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General Certificate of Education

Mathematics 6360
2013

Material accompanying this Specification

- Specimen and Past Papers and Mark Schemes
- Reports on the Examination
- Teachers' Guide

SPECIFICATION

This specification will be published annually on the AQA Website (www.aqa.org.uk). If there are any changes to the specification centres will be notified in print as well as on the Website. The version on the Website is the definitive version of the specification.

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Background Information

1 Advanced Subsidiary and Advanced Level Specifications

1.1 Advanced Subsidiary (AS)

Advanced Subsidiary courses were introduced in September 2000 for the award of the first qualification in August 2001. They may be used in one of two ways:

- as a final qualification, allowing candidates to broaden their studies and to defer decisions about specialism;
- as the first half (50%) of an Advanced Level qualification, which must be completed before an Advanced Level award can be made.

Advanced Subsidiary is designed to provide an appropriate assessment of knowledge, understanding and skills expected of candidates who have completed the first half of a full Advanced Level qualification. The level of demand of the AS examination is that expected of candidates half-way through a full A Level course of study.

1.2 Advanced Level (AS+A2)

The Advanced Level examination is in two parts:

- Advanced Subsidiary (AS) – 50% of the total award;
- a second examination, called A2 – 50% of the total award.

Most Advanced Subsidiary and Advanced Level courses are modular. The AS comprises three teaching and learning modules and the A2 comprises a further three teaching and learning modules. Each teaching and learning module is normally assessed through an associated assessment unit. The specification gives details of the relationship between the modules and assessment units.

With the two-part design of Advanced Level courses, centres may devise an assessment schedule to meet their own and candidates' needs. For example:

- assessment units may be taken at stages throughout the course, at the end of each year or at the end of the total course;
- AS may be completed at the end of one year and A2 by the end of the second year;
- AS and A2 may be completed at the end of the same year.

Details of the availability of the assessment units for each specification are provided in Section 3.

2 Specification at a Glance

2.1 General

All assessment units are weighted at 16.7% of an A Level (33.3% of an AS). Three units are required for an AS subject award, and six for an A Level subject award. Each unit has a corresponding teaching module. The subject content of the modules is specified in Section 11 and following sections of this specification.

The unit Statistics1 is available with coursework. This unit has an equivalent unit without coursework. The same teaching module is assessed, whether the assessment unit with or without coursework is chosen. So, Module Statistics 1 (Section 20) can be assessed by either unit MS1A or unit MS1B. For the unit with coursework, the coursework contributes 25% towards the marks for the unit, and the written paper 75% of the marks.

Pure Core, Further Pure, Mechanics and Decision Mathematics units do not have coursework.

The papers for units without coursework are 1 hour 30 minutes in duration and are worth 75 marks.

The paper for MS1A (with coursework) is 1 hour 15 minutes in duration and is worth 60 marks.

For units in which calculators are allowed (ie all except MPC1) the rules (http://web.aqa.org.uk/admin/p_conduct.php) regarding what is permitted for GCE Maths and GCE Statistics are the same as for any other GCE examination.

Most models of scientific or graphical calculator are allowed. However, calculators that feature a 'Computer Algebra System' (CAS) are **not** allowed. It is usually clear from the manufacturer's specifications whether a model has this feature.

2.2 List of units for AS/A Level Mathematics

The following units can be used towards subject awards in AS Mathematics and A Level Mathematics. Allowed combinations of these units are detailed in the sections 2.3 and 2.4.

Pure Core 1	MPC1	AS
Pure Core 2	MPC2	AS
Pure Core 3	MPC3	A2
Pure Core 4	MPC4	A2
Statistics 1A	MS1A	AS with coursework
Statistics 1B	MS1B	AS without coursework
Statistics 2B	MS2B	A2
Mechanics 1B	MM1B	AS
Mechanics 2B	MM2B	A2
Decision 1	MD01	AS
Decision 2	MD02	A2

2.3 AS Mathematics

Comprises 3 AS units. Two units are compulsory.

MPC1* + MPC2 + MS1A or MS1B or MM1B or MD01

2.4 A Level Mathematics

Comprises 6 units, of which 3 or 4 are AS units. Four units are compulsory.

