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Mathematics **MFP2**

(Specification 6360)

Further Pure 2

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

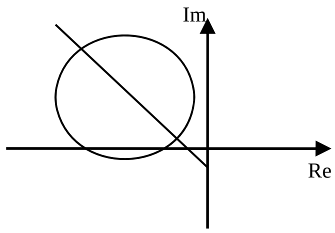
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP2

Q	Solution	Marks	Total	Comments
1(a)	$\cosh x = \frac{1}{2}(e^x + e^{-x})$			*or* $12\cosh x = 6(e^x + e^{-x})$
	or $\sinh x = \frac{1}{2}(e^x - e^{-x})$	M1		*or* $4\sinh x = 2(e^x - e^{-x})$
	$12\cosh x - 4\sinh x =$ $6(e^x + e^{-x}) - 2(e^x - e^{-x})$			
	$12\cosh x - 4\sinh x = 4e^x + 8e^{-x}$	A1 **cso**	2	**AG**
(b)	$4e^x + 8e^{-x} = 33$			
	$\Rightarrow 4e^{2x} - 33e^x + 8 \quad (=0)$	M1		attempt to multiply by e^x to form quadratic in e^x
	$\Rightarrow (e^x - 8)(4e^x - 1) \quad (=0)$	m1		factorisation attempt (see below) or correct use of formula
	$\Rightarrow \left(e^x =\right) \; 8, \; \left(e^x =\right) \; \frac{1}{4}$	A1		correct roots
	$\left(x =\right) \; 3\ln 2$	A1		
	$\left(x =\right) \; -2\ln 2$	A1	5	
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\lvert 4-4\mathrm{i}\rvert = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$	B1		verification that $\lvert -2+\mathrm{i}+6-5\mathrm{i}\rvert = 4\sqrt{2}$
	$\arg(-2+2\mathrm{i}) = \pi - \tan^{-1}(1) = \frac{3\pi}{4}$	B1	2	verification that $\arg(z+\mathrm{i}) = \frac{3\pi}{4}$
				
(b)	Circle	M1		freehand circle sketched
	Centre at $-6+5\mathrm{i}$	A1		clear from diagram or centre stated
	Cutting Re axis but not cutting Im axis	A1		
	"Straight" line	M1		freehand line
	Half line from $0-\mathrm{i}$	A1		not horizontal or vertical but end point at $0-\mathrm{i}$ must be clear from diagram/stated
	gradient –1 (approx)	A1	6	making $45°$ to negative Re axis and positive Im axis
(c)	Calculation based on fact that L_2 passes through centre of L_1	M1		idea of vector $\begin{bmatrix}-4\\4\end{bmatrix}$ from centre
	Q represents $-10+9\mathrm{i}$	A1	2	must write as a complex number
	Total		**10**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\frac{1}{5r-2}-\frac{1}{5r+3}=\frac{5r+3-(5r-2)}{(5r-2)(5r+3)}$	M1		condone omission of brackets for M1
	$=\frac{5}{(5r-2)(5r+3)}$	A1cso	2	$A = 5$
(b)	Attempt to use method of differences	M1		at least 2 terms of correct form seen
	$k\left\{\frac{1}{3}-\frac{1}{5n+3}\right\}$	A1		correct cancellation leaving correct two fractions
	$k\left\{\frac{(5n+3)-3}{3(5n+3)}\right\}$	m1		attempt to write with common denominator
	$S_n=\frac{1}{5}\left\{\frac{(5n+3)-3}{3(5n+3)}\right\}=\frac{n}{3(5n+3)}$	A1cso	4	**AG** $k=\frac{1}{5}$ used correctly throughout
(c)	$S_\infty=\frac{1}{15}$	B1	1	
	Total		**7**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\alpha+\beta+\gamma=5$	B1		
	$\alpha\beta\gamma=4$	B1	2	
(ii)	$\alpha\beta\gamma^2+\alpha\beta^2\gamma+\alpha^2\beta\gamma=\alpha\beta\gamma(\alpha+\beta+\gamma)$	M1		
	$=5\times4=20$	A1√	2	FT their results from (a)(i)
(b)(i)	If α,β,γ are all real then $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2\geqslant0$			
	Hence α,β,γ cannot all be real	E1	1	argument must be sound
(ii)	$\alpha\beta+\beta\gamma+\gamma\alpha=k$	B1		$\sum\alpha\beta=k$ PI
	$(\alpha\beta+\beta\gamma+\gamma\alpha)^2$ $=\sum\alpha^2\beta^2+2(\alpha\beta\gamma^2+\alpha\beta^2\gamma+\alpha^2\beta\gamma)$	M1		correct identity for $\left(\sum\alpha\beta\right)^2$
	$=-4+2(20)$	A1√		substituting their result from (a)(ii)
	$k=\pm6$	A1 **cso**	4	must see k=…
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x = \tanh y = \dfrac{e^y - e^{-y}}{e^y + e^{-y}}$			
	$xe^y + xe^{-y} = e^y - e^{-y}$	M1		*or* $xe^{2y} + x = e^{2y} - 1$
	$\Rightarrow (x+1)e^{-y} = e^y(1-x)$			
	$\Rightarrow (x+1) = e^{2y}(1-x)$	A1		
	$e^{2y} = \dfrac{1+x}{1-x} \Rightarrow y = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$	A1cso	3	**AG**
(b)	$y = \dfrac{1}{2}\ln(1+x) - \dfrac{1}{2}\ln(1-x)$	M1		
	$\dfrac{dy}{dx} = \dfrac{1}{2(1+x)} + \dfrac{1}{2(1-x)}$	A1		
	$= \dfrac{1-x+1+x}{2(1+x)(1-x)} = \dfrac{2}{2(1-x^2)} = \dfrac{1}{1-x^2}$	A1cso	3	**AG**
				Alternative 1 $\dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{(1-x)}{(1+x)} \times \dfrac{d}{dx}\left(\dfrac{1+x}{1-x}\right)$ M1 $\dfrac{dy}{dx} = \dfrac{1}{2} \times \dfrac{(1-x)}{(1+x)} \times \dfrac{(1-x)+(1+x)}{(1-x)^2}$ A1 $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{1-x^2}$ A1 cso
(c)	$\int 4\tanh^{-1}x\,dx = 4x\tanh^{-1}x - \int \dfrac{4x}{1-x^2}\,dx$	M1		
	$4x\tanh^{-1}x + 2\ln(1-x^2)$	A1		
	$\tanh^{-1}\tfrac{1}{2} = \tfrac{1}{2}\ln 3$	B1		must simplify logarithm to ln3
	Value of integral = $\ln 3 + 2\ln\tfrac{3}{4}$	A1		any correct form
	$\ln\left(\dfrac{3^3}{2^4}\right)$	A1cso	5	all working must be correct
	Total		**11**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{dx}{dt}=3t^2 \quad \frac{dy}{dt}=12t$	B1		both correct
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 9t^4+144t^2$	M1		'their' $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$
	$s=\int \sqrt{9t^4+144t^2}\,(dt)$	A1		OE
	$s=\int_0^3 3t\sqrt{t^2+16}\,dt$	A1cso	4	$A = 16$
(b)	$k(t^2+A)^{\frac{3}{2}}$	M1		where k is a constant; ft their A
	$(t^2+16)^{\frac{3}{2}}$	A1		
	$25^{\frac{3}{2}}-16^{\frac{3}{2}}$	m1		F(3) – F(0)
	$= 61$	A1 **cso**	4	**AG**
	Total		**8**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\text{p}(k+1)-\text{p}(k)=k^3+(k+1)^3+(k+2)^3$ $-(k-1)^3-k^3-(k+1)^3$	M1		
	$=(k+2)^3-(k-1)^3$			
	$=k^3+6k^2+12k+8-(k^3-3k^2+3k-1)$	A1		multiplied out & correct unsimplified
	$=9k^2+9k+9 \quad =9(k^2+k+1)$ which is a multiple of 9 (since k^2+k+1 is an integer)	A1cso	3	correct algebra plus statement
(ii)	p(1) = 1 + 8 = 9 $\Rightarrow$ p(1) is a multiple of 9	B1		result true for $n = 1$
	$\text{p}(k+1) = \text{p}(k) + 9(k^2+k+1)$ *or* $\text{p}(k+1) = \text{p}(k) + 9N$	M1		p(k+1) = … **and** result from part (i) considered **and** mention of divisible by 9
	Assume p(k) is a multiple of 9 so p(k) = 9M, where M is integer $\Rightarrow \text{p}(k+1) = 9M + 9N = 9(M+N)$			must have word such as "assume" for A1
	$\Rightarrow \text{p}(k+1)$ is a multiple of 9	A1		convincingly shown
	Result true for n = 1 therefore true for n = 2, n = 3 etc by induction. (*or* p(n) is a multiple of 9 for all integers $n \geqslant 1$)	E1	4	must earn previous 3 marks before E1 is scored
(b)	$\text{p}(n)=(n-1)^3+n^3+(n+1)^3$ $= 3n^3+6n$	B1		need to see this OE as evidence *or* $3n(n^2+2)$
	$\text{p}(n)=3n(n^2+2)$ & p(n) is a multiple of 9.			both of these required
	Therefore $n(n^2+2)$ is a multiple of 3 (for any positive integer n.)	E1	2	plus concluding statement
	Total		**9**	

