formula used with attempt at a vector product of any 2 of the above (ignore missing $\frac{1}{2}$ for now)
	$= \frac{1}{2}\sqrt{4^2 + 2^2 + 4^2}$	M1		Method for finding magnitude of their relevant vector
	$= 3$	A1	4	CSO
	Total		**6**	
2(a)	$\mathbf{AB} = \begin{bmatrix} 2x+1 & 2x-1 \\ 8 & 4 \end{bmatrix}$	M1		Good attempt (at least one entry in R_1✓)
		A1	2	All four correct
(b)	$\mathbf{B}^\mathrm{T}\mathbf{A}^\mathrm{T} = (\mathbf{AB})^\mathrm{T}$ **Or** $\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ x & 3 \end{bmatrix}$	M1		
	$= \begin{bmatrix} 2x+1 & 8 \\ 2x-1 & 4 \end{bmatrix}$	A1√		Ft their (a) **Or** CAO
	$2x + 1 = 4 - 4x$ **Or** $2x - 1 = 8x - 4$	M1		Ft previous answers
	$x = \frac{1}{2}$	A1		CAO
	Checking/noting $x = \frac{1}{2}$ in other eqn.	B1	5	*Visibly*
	Total		**7**	
3(a)	Clearly identifying $\mathbf{n} = \begin{bmatrix} 9 \\ -8 \\ 72 \end{bmatrix}$	B1		
	$d = \begin{bmatrix} 9 \\ -8 \\ 72 \end{bmatrix} \bullet \begin{bmatrix} 2 \\ 10 \\ 1 \end{bmatrix} = 10$	M1 A1	3	
(b)	Use of $\dfrac{\text{Sc.prod. of normals}}{\text{prod. of their moduli}}$	M1		Must be $(9\mathbf{i} - 8\mathbf{j} + 72\mathbf{k})$, $(\mathbf{i} + \mathbf{j} + \mathbf{k})$ or their $\mathbf{n}$ from (a)
	$\mathrm{N^r.} = 73$ $\mathrm{D^r.} = 73\sqrt{3}$ or $\sqrt{15987}$	B1√ B1√		Ft their $\mathbf{n}$ from (a) only
	$\cos\theta = \dfrac{1}{\sqrt{3}}$	A1	4	CAO Allow unsimplified exact forms
	Total		**7**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)	(**v** =) $\pm(\mathbf{a}-\mathbf{b}) = \pm\begin{bmatrix}1\\-5\\4\end{bmatrix}$	M1 A1		M1 A0 if $\pm\overrightarrow{AB}$ found but not stated/shown this is **v**
	$\mathbf{u} = \begin{bmatrix}3\\-4\\1\end{bmatrix}$ or $\begin{bmatrix}2\\1\\-3\end{bmatrix}$	B1	3	
(b)	$\mathbf{b}\times\mathbf{c} = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 1 & -3\\ 2-t & t & 5\end{vmatrix} = \begin{bmatrix}3t+5\\3t-16\\3t-2\end{bmatrix}$	M1 A3,2,1	4	
(c) (i)	$\mathbf{a}\bullet\mathbf{b}\times\mathbf{c} = \begin{bmatrix}3\\-4\\1\end{bmatrix}\bullet\begin{bmatrix}3t+5\\3t-16\\3t-2\end{bmatrix} = 77$	M1		Or starting again: $\begin{vmatrix}3 & -4 & 1\\ 2 & 1 & -3\\ 2-t & t & 5\end{vmatrix}$
		A1	2	CAO
(ii)	*C* never lies in plane of *O*, *A*, *B* (or is a fixed distance from it) **or** Vol. //ppd. *OABC* always = 77 **or** Vol. tetrhdrn. *OABC* always = $\frac{77}{6}$ **or** *O* is never in plane of *A*, *B*, *C* **or** $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ never co-planar	B1	1	Any suitable geometrical comment Vectors ✓; points ✗
	Total		**10**	
5	$\Delta = \begin{vmatrix}x & y & z\\ x^2 & y^2 & z^2\\ yz & zx & xy\end{vmatrix}$			
	$= \begin{vmatrix}x & y-x & z-x\\ x^2 & y^2-x^2 & z^2-x^2\\ yz & z(x-y) & y(x-z)\end{vmatrix}$	M1 M1		By $C_2{}' = C_2 - C_1$ (eg) $C_3{}' = C_3 - C_1$ (eg)
	$= (y-x)(z-x)\begin{vmatrix}x & 1 & 1\\ x^2 & y+x & z+x\\ yz & -z & -y\end{vmatrix}$	A1 A1		First two factors extracted (what's left has to be correct also)
	$= (y-x)(z-x)\begin{vmatrix}x & 1 & 0\\ x^2 & y+x & z-y\\ yz & -z & z-y\end{vmatrix}$	M1		By $C_3{}' = C_3 - C_2$ (e.g.)