MPC1* + MPC2 + MS1A or MS1B or MM1B or MD01

together with

MPC3 + MPC4 + MS1A or MS1B or MM1B or MD01

or MS2B or MM2B or MD02

Notes

* – calculator not allowed

[shaded box] unit includes coursework assessment

Many combinations of AS and A2 optional Applied units are permitted for A Level Mathematics.

However, the two units chosen must assess different teaching modules. For example, units MS1B and MM1B assess different teaching modules and this is an allowed combination. However, units MS1A and MS1B both assess module Statistics 1, and therefore MS1A and MS1B is not an allowed combination.

Also a second Applied unit (MS2B, MM2B and MD02) can only be chosen in combination with a first Applied unit in the same application. For example, MS2B can be chosen with MS1A (or MS1B), but not with MM1B or MD01.

2.5 List of units for AS/A Level Pure Mathematics

The following units can be used towards subject awards in AS Pure Mathematics and A Level Pure Mathematics. Allowed combinations of these units are detailed in the sections 2.6 and 2.7.

Pure Core 1	MPC1	AS
Pure Core 2	MPC2	AS
Pure Core 3	MPC3	A2
Pure Core 4	MPC4	A2
Further Pure	MFP1	AS
Further Pure	MFP2	A2
Further Pure	MFP3	A2
Further Pure	MFP4	A2

2.6 AS Pure Mathematics

Comprises 3 compulsory AS units.

MPC1* + MPC2 + MFP1

2.7 A Level Pure Mathematics

Comprises 3 AS units and 3 A2 units. Five are compulsory.

MPC1* + MPC2 + MFP1

together with

MPC3 + MPC4 + MFP2 or MFP3 or MFP4

Notes

* – calculator not allowed

The units in AS/A Level Pure Mathematics are common with those for AS/A Level Mathematics and AS/A Level Further Mathematics. Therefore there are restrictions on combinations of subject awards that candidates are allowed to enter. Details are given in section 3.4.

2.8 AS and A Level Further Mathematics

Many combinations of units are allowed for AS and A Level Further Mathematics. Four Further Pure units are available. (Pure Core Units cannot be used towards AS/A Level Further Mathematics.) Any of the Applied units listed for AS/A Level Mathematics may be used towards AS/A Level Further Mathematics and there are additional Statistics and Mechanics units available only for Further Mathematics.

Some units which are allowed to count towards AS/A Level Further Mathematics are common with those for AS/A Level Mathematics and AS/A Level Pure Mathematics. Therefore there are restrictions on combinations of subject awards that candidates are allowed to enter. Details are given in section 3.4.

The subject award AS Further Mathematics requires three units, one of which is chosen from MFP1, MFP2, MFP3 and MFP4, and two more units chosen from the list below. All three units can be at AS standard: for example, MFP1, MM1B and MS1A could be chosen. All three units can be in Pure Mathematics: for example, MFP1, MFP2 and MFP4 could be chosen.

The subject award A Level Further Mathematics requires six units, two of which are chosen from MFP1, MFP2, MFP3 and MFP4, and four more units chosen from the list below. At least three of the six units for A Level Further Mathematics must be at A2 standard, and at least two must be in Pure Mathematics.

2.9 List of units for AS/A Level Further Mathematics

The following units can be used towards subject awards in AS Further Mathematics and A Level Further Mathematics.

Further Pure 1	MFP1	AS
Further Pure 2	MFP2	A2
Further Pure 3	MFP3	A2
Further Pure 4	MFP4	A2
Statistics 1A	MS1A	AS with coursework
Statistics 1B	MS1B	AS without coursework
Statistics 2B	MS2B	A2
Statistics 3	MS03	A2
Statistics 4	MS04	A2
Mechanics 1B	MM1B	AS
Mechanics 2B	MM2B	A2
Mechanics 3	MM03	A2
Mechanics 4	MM04	A2
Mechanics 5	MM05	A2
Decision 1	MD01	AS
Decision 2	MD02	A2

Notes

Only one unit from MS1A and MS1B can be counted towards a subject award in AS or A Level Further Mathematics.

MFP2, MFP3 and MFP4 are independent of each other, so they can be taken in any order.

MS03 and MS04 are independent of each other, so they can be taken in any order.

MM03, MM04 and MM05 are independent of each other, so they can be taken in any order.

