| Centre Number | | | | | | Candidate Number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Surname | | | | | | | | | | |
| Other Names | | | | | | | | | | |
| Candidate Signature | | | | | | | | | | |

| For Examiner's Use | |
|---|---|
| Examiner's Initials | |
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| TOTAL | |

General Certificate of Education
Advanced Subsidiary Examination
January 2011

# Mathematics MFP1

## Unit Further Pure 1

**Friday 14 January 2011 1.30 pm to 3.00 pm**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

J A N 1 1 M F P 1 0 1

P38265/Jan11/MFP1 6/6/

Answer **all** questions in the spaces provided.

**1** The quadratic equation $x^2 - 6x + 18 = 0$ has roots $\alpha$ and $\beta$.

**(a)** Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

**(b)** Find a quadratic equation, with integer coefficients, which has roots $\alpha^2$ and $\beta^2$. *(4 marks)*

**(c)** Hence find the values of $\alpha^2$ and $\beta^2$. *(1 mark)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

**2 (a)** Find, in terms of $p$ and $q$, the value of the integral $\int_p^q \frac{2}{x^3}\,dx$. *(3 marks)*

**(b)** Show that only one of the following improper integrals has a finite value, and find that value:

**(i)** $\int_0^2 \frac{2}{x^3}\,dx$;

**(ii)** $\int_2^\infty \frac{2}{x^3}\,dx$. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

**3 (a)** Write down the $2 \times 2$ matrix corresponding to each of the following transformations:

**(i)** a rotation about the origin through 90° clockwise; *(1 mark)*

**(ii)** a rotation about the origin through 180°. *(1 mark)*

**(b)** The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$$

**(i)** Calculate the matrix $\mathbf{AB}$. *(2 marks)*

**(ii)** Show that $(\mathbf{A} + \mathbf{B})^2 = k\mathbf{I}$, where **I** is the identity matrix, for some integer $k$. *(3 marks)*

**(c)** Describe the single geometrical transformation, or combination of two geometrical transformations, represented by each of the following matrices:

**(i)** $\mathbf{A} + \mathbf{B}$; *(2 marks)*

**(ii)** $(\mathbf{A} + \mathbf{B})^2$; *(2 marks)*

**(iii)** $(\mathbf{A} + \mathbf{B})^4$. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

QUESTION
PART
REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

**4** Find the general solution of the equation

$$\sin\left(4x - \frac{2\pi}{3}\right) = -\frac{1}{2}$$

giving your answer in terms of $\pi$ . *(6 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

**5 (a)** It is given that $z_1 = \frac{1}{2} - \mathrm{i}$.

**(i)** Calculate the value of $z_1{}^2$, giving your answer in the form $a + b\mathrm{i}$. *(2 marks)*

**(ii)** Hence verify that $z_1$ is a root of the equation

$$z^2 + z^* + \frac{1}{4} = 0$$

*(2 marks)*

**(b)** Show that $z_2 = \frac{1}{2} + \mathrm{i}$ also satisfies the equation in part **(a)(ii)**. *(2 marks)*

**(c)** Show that the equation in part **(a)(ii)** has two equal **real** roots. *(2 marks)*

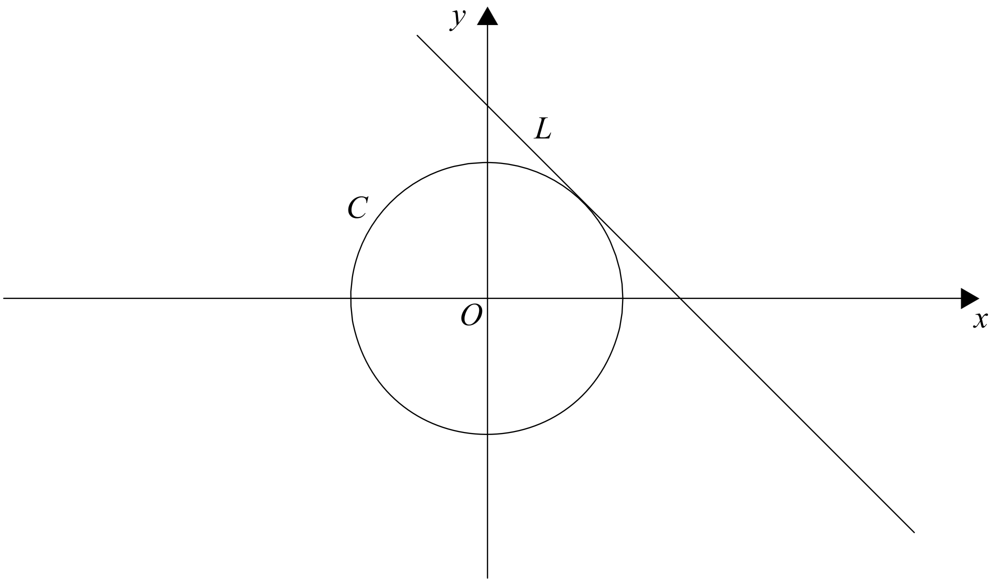
QUESTION PART REFERENCE

QUESTION PART REFERENCE

**6** The diagram shows a circle $C$ and a line $L$, which is the tangent to $C$ at the point $(1, 1)$. The equations of $C$ and $L$ are

$$x^2 + y^2 = 2 \quad \text{and} \quad x + y = 2$$

respectively.