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$r = 8$	B1		
	$\tan^{-1} \pm \frac{4\sqrt{3}}{4}$ or $\pm\frac{\pi}{3}$ seen	M1		or $\frac{\pi}{6}$ marked as angle to Im axis with "vector" in second quadrant on Arg diag
	$\Rightarrow \theta = \frac{2\pi}{3}$	A1	3	$-4 + 4\sqrt{3}\mathrm{i} = 8\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$
(b)(i)	modulus of each root = 2	B1√		
		M1		use of De Moivre – dividing argument by 3
	$\Rightarrow \theta = -\frac{4\pi}{9}, \frac{2\pi}{9}, \frac{8\pi}{9}$	A2	4	A1 if 3 "correct" values not all in requested interval
				$2\,\mathrm{e}^{-\mathrm{i}\frac{4\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{8\pi}{9}}$
(ii)	Area = $3 \times \frac{1}{2} \times PO \times OR \times \sin\frac{2\pi}{3}$	M1		Correct expression for area of triangle *PQR*
	$= 3 \times \frac{1}{2} \times 2 \times 2 \times \sin\frac{2\pi}{3}$	A1		correct values of lengths in formula
	$= 3\sqrt{3}$	A1cso	3	
(c)	Sum of roots (of cubic) = 0	E1		must be stated explicitly
	Sum of 3 roots including Im terms	M1		in form $r(\cos\theta + i\sin\theta)$
	$2\left(\cos\frac{(-)4\pi}{9} + \cos\frac{2\pi}{9} + \cos\frac{8\pi}{9}\right)$	A1		isolating real terms ; correct and with "2"
	$\mathrm{e}^{-\mathrm{i}\frac{4\pi}{9}} = \cos\frac{4\pi}{9} - \mathrm{i}\sin\frac{4\pi}{9}$ seen earlier			*or* $\cos\frac{-4\pi}{9} = \cos\frac{4\pi}{9}$ explicitly stated to earn final A1 mark
	$\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{8\pi}{9} = 0$	A1**cso**	4	**AG**
	Total		**14**	
	TOTAL		**75**	