3 Availability of Assessment Units and Entry Details

3.1 Availability of Assessment Units

Examinations based on this specification are available as follows:

	Availability of Units		Availability of Qualification	
	AS	**A2**	**AS**	**A Level**
January series	MPC1 MPC2 MS1A MS1B MM1B MD01 MFP1	MPC3 MPC4 MS2B MM2B MD02 MFP2 MFP3 MFP4	All	All
June series	All	All	All	All

3.2 Sequencing of Units

There are no restrictions on the order in which assessment units are taken. However, later teaching modules assume some or all of the knowledge, understanding and skills of earlier modules. For example, some material in MPC2 depends on material in MPC1 and some material in MPC4 depends on material in MPC3. Some of the additional units available for Further Mathematics are exceptions to this general rule (see Section 2.9). Details of the prerequisites for each module are given in the introductions to the individual modules. It is anticipated that teachers will use this and other information to decide on a teaching sequence.

3.3 Entry Codes

Normal entry requirements apply, but the following information should be noted.
The following unit entry codes should be used.

Module assessed by Unit	Standard of assessment	With or without coursework	Unit Entry Code
Pure Core 1	AS	without	MPC1
Pure Core 2	AS	without	MPC2
Pure Core 3	A2	without	MPC3
Pure Core 4	A2	without	MPC4
Further Pure 1	AS	without	MFP1
Further Pure 2	A2	without	MFP2
Further Pure 3	A2	without	MFP3
Further Pure 4	A2	without	MFP4
Statistics 1	AS	with	MS1A
Statistics 1	AS	without	MS1B
Statistics 2	A2	without	MS2B
Statistics 3	A2	without	MS03
Statistics 4	A2	without	MS04
Mechanics 1	AS	without	MM1B
Mechanics 2	A2	without	MM2B
Mechanics 3	A2	without	MM03
Mechanics 4	A2	without	MM04
Mechanics 5	A2	without	MM05
Decision 1	AS	without	MD01
Decision 2	A2	without	MD02

The **Subject Code** for entry to the Mathematics AS only award is 5361.

The **Subject Code** for entry to the Pure Mathematics AS only award is 5366.

The **Subject Code** for entry to the Further Mathematics AS only award is 5371.

The **Subject Code** for entry to the Mathematics Advanced Level award is 6361.

The **Subject Code** for entry to the Pure Mathematics Advanced Level award is 6366.

The **Subject Code** for entry to the Further Mathematics Advanced Level award is 6371.

3.4 Rules for Combinations of Awards and Unit Entries

Combinations of subject awards for this specification are subject to the following restrictions.

- Awards in the following pairs of subjects titles will not be allowed:

  AS Mathematics and AS Pure Mathematics;
  AS Mathematics and A Level Pure Mathematics;
  A Level Mathematics and AS Pure Mathematics;
  A Level Mathematics and A Level Pure Mathematics.

- Awards in the following pairs of subjects titles will not be allowed:

  AS Pure Mathematics and AS Further Mathematics;
  AS Pure Mathematics and A Level Further Mathematics;
  A Level Pure Mathematics and AS Further Mathematics;
  A Level Pure Mathematics and A Level Further Mathematics.

Units that contribute to an award in A Level Mathematics may not also be used for an award in Further Mathematics.

- Candidates who are awarded certificates in both A Level Mathematics and A Level Further Mathematics must use unit results from 12 different teaching modules.
- Candidates who are awarded certificates in both A Level Mathematics and AS Further Mathematics must use unit results from 9 different teaching modules.
- Candidates who are awarded certificates in both AS Mathematics and AS Further Mathematics must use unit results from 6 different teaching modules.

Note that AQA advises against the award of certificates in AS Further Mathematics in the first year of a two-year course, because early certification of AS Further Mathematics can make it difficult for a candidate to obtain their best grade for A Level Mathematics.

There are also restrictions on combinations of unit entries for this Specification and AQA GCE Statistics. Concurrent entries for the following pairs of units will not be accepted:

MS1A	and	SS1A
MS1A	and	SS1B
MS1B	and	SS1A
MS1B	and	SS1B

In addition, concurrent entries for:

MS1A	and	MS1B

will not be accepted.

