



General Certificate of Education

Mathematics 6360

MFP2 Further Pure 2

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A _{2,1}	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

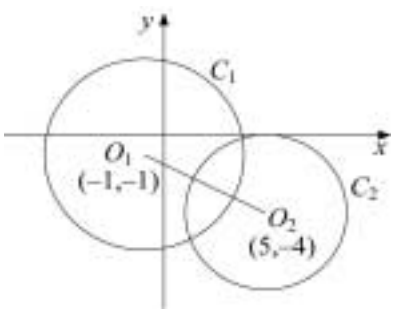
Q	Solution	Marks	Total	Comments
1(a)	$z^4 = 16e^{\frac{4\pi i}{12}}$ $= 16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ $= 8 + 8\sqrt{3}i; a = 8$	M1 A1 A1F	3	Allow M1 if $z^4 = 2e^{\frac{4\pi i}{12}}$ OE could be $2ae^{\frac{\pi i}{3}}$ or $2a\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ ft errors in 2^4
(b)	For other roots, $r = 2$ $\theta = \frac{\pi}{12} + \frac{2k\pi}{4}$ Roots are $2e^{\frac{7\pi i}{12}}, 2e^{\frac{-5\pi i}{12}}, 2e^{\frac{-11\pi i}{12}}$	B1 M1A1 A2,1, 0 F	5	for realising roots are of form $2 \times e^{i\theta}$ M1 for strictly correct θ i.e must be $\left(\text{their } \frac{\pi}{3} + 2k\pi\right) \times \frac{1}{4}$ ft error in $\frac{\pi}{12}$ or r [accept $2e^{\left(\frac{\pi}{12} + \frac{2k\pi}{4}\right)i}$ $k = -1, -2, 1$]
Total			8	
2(a)	$A = \frac{1}{2}, B = -\frac{1}{2}$	B1, B1F	2	For either A or B For the other
(b)	Method of differences clearly shown $\text{Sum} = \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$ $= \frac{n}{2n+1}$	M1 A1 A1	3	AG
(c)	$\frac{1}{2(2n+1)} < 0.001 \text{ or } \frac{n}{2n+1} > 0.499$ $1 < 0.004n + 0.002 \text{ or } n > 0.998n + 0.499$ $n > \frac{0.998}{0.004} \text{ or } 0.004n > 0.998$ $n = 250$	M1 A1 A1F	3	Condone use of equals sign OE ft if say 0.4999 used If method of trial and improvement used, award full marks for a completely correct solution showing working
Total			8	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$2 + 3i$	B1	1	
(b)(i)	$\alpha\beta = 13$	B1	1	
(ii)	$\alpha\beta + \beta\gamma + \gamma\alpha = 25$ $\gamma(\alpha + \beta) = 12$ $\gamma = 3$	M1 A1F A1F	3	M1A0 for -25 (no ft) ft error in $\alpha\beta$
(iii)	$p = -\sum \alpha = -7$ $q = -\alpha\beta\gamma = -39$	M1 A1F A1F	3	M1 for a correct method for either p or q ft from previous errors p and q must be real for sign errors in p and q allow M1 but A0
	Alternative for (b)(ii) and (iii):			
(ii)	Attempt at $(z - 2 + 3i)(z - 2 - 3i)$ $z^2 - 4z + 13$ cubic is $(z^2 - 4z + 13)(z - 3) \therefore \gamma = 3$	(M1) (A1) (A1)	(3)	
(iii)	Multiply out or pick out coefficients $p = -7, q = -39$	(M1) (A1, A1)	(3)	
	Total		8	
4(a)	Sketch, approximately correct shape Asymptotes at $y = \pm 1$	B1 B1	2	B0 if curve touches asymptotes lines of answer booklet could be used for asymptotes be strict with sketch
(b)	Use of $u = \frac{\sinh x}{\cosh x}$ $= \frac{e^x - e^{-x}}{e^x + e^{-x}}$ or $\frac{e^{2x} - 1}{e^{2x} + 1}$ $u(e^x + e^{-x}) = e^x - e^{-x}$ $e^{-x}(1+u) = e^x(1-u)$ $e^{2x} = \frac{1+u}{1-u}$ $x = \frac{1}{2} \ln \left(\frac{1+u}{1-u} \right)$	M1 A1 M1 A1 m1 A1	6	M1 for multiplying up A1 for factorizing out e's or M1 for attempt at $1+u$ and $1-u$ in terms of e^x AG