	$= (y-x)(z-x)(z-y)\begin{vmatrix}x & 1 & 0\\ x^2 & y+x & 1\\ yz & -z & 1\end{vmatrix}$	A1		3rd factor extracted
	$= (x-y)(y-z)(z-x)(xy+yz+zx)$	M1		Further *R*/*C* ops or expansion of remaining det (almost a dM1)
		A1	8	CAO up to equivalents due to re-positioning of the signs
	Alternatives using *Cyclic Symmetry* and the *Factor Theorem* are fine			
	Total		**8**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\bullet = \sqrt{6^2+2^2+9^2}$ attempted *and* $\pm\left(\frac{6}{\bullet}, \frac{2}{\bullet}, \frac{-9}{\bullet}\right)$	M1		
	$\bullet$ = 11 and all correct	A1	2	$\pm$(0.545, 0.182, – 0.818) ok
(ii)	**Either** $\begin{bmatrix}5\\3\\4\end{bmatrix}\times\begin{bmatrix}1\\6\\2\end{bmatrix}=-3\begin{bmatrix}6\\2\\-9\end{bmatrix}$	M1 A1		Correct vector product only here
	Explaining that d.v. of L is in dirn. of Π's nml. $\Rightarrow L \perp^r \Pi$	B1		
	Or $\begin{bmatrix}6\\2\\-9\end{bmatrix}\bullet\begin{bmatrix}5\\3\\4\end{bmatrix}=0$ and $\begin{bmatrix}6\\2\\-9\end{bmatrix}\bullet\begin{bmatrix}1\\6\\2\end{bmatrix}=0$	(M1) (A1)		
	Explaining that d.v. of L is $\perp^r$ to 2 (non-//) vectors in $\Pi \Rightarrow L \perp^r \Pi$	(B1)	3	Not just stating
(b)	E.g. 6×① – ②: $46 = 34p + 27q$	M1		Eliminating r from any pair of eqns.
	2×① – ③: $-57 = 21p + 6q$ ② – 3×③: $-217 = 29p - 9q$	A1 A1		Any 2 correct eqns (1 mark each)
	2×④ + 9×⑤: $605 = -121p$	M1		Solving a 2×2 system (any means) in order to get values for p, q, r
	$p = -5,\ q = 8,\ r = -1$	A1	5	All 3 ✓ CAO
(c)	**(i)** $\begin{aligned}7+6t &= -2+5\lambda+\mu\\ 8+2t &= 0+3\lambda+6\mu\\ 50-9t &= -25+4\lambda+2\mu\end{aligned}$			
	$\rightarrow \begin{aligned}9 &= -6t+5\lambda+\mu\\ 8 &= -2t+3\lambda+6\mu\\ 75 &= 9t+4\lambda+2\mu\end{aligned}$	M1		Including re-arrangement attempt
	i.e. the above system with $p = -t,\ q = \lambda$ and $r = \mu$	A1	2	
	(ii) Subst^g. t = 5 into L's eqn.			