3.5 Classification Code

Every specification is assigned to a national classification code indicating the subject area to which it belongs.

The classification codes for this specification are:

2210 Advanced Subsidiary GCE in Mathematics

2330 Advanced Subsidiary GCE in Further Mathematics

2230 Advanced Subsidiary GCE in Pure Mathematics

2210 Advanced GCE in Mathematics

2330 Advanced GCE in Further Mathematics

2230 Advanced GCE in Pure Mathematics

It should be noted that, although Pure Mathematics qualifications have a different classification code, they are discounted against the other two subjects for the purpose of the School and College Performance Tables. This means that any candidate with AS/A level Pure Mathematics plus either AS/A level Mathematics or AS/A level Further Mathematics will have only one grade (the highest) counted for the purpose of the Performance Tables. Any candidate with all three qualifications will have either the Mathematics and Further Mathematics grades or the Pure Mathematics grade only counted, whichever is the more favourable.

3.6 Private Candidates

This specification is available to private candidates.

Private candidates who have previously entered this specification can enter units with coursework (as well as units without coursework) providing they have a coursework mark which can be carried forward.

Private candidates who have not previously entered for this specification can enter units without coursework only.

Private candidates should write to AQA for a copy of *'Supplementary Guidance for Private Candidates'*.

3.7 Access Arrangements and Special Consideration

We have taken note of equality and discrimination legislation and the interests of minority groups in developing and administering this specification.

We follow the guidelines in the Joint Council for Qualifications (JCQ) document: *Access Arrangements, Reasonable and Special Consideration: General and Vocational Qualifications*. This is published on the JCQ website (**http://www.jcq.org.uk**) or you can follow the link from our website (**http://www.aqa.org.uk**).

Applications for access arrangements and special consideration should be submitted to AQA by the Examinations Officer at the centre.

3.8 Language of Examinations

All Assessment Units in this subject are provided in English only.

Scheme of Assessment

4

Introduction

AQA offers one specification in GCE Mathematics, and a separate specification in GCE Statistics.
This specification is a development from both the AQA GCE Mathematics Specification A (6300) (which was derived from the School Mathematics Project (SMP) 16–19 syllabus) and the AQA GCE Mathematics and Statistics Specification B (6320).
It includes optional assessed coursework in one Statistics unit, but coursework is not a compulsory feature.

This specification is designed to encourage candidates to study mathematics post-16. It enables a variety of teaching and learning styles, and provides opportunities for students to develop and be assessed in five of the six Key Skills.

This GCE Mathematics specification complies with:

- the Common Criteria;
- the Subject Criteria for Mathematics;
- the GCSE, GCE, Principal Learning and Project Code of Practice, April 2011;
- the GCE Advanced Subsidiary and Advanced Level Qualification-Specific Criteria.

The qualifications based on this specification are a recognised part of the National Qualifications Framework. As such, AS and A Level provide progression from Key Stage 4, through post-16 studies and form the basis of entry to higher education or employment.

Prior Level of Attainment

Mathematics is, inherently, a sequential subject. There is a progression of material through all levels at which the subject is studied. The Subject Criteria for Mathematics and therefore this specification build on the knowledge, understanding and skills established at GCSE Mathematics.

There is no specific prior requirement, for example, in terms of tier of GCSE entry or grade achieved. Teachers are best able to judge what is appropriate for different candidates and what additional support, if any, is required.

5 Aims

The aims set out below describe the educational purposes of following a course in Mathematics/Further Mathematics/Pure Mathematics and are consistent with the Subject Criteria. They apply to both AS and Advanced specifications. Most of these aims are reflected in the assessment objectives; others are not because they cannot be readily translated into measurable objectives.
The specification aims to encourage candidates to:

a. develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment;
b. develop abilities to reason logically and to recognise incorrect reasoning, to generalise and to construct mathematical proofs;
c. extend their range of mathematical skills and techniques and use them in more difficult unstructured problems;
d. develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected;
e. recognise how a situation may be represented mathematically and understand the relationship between 'real world' problems and standard and other mathematical models and how these can be refined and improved;
f. use mathematics as an effective means of communication;
g. read and comprehend mathematical arguments and articles concerning applications of mathematics;
h. acquire the skills needed to use technology such as calculators and computers effectively, to recognise when such use may be inappropriate and to be aware of limitations;
i. develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general;
j. take increasing responsibility for their own learning and the evaluation of their own mathematical development.