Q	Solution	Marks	Total	Comments
4(c)(i)	Use of $\tanh^2 x = 1 - \operatorname{sech}^2 x$ Printed answer	M1 A1	2	
(ii)	$(3 \tanh x - 1)(\tanh x - 2) = 0$ $\tanh x \neq 2$ $\tanh x = \frac{1}{3}$ $x = \frac{1}{2} \ln 2$	M1 E1 A1 M1 A1F	5	Attempt to factorise Accept $\tanh x \neq 2$ written down but not ignored or just crossed out ft
Total			15	
5(a)	$(\cos \theta + i \sin \theta)^{k+1} =$ $(\cos k\theta + i \sin k\theta)(\cos \theta + i \sin \theta)$ Multiply out $= \cos(k+1)\theta + i \sin(k+1)\theta$ True for $n = 1$ shown $P(k) \Rightarrow P(k+1)$ and $P(1)$ true	M1 A1 A1 B1 E1	5	Any form Clearly shown provided previous 4 marks earned
(b)	$\frac{1}{z^n} = \frac{1}{\cos n\theta + i \sin n\theta} = \cos n\theta - i \sin n\theta$ $z^n + \frac{1}{z^n} = 2 \cos n\theta$	M1A1 A1	3	or $z^{-n} = \cos(-n\theta) + i \sin(-n\theta)$ SC $(\cos \theta + i \sin \theta)^{-n}$ quoted as $\cos n\theta - i \sin n\theta$ earns M1A1 only AG
(c)	$z + \frac{1}{z} = \sqrt{2}$ $2 \cos \theta = \sqrt{2}$ $\theta = \frac{\pi}{4}$ $z^{10} + \frac{1}{z^{10}} = 2 \cos\left(\frac{10\pi}{4}\right)$ $= 0$	M1 A1 M1 A1F	4	M0 for merely writing $z^{10} + \frac{1}{z^{10}} = 2 \cos 10\theta$
Total			12	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Centre $-1-i$ or $(-1, -1)$ Radius 5 $ z+1+i =5$ or $ z-(-1-i) =5$	B1 M1 A1F A1F	4	ft incorrect centre if used ft $ z+1+i =10$ earns M0B1
(b)	 <p>C_1 correct centre, correct radius C_2 correct centre, correct radius Touching x-axis</p>	B1F B1 B1F	3	ft errors in (a) but fit circles need to intersect and C_1 enclose $(0,0)$ error in plotting centre
(c)	$O_1O_2 = 3\sqrt{5}$ Correct length identified Length is $9 + 3\sqrt{5}$	M1A1 m1 M1 A1F	5	allow if circles misplaced but O_1O_2 is still $3\sqrt{5}$ ft if r is taken as 10
	Total		12	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \left(\frac{s}{2}\right)^2}$ $= \frac{1}{2}\sqrt{4 + s^2}$	M1A1 A1	3	Allow M1 for $s = \int \sqrt{1 + \left(\frac{s}{2}\right)^2} dx$ then A1 for $\frac{dy}{dx}$ AG
(ii)	$\int \frac{ds}{\sqrt{4 + s^2}} = \int \frac{1}{2} dx$ $\sinh^{-1} \frac{s}{2} = \frac{1}{2}x + C$ $C = 0$ $s = 2 \sinh \frac{1}{2}x$	M1 A1 A1 A1	4	For separation of variables; allow without integral sign Allow if C is missing AG if C not mentioned allow $\frac{3}{4}$ SC incomplete proof of (a)(ii), differentiating $s = 2 \sinh \frac{x}{2}$ to arrive at $\frac{ds}{dx} = \frac{1}{2}\sqrt{4 + s^2}$ allow M1A1 only $\left(\frac{2}{4}\right)$
(iii)	$\frac{dy}{dx} = \sinh \frac{1}{2}x$ $y = 2 \cosh \frac{1}{2}x + C$ $C = 0$	M1 A1 A1	3	Allow if C is missing Must be shown to be zero and CAO
(b)	$y^2 = 4 \left(1 + \sinh^2 \frac{x}{2}\right)$ $= 4 + s^2$	M1 A1	2	Use of $\cosh^2 = 1 + \sinh^2$ AG
	Total		12	
	TOTAL		75	