The circle $C$ is now transformed by a stretch with scale factor 2 parallel to the $x$-axis. The image of $C$ under this stretch is an ellipse $E$.

**(a)** **On the diagram below**, sketch the ellipse $E$, indicating the coordinates of the points where it intersects the coordinate axes. *(4 marks)*

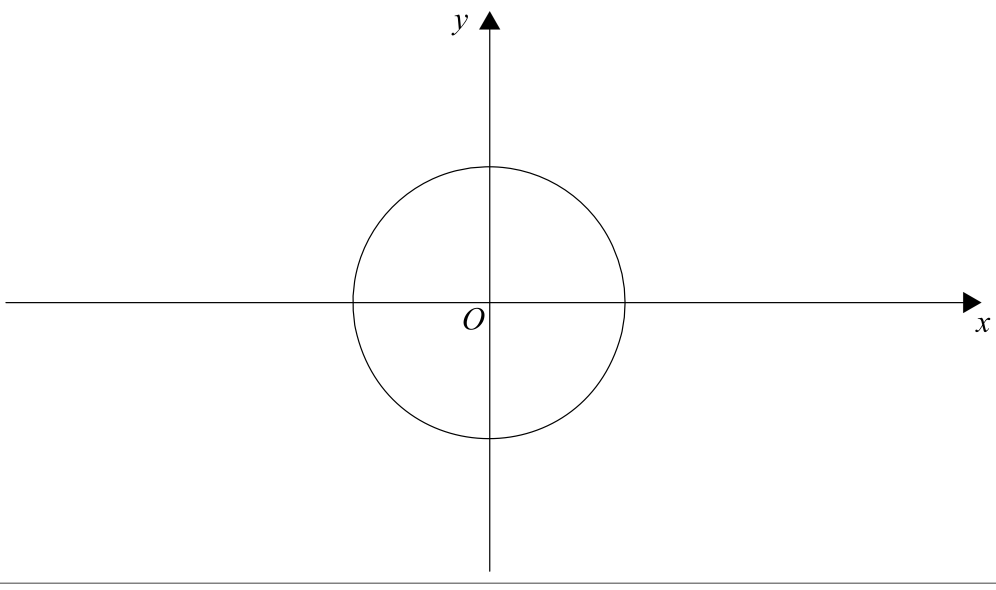
**(b)** Find equations of:

**(i)** the ellipse $E$; *(2 marks)*

**(ii)** the tangent to $E$ at the point $(2, 1)$. *(2 marks)*

QUESTION PART REFERENCE

**(a)**



QUESTION PART REFERENCE

Do not write outside the box

**7** A graph has equation

$$y = \frac{x-4}{x^2+9}$$

**(a)** Explain why the graph has no vertical asymptote and give the equation of the horizontal asymptote. *(2 marks)*

**(b)** Show that, if the line $y = k$ intersects the graph, the $x$-coordinates of the points of intersection of the line with the graph must satisfy the equation

$$kx^2 - x + (9k + 4) = 0$$ *(2 marks)*

**(c)** Show that this equation has real roots if $-\frac{1}{2} \leqslant k \leqslant \frac{1}{18}$. *(5 marks)*

**(d)** Hence find the coordinates of the two stationary points on the curve.

(No credit will be given for methods involving differentiation.) *(6 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

QUESTION PART REFERENCE

**8 (a)** The equation

$$x^3 + 2x^2 + x - 100\,000 = 0$$

has one real root. Taking $x_1 = 50$ as a first approximation to this root, use the Newton-Raphson method to find a second approximation, $x_2$, to the root. *(3 marks)*

**(b) (i)** Given that $S_n = \sum_{r=1}^{n} r(3r+1)$, use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$S_n = n(n+1)^2$$ *(5 marks)*

**(ii)** The lowest integer $n$ for which $S_n > 100\,000$ is denoted by $N$.

Show that

$$N^3 + 2N^2 + N - 100\,000 > 0$$ *(1 mark)*

**(c)** Find the value of $N$, justifying your answer. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

QUESTION PART REFERENCE

**END OF QUESTIONS**

*Do not write outside the box*

**There are no questions printed on this page**

**DO NOT WRITE ON THIS PAGE**
**ANSWER IN THE SPACES PROVIDED**