6 Assessment Objectives

The assessment objectives are common to both AS and A Level.
The schemes of assessment will assess candidates' ability to:

AO1 recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts;

AO2 construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form;

AO3 recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models;

AO4 comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications;

AO5 use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations; give answers to appropriate accuracy.

The use of clear, precise and appropriate mathematical language is expected as an inherent part of the assessment of AO2.

7 Scheme of Assessment
Mathematics
Advanced Subsidiary (AS)

The Scheme of Assessment has a modular structure. The Advanced Subsidiary (AS) award comprises two compulsory core units and one optional Applied unit. All assessment is at AS standard.
For the written papers, each candidate will require a copy of the AQA Booklet of formulae and statistical tables issued for this specification.

7.1 Compulsory Assessment Units

Unit MPC1 $33^1/_3\%$ of the total AS marks	Written Paper	1hour 30 minutes 75 marks

Core 1
All questions are compulsory. Calculators are **not** permitted.

Unit MPC2 $33^1/_3\%$ of the total AS marks	Written Paper	1 hour 30 minutes 75 marks

Core 2
All questions are compulsory. A graphics calculator may be used.

7.2 Optional Assessment Units

Unit MS1A $33^1/_3\%$ of the total AS marks	Written Paper + Coursework	1 hour 15 minutes 60 marks

Statistics 1A
The written paper comprises 25% of the AS marks. All questions are compulsory. A graphics calculator may be used.
The coursework comprises $8^1/_3\%$ of the AS marks. One task is required.

Unit MS1B 331/3% of the total AS marks	Written Paper	1 hour 30 minutes 75 marks

Statistics 1B
All questions are compulsory. A graphics calculator may be used.

Unit MM1B $33^1/_3\%$ of the total AS marks	Written Paper	1 hour 30 minutes 75 marks

Mechanics 1B
All questions are compulsory. A graphics calculator may be used.

Unit MD01 $33^1/_3\%$ of the total AS marks	Written Paper	1 hour 30 minutes 75 marks

Decision 1
All questions are compulsory. A graphics calculator may be used.

7.3 Weighting of Assessment Objectives for AS

The approximate relationship between the relative percentage weighting of the Assessment Objectives (AOs) and the overall Scheme of Assessment is shown in the following table:

Assessment Objectives	**Unit Weightings (range %)**			**Overall Weighting of AOs (range %)**
	MPC1	MPC2	Applied unit	
AO1	14–16	12–14	6–10	32–40
AO2	14–16	12–14	6–10	32–40
AO3	0	0	10–12	10–12
AO4	2–4	2–4	2–4	6–12
AO5	0	2–4	3–5	5–9
Overall Weighting of Units (%)	$33^1/_3$	$33^1/_3$	$33^1/_3$	100 (maximum)

Candidates' marks for each assessment unit are scaled to achieve the correct weightings.

7.4 Progression to Advanced GCE in Mathematics

Unit results counted towards an AS award in Mathematics may also be counted towards an Advanced award in Mathematics. Candidates who have completed the units needed for the AS qualification and who have taken the additional units necessary are eligible for an Advanced award.

8 Scheme of Assessment *Mathematics* *Advanced Level (AS + A2)*

The Scheme of Assessment has a modular structure. The A Level award comprises four compulsory Core units, one optional Applied unit from the AS scheme of assessment, and one optional Applied unit either from the AS scheme of assessment or from the A2 scheme of assessment. See section 2.4 on page 8 for permitted combinations. For the written papers, each candidate will require a copy of the AQA Booklet of formulae and statistical tables issued for this specification.

8.1 AS Compulsory Assessment Units

Unit MPC1 $16^2/_3$% of the total A level marks	Written Paper	1hour 30 minutes 75 marks

Core 1
All questions are compulsory. Calculators are **not** permitted.

Unit MPC2 $16^2/_3$% of the total A level marks	Written Paper	1hour 30 minutes 75 marks

Core 2
All questions are compulsory. A graphics calculator may be used.