	Or λ = 8 ***and*** μ = – 1 into Π's eqn.	M1		
	P = (37, 18, 5)	A1	2	CAO
	Total		**14**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	Evals $\lambda = 27$ 1	B1		Both
	Evecs $(\alpha)\begin{bmatrix}4\\1\end{bmatrix}$ and $(\beta)\begin{bmatrix}-1\\3\end{bmatrix}$	B1 B1		
		B1	4	Correctly matched up with evals (look out for λ_1, $\mathbf{v}_1$ notations)
(ii)	$y = -3x$	B1√		Ft $4y = x$ if evecs mis-matched
	from $\lambda = 1$	B1	2	**Must** say why they have chosen this one
(b)	$\mathbf{U}^{-1} = \frac{1}{13}\begin{bmatrix}3 & 1\\-1 & 4\end{bmatrix}$	B1 B1		Det; mtx
	$\mathbf{M} = \mathbf{U}\,\mathbf{D}\,\mathbf{U}^{-1}$	M1		Including attempt to multiply (at least **U D …**)
	$= \frac{1}{13}\begin{bmatrix}4 & -1\\1 & 3\end{bmatrix}\begin{bmatrix}27 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 4\end{bmatrix}$			
	$= \frac{1}{13}\begin{bmatrix}4 & -1\\1 & 3\end{bmatrix}\begin{bmatrix}81 & 27\\-1 & 4\end{bmatrix}$ **or** $\frac{1}{13}\begin{bmatrix}108 & -1\\27 & 3\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 4\end{bmatrix}$	A1		Ft incorrect/missing $\mathbf{U}^{-1}$ for one product; ignore missing $\frac{1}{13}$ until the end
	$= \begin{bmatrix}25 & 8\\6 & 3\end{bmatrix}$	A1	5	CAO
(c)	$\mathbf{M}^n = \mathbf{U}\,\mathbf{D}^n\,\mathbf{U}^{-1}$	M1		Including attempt to multiply
	$\mathbf{D}^n = \begin{bmatrix}27^n & 0\\0 & 1\end{bmatrix}$	B1		
	$\mathbf{M}^n\,(1,1) = \frac{1}{13}\left(12\times 27^n + 1\right)$	A1		
	So $4\times 3\times 3^{3n} + 1 = 4\times 3^{3n+1} + 1$ div. by 13	E1		Legitimately so from their working, from fact that the element is an integer
	Since **M** has all integer elements, each element of $\mathbf{M}^n$ is an integer also	E1	5	Explaining *why* it must be an integer
	Total		**16**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
8	det **W** = 12.36 + 9.16 = 576 = k^2	M1		Attempt at det. = k^2
	$\Rightarrow\ k = 24$	A1	2	
	$\frac{1}{24}\mathbf{W} = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}$	B1		
	$\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x + \frac{2}{3}y \\ \frac{3}{2}y - \frac{3}{8}x \end{bmatrix}$	M1 A1		
	Equating this to $\begin{bmatrix} x \\ y \end{bmatrix} =$	M1		
	$y = \frac{3}{4}x$	A1		CAO
	ALT. 1			
	$\frac{1}{24}\mathbf{W} = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}$	(B1)		
	$\begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ -\frac{3}{8} & \frac{3}{2} \end{bmatrix}\begin{bmatrix} x \\ mx \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2} + \frac{2}{3}m\right)x \\ \left(\frac{3}{2}m - \frac{3}{8}\right)x \end{bmatrix}$	(M1) (A1)		
	Setting $y' = mx'$ and solving for m	(M1)		Get $(4m-3)^2 = 0$
	$y = \frac{3}{4}x$	(A1)		CAO
	ALT. 2			
	$\lambda^2 - 2\lambda + 1 = 0$	(M1)		This may simply be stated or assumed
	$\Rightarrow\ \lambda = 1$ (twice)	(A1)		
	$\lambda = 1 \Rightarrow \begin{matrix} -\frac{1}{2}x + \frac{2}{3}y = 0 \\ -\frac{3}{8}x + \frac{1}{2}y = 0 \end{matrix}$	(M1) (A1)		
	$y = \frac{3}{4}x$	(A1)	5	
	Total		**7**	
	TOTAL		**75**	