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General Certificate of Education (A-level)
June 2011

Mathematics MFP2

(Specification 6360)

Further Pure 2

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

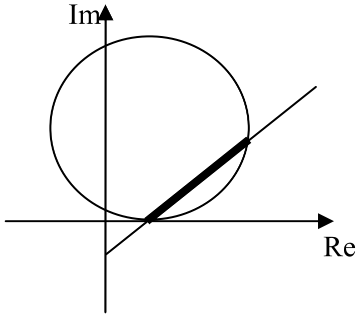
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)				Use average of whole question if 2 diagrams used
(i)	Circle correct centre touching x-axis	B1 B1 B1F	3	Circle in any position Must be shown ft incorrect centre
(ii)	half-line through (0, –2) through point of contact of circle with x-axis	B1 B1 B1	3	 Can be inferred
(b)	Inside circle On line	B1 B1F	2	ft errors in position of line and circle
	Total		**8**	
2(a)	$\frac{\left(e^x+e^{-x}\right)}{2}\frac{\left(e^y+e^{-y}\right)}{2}-\frac{\left(e^x-e^{-x}\right)}{2}\frac{\left(e^y-e^{-y}\right)}{2}$	M1A1		M0 if sinh and cosh confused M1 for formula quoted correctly
	Correct expansions	A1		Use of e^{xy} A0
	$=\frac{1}{2}\left(e^{x-y}+e^{-(x-y)}\right)=\cosh(x-y)$	A1	4	AG
(b)(i)	$\cosh(x-\ln 2)=\cosh x\cosh(\ln 2)$ $-\sinh x\sinh(\ln 2)$	M1		**Alternative:** $\frac{e^{x-\ln 2}+e^{-x+\ln 2}}{2}=\frac{e^x-e^{-x}}{2}$ M1
	$\left.\begin{array}{l}\cosh(\ln 2)=\frac{5}{4}\\ \sinh(\ln 2)=\frac{3}{4}\end{array}\right\}$ any method	B1		Both correct $e^{x-\ln 2}=\frac{e^x}{2}$ or $e^{-x+\ln 2}=2e^{-1}$ used B1
	$\frac{5}{4}\cosh x=\frac{7}{4}\sinh x$	A1F		$e^x=\sqrt{6}$ A1
	$\tanh x=\frac{5}{7}$	A1	4	AG $\tanh x=\frac{5}{7}$ A1
(ii)	$x=\frac{1}{2}\ln\left(\frac{1+\frac{5}{7}}{1-\frac{5}{7}}\right)$ or $\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{5}{7}$	M1		Could be embedded in (b)(i)
	$=\frac{1}{2}\ln 6$	A1	2	
	Total		**10**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$(r+1)! = (r+1)r(r-1)!$	M1		
	Result	A1	2	AG
(b)	**Attempt** to use method of differences	M1		
	$\sum_{r=1}^{n}(r^2+r-1)(r-1)! = (n+1)! + n! - 1! - 0!$	A1		
	$(n+1)! = (n+1)n!$	m1		Must be seen
	$(n+2)n! - 2$	A1	4	AG
	Total		**6**	
4(a)(i)	$\sum \alpha = 2$	B1		
	$\sum \alpha\beta = 0$	B1	2	
(ii)	$\sum \alpha^2 = (\sum \alpha)^2 - 2\sum \alpha\beta$	M1		Used. Watch $\sum \alpha = -2$ (M1A0)
	$= 4$	A1	2	AG
(iii)	Clear explanation	E1	1	eg α satisfies the cubic equation since it is a root. Accept $z = \alpha$
(iv)	$\sum \alpha^3 = 2\sum \alpha^2 - 3k$	M1		Or $\sum \alpha^3 = (\sum \alpha)^3 - 3\sum \alpha \sum \alpha\beta + 3\alpha\beta\gamma$
	$= 8 - 3k$	A1	2	AG
(b)(i)	$\alpha^4 = 2\alpha^3 - k\alpha$	B1		
	$\sum \alpha^4 = 2\sum \alpha^3 - k\sum \alpha$	M1		Or $\sum \alpha^4 = (\sum \alpha^2)^2 - 2(\sum \alpha\beta)^2 + 4\alpha\beta\gamma\sum \alpha$
	$= 2(8-3k) - 2k$	A1		ft on $\sum \alpha = -2$
	$k = 2$	A1	4	AG
(ii)	$\sum \alpha^5 = 2\sum \alpha^4 - k\sum \alpha^2$	M1		
	Substitution of values	A1		
	$= -8$	A1	3	
	Total		**14**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2x$	B1		Or $\frac{\mathrm{d}y}{\mathrm{d}x} = x\left(x^2+8\right)^{-\frac{1}{2}}$
	$S = 2\pi\int_0^6 y\sqrt{1+\frac{x^2}{y^2}}\ \mathrm{d}x$	M1 A1F		M1 for use of formula provided $\frac{\mathrm{d}y}{\mathrm{d}x}$ is a function of x A1 for substitution for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (one slip)