8.2 AS Optional Assessment Units

Unit MS1A $16^2/_3$% of the total A level marks	Written Paper + Coursework	1 hour 15 minutes 60 marks

Statistics 1A
The written paper comprises 12½% of the A Level marks. All questions are compulsory. A graphics calculator may be used.
The coursework comprises $4^1/_6$% of the A Level marks. One task is required.

Unit MS1B $16^2/_3$% of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Statistics 1B
All questions are compulsory. A graphics calculator may be used.

Unit MM1B $16^2/_3$% of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Mechanics 1B
All questions are compulsory. A graphics calculator may be used.

Unit MD01 $16^{2}/_{3}\%$ of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Decision 1
All questions are compulsory. A graphics calculator may be used.

8.3 A2 Compulsory Assessment Units

Unit MPC3 $16^{2}/_{3}\%$ of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Core 3
All questions are compulsory. A graphics calculator may be used.

Unit MPC4 $16^{2}/_{3}\%$ of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Core 4
All questions are compulsory. A graphics calculator may be used.

8.4 A2 Optional Assessment Units

Unit MS2B $16^{2}/_{3}\%$ of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Statistics 2
All questions are compulsory. A graphics calculator may be used.

Unit MM2B $16^{2}/_{3}\%$ of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Mechanics 2
All questions are compulsory. A graphics calculator may be used.

Unit MD02 $16^{2}/_{3}\%$ of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Decision 2
All questions are compulsory. A graphics calculator may be used.

8.5 Synoptic Assessment

The GCE Advanced Subsidiary and Advanced Level Qualification-specific Criteria state that A Level specifications must include synoptic assessment (representing at least 20% of the total A Level marks).

Synoptic assessment in mathematics addresses candidates' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the A level course, focusing on the use and application of methods developed at earlier stages of the course to the solution of problems. Making and understanding connections in this way is intrinsic to learning mathematics.

The requirement for 20% synoptic assessment is met by synoptic assessment in: Core 2, Core 3, Core 4.

There is no restriction on when synoptic units may be taken.

8.6 Weighting of Assessment Objectives for A Level

The approximate relationship between the relative percentage weighting of the Assessment Objectives (AOs) and the overall Scheme of Assessment is shown in the following table.

A Level Assessment Units (AS + A2)

Assessment Objectives	**Unit Weightings (range %)**						**Overall Weighting of AOs (range %)**
	MPC1	MPC2	Applied unit	MPC3	MPC4	Applied unit	
AO1	7–8	6–7	3–5	6–7	6–7	3–5	32–40
AO2	7–8	6–7	3–5	6–7	6–7	3–5	32–40
AO3	0	0	5–6	0	0	5–6	10–12
AO4	1–2	1–2	1–2	1–2	1–2	1–2	6–12
AO5	0	1–2	1½–2½	1–2	1–2	1½–2½	6–11
Overall Weighting of Units (%)	$16^2/_3$	$16^2/_3$	$16^2/_3$	$16^2/_3$	$16^2/_3$	$16^2/_3$	100 (maximum)

Candidates' marks for each assessment unit are scaled to achieve the correct weightings.

9 Scheme of Assessment
Pure Mathematics
Advanced Subsidiary (AS)
Advanced Level (AS and A2)

The Pure Mathematics Advanced Subsidiary (AS) award comprises three compulsory assessment units.
The Pure Mathematics A Level (AS and A2) award comprises five compulsory assessment units, and one optional unit chosen from three Further Pure assessment units.
For the written papers, each candidate will require a copy of the AQA Booklet of formulae and statistical tables issued for this specification.

9.1 AS Assessment Units

Unit MPC1 $33^1/_3$% of the total AS marks	Written Paper	1hour 30 minutes 75 marks

Core 1
All questions are compulsory. Calculators are **not** permitted.

Unit MPC2 $33^1/_3$% of the total AS marks	Written Paper	1hour 30 minutes 75 marks

Core 2
All questions are compulsory. A graphics calculator may be used.

Unit MFP1 $33^1/_3$% of the total AS marks	Written Paper	1hour 30 minutes 75 marks

Further Pure 1
All questions are compulsory. A graphics calculator may be used.

9.2 A2 Compulsory Assessment Units

Unit MPC3 $16^2/_3$% of the total A Level marks	Written Paper	1hour 30 minutes 75 marks

Core 3
All questions are compulsory. A graphics calculator may be used.

Unit MPC4 $16^2/_3$% of the total A Level marks	Written Paper	1hour 30 minutes 75 marks

Core 4
All questions are compulsory. A graphics calculator may be used.

9.3 A2 Optional Assessment Units

Unit MFP2 Written Paper 1hour 30 minutes
$16^2/_3$% of the total A Level marks 75 marks

Further Pure 2
All questions are compulsory. A graphics calculator may be used.

Unit MFP3 Written Paper 1hour 30 minutes
$16^2/_3$% of the total A Level marks 75 marks

Further Pure 3
All questions are compulsory. A graphics calculator may be used.

Unit MFP4 Written Paper 1hour 30 minutes
$16^2/_3$% of the total A Level marks 75 marks

Further Pure 4
All questions are compulsory. A graphics calculator may be used.

9.4 Synoptic Assessment

The GCE Advanced Subsidiary and Advanced Level Qualification-specific Criteria state that A Level specifications must include synoptic assessment (representing at least 20% of the total A Level marks).

Synoptic assessment in mathematics addresses candidates' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the A level course, focusing on the use and application of methods developed at earlier stages of the course to the solution of problems. Making and understanding connections in this way is intrinsic to learning mathematics.

The requirement for 20% synoptic assessment is met by synoptic assessment in: Core 2, Core 3, Core 4.

There is no restriction on when synoptic units may be taken.

9.5 Weighting of Assessment Objectives for AS

The approximate relationship between the relative percentage weighting of the Assessment Objectives (AOs) and the overall Scheme of Assessment is shown in the following table:

Assessment Objectives	**Unit Weightings (range %)**			**Overall Weighting of AOs (range %)**
	MPC1	MPC2	MFP1	
AO1	14–16	12–14	12–14	38–44
AO2	14–16	12–14	12–14	38–44
AO3	0	0	0	0
AO4	2–4	2–4	2–4	6–12
AO5	0	2–4	2–4	4–8
Overall Weighting of Units (%)	$33^1/_3$	$33^1/_3$	$33^1/_3$	100 (maximum)

Candidates' marks for each assessment unit are scaled to achieve the correct weightings.

9.6 Weighting of Assessment Objectives for A Level

The approximate relationship between the relative percentage weighting of the Assessment Objectives (AOs) and the overall Scheme of Assessment is shown in the following table:

Assessment Objectives	**Unit Weightings (range %)**		**Overall Weighting of AOs (range %)**
	MPC1	All other units	
AO1	7–8	6–7	37–43
AO2	7–8	6–7	37–43
AO3	0	0	0
AO4	1–2	1–2	6–12
AO5	0	1–2	5–10
Overall Weighting of Units (%)	$16^2/_3$	$16^2/_3$	100 (maximum)

Candidates' marks for each assessment unit are scaled to achieve the correct weightings.

10 Scheme of Assessment

Further Mathematics
Advanced Subsidiary (AS)
Advanced Level (AS + A2)

Candidates for AS and/or A Level Further Mathematics are expected to have already obtained (or to be obtaining concurrently) an AS and/or A Level award in Mathematics.

The Advanced Subsidiary (AS) award comprises three units chosen from the full suite of units in this specification, except that the Core units cannot be included. One unit must be chosen from MFP1, MFP2, MFP3 and MFP4. All three units can be at AS standard; for example, MFP1, MM1B and MS1A could be chosen. All three units can be in Pure Mathematics; for example, MFP1, MFP2 and MFP4 could be chosen.

The Advanced (A Level) award comprises six units chosen from the full suite of units in this specification, except that the Core units cannot be included. The six units must include at least two units from MFP1, MFP2, MFP3 and MFP4. All four of these units could be chosen. At least three of the six units counted towards A Level Further Mathematics must be at A2 standard.

Details of the units which can be used towards AS/A Level Mathematics or AS/A Level Further Mathematics are given in section 8. Details of the additional units available for Further Mathematics, but not Mathematics, are given in sections 10.1 and 10.2.

Units that contribute to an award in A Level Mathematics may not also be used for an award in Further Mathematics.

- Candidates who are awarded certificates in both A Level Mathematics and A Level Further Mathematics must use unit results from 12 different teaching modules.
- Candidates who are awarded certificates in both A Level Mathematics and AS Further Mathematics must use unit results from 9 different teaching modules.
- Candidates who are awarded certificates in both AS Mathematics and AS Further Mathematics must use unit results from 6 different teaching modules.

For the written papers, each candidate will require a copy of the AQA Booklet of formulae and statistical tables issued for this specification.

10.1 Further Mathematics Assessment Units (Pure)

Unit MFP1 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1hour 30 minutes 75 marks

Further Pure 1
All questions are compulsory. A graphics calculator may be used.

Unit MFP2 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1hour 30 minutes 75 marks

Further Pure 2
All questions are compulsory. A graphics calculator may be used.

Unit MFP3 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1hour 30 minutes 75 marks

Further Pure 3
All questions are compulsory. A graphics calculator may be used.

Unit MFP4 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Further Pure 4
All questions are compulsory. A graphics calculator may be used.

10.2 Further Mathematics Assessment Units (Applied)

Unit MS03 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Statistics 3
All questions are compulsory. A graphics calculator may be used.

Unit MS04 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Statistics 4
All questions are compulsory. A graphics calculator may be used.

Unit MM03 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Mechanics 3
All questions are compulsory. A graphics calculator may be used.

Unit MM04 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Mechanics 4
All questions are compulsory. A graphics calculator may be used.

Unit MM05 $33^{1}/_{3}$% of the total AS marks $16^{2}/_{3}$% of the total A level marks	Written Paper	1 hour 30 minutes 75 marks

Mechanics 5
All questions are compulsory. A graphics calculator may be used.

10.3 Weighting of Assessment Objectives

The approximate relationship between the relative percentage weighting of the Assessment Objectives (AOs) and the overall Scheme of Assessment is shown in the following tables:

Further Mathematics AS

Assessment Objectives	Unit Weightings (range %)		Overall Weighting of AOs (range %)
	Further Pure Units	Applied Units	
AO1	12 14	6 10	24 42
AO2	12 14	6 10	24 42
AO3	0	10 12	0 36
AO4	2 4	2 4	6–12
AO5	2 4	3 5	6–14
Overall Weighting of Units (%)	$33^{1}/_{3}$	$33^{1}/_{3}$	100 (maximum)

Further Mathematics Advanced

Assessment Objectives	Unit Weightings (range %)		Overall Weighting of AOs (range %)
	Further Pure Units	Applied Units	
AO1	6 7	3 5	24 38
AO2	6 7	3 5	24 38
AO3	0	5 6	10 24
AO4	1 2	1 2	6 12
AO5	1 2	1½ 2½	7 14
Overall Weighting of Units (%)	$16^{2}/_{3}$	$16^{2}/_{3}$	100 (maximum)

Subject Content

11 Summary of Subject Content

11.1 Pure Core Modules

AS MODULE – Pure Core 1

Algebra
Coordinate Geometry
Differentiation
Integration

AS MODULE – Pure Core 2

Algebra and Functions
Sequences and Series
Trigonometry
Exponentials and logarithms
Differentiation
Integration

A2 MODULE – Pure Core 3

Algebra and Functions
Trigonometry
Exponentials and Logarithms
Differentiation
Integration
Numerical Methods

A2 MODULE – Pure Core 4

Algebra and Functions
Coordinate Geometry in the (x, y) plane
Sequences and Series
Trigonometry
Exponentials and Logarithms
Differentiation and Integration
Vectors

11.2 Further Pure Modules

AS MODULE – Further Pure 1

Algebra and Graphs
Complex Numbers
Roots and Coefficients of a quadratic equation
Series
Calculus
Numerical Methods
Trigonometry
Matrices and Transformations

A2 MODULE – Further Pure 2

Roots of Polynomials
Complex Numbers
De Moivre's Theorem
Proof by Induction
Finite Series
The Calculus of Inverse Trigonometrical Functions
Hyperbolic Functions
Arc Length and Area of surface of revolution about the x-axis

A2 MODULE – Further Pure 3

Series and Limits
Polar Coordinates
Differential Equations
Differential Equations – First Order
Differential Equations – Second Order

A2 MODULE – Further