	Eliminating all y	m1		
	$= 2\sqrt{2}\pi\int_0^6\sqrt{x^2+4}\ \mathrm{d}x$	A1	5	AG
(b)	$\mathrm{d}x = 2\cosh\theta\ \mathrm{d}\theta$ or $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cosh\theta$	B1		
	$S = 2\sqrt{2}\,\pi\int\sqrt{4\sinh^2 x+4}\,.\,2\cosh\theta\ \mathrm{d}\theta$	M1		For eliminating x completely and use of $\mathrm{d}\theta$, ie $\mathrm{d}\theta$ attempted
	$S = \left(2\sqrt{2}\right)\pi\int 2\cosh\theta\,.\,2\cosh\theta\ \mathrm{d}\theta$	m1		Use of $\cosh^2\theta-\sinh^2\theta=1$ (ignore limits)
	$= 4\sqrt{2}\,\pi\int\left(\cosh2\theta+1\right)\mathrm{d}\theta$	m1		Use of formula for $\cosh 2\theta$; must be correct
	$= 4\sqrt{2}\,\pi\left[\frac{\sinh 2\theta}{2}+\theta\right]$	B1F		Correct integration of $a\cosh 2\theta+b$
	$= 4\sqrt{2}\,\pi\left[\sinh\theta\cosh\theta+\theta\right]$	m1		Use of $\sinh 2\theta = 2\sinh\theta\cosh\theta$ Must be seen
	$= 4\sqrt{2}\,\pi\left[\frac{x}{2}\sqrt{\frac{x^2}{4}+1}+\sinh^{-1}\frac{x}{2}\right]_0^6$	M1		Or change limits
	$= \pi\left[24\sqrt{5}+4\sqrt{2}\ \sinh^{-1}3\right]$	A1	8	AG
	Total		**13**	
6(a)	Expansion of $\left(k+1\right)\left(4\left(k+1\right)^2-1\right)$	M1		Any valid method – first step correct
	$= 4k^3+12k^2+11k+3$	A1	2	AG
(b)	Assume true for $n=k$ For $n=k+1$:			
	$\sum_{r=1}^{k+1}(2r-1)^2 = \frac{1}{3}k\left(4k^2-1\right)+\left(2k+1\right)^2$	M1A1		No LHS M1A0
	$= \frac{1}{3}\left(4k^3+12k^2+11k+3\right)$	A1F		ft error in $(2k+1)$
	$= \frac{1}{3}\left(k+1\right)\left(4\left(k+1\right)^2-1\right)$	A1		Using part (a)
	True for $n=1$ shown	B1		
	Proof by induction set out properly (if factorised by 3 linear factors, allow A1 at this particular point)	E1	6	Dependent on all marks correct
	Total		**8**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\cos 5\theta + \mathrm{i}\sin 5\theta = (\cos\theta + \mathrm{i}\sin\theta)^5$	M1		Attempt to expand 3 correct terms
	Expansion in any form	A1		Correct simplification
	Equate real parts:	m1		
	$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	A1		AG
	Equate imaginary parts:			
	$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$	A1	5	CAO
(ii)	$\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta}$	M1		Used
	Division by $\cos^5\theta$ or by $\cos^4\theta$	m1		
	$\tan 5\theta = \dfrac{\tan\theta(5 - 10\tan^2\theta + \tan^4\theta)}{1 - 10\tan^2\theta + 5\tan^4\theta}$	A1	3	AG
(b)	$\theta = \dfrac{\pi}{5} \Rightarrow \tan 5\theta = 0$	M1		Or for $\tan^4\theta - 10\tan^2\theta + 5 = 0$
	$\therefore \tan\dfrac{\pi}{5}$ satisfies $t^4 - 10t^2 + 5 = 0$	A1		Or for $\tan 5\theta = 0$
	Other roots $\tan\dfrac{k\pi}{5}$ $k = 2, 3, 4$	B1	3	OE
(c)	Product of roots = 5	M1		
	$\tan\dfrac{\pi}{5} = -\tan\dfrac{4\pi}{5}$	B1		Or $\tan\dfrac{2\pi}{5} = -\tan\dfrac{3\pi}{5}$
	$\tan^2\dfrac{\pi}{5}\tan^2\dfrac{2\pi}{5} = 5$	A1		
	$\tan\dfrac{\pi}{5}\tan\dfrac{2\pi}{5} = +\sqrt{5}$	A1		
	– sign rejected with reason	E1	5	**Alternative (c)** Use of quadratic formula M1 $t^2 = 5 \pm 2\sqrt{5}$ A1 $t = \pm\sqrt{5 \pm 2\sqrt{5}}$ B1 Correct selection of +ve values E1 Multiplied together to get $\sqrt{5}$ A1
	Total		**16**	
	TOTAL		**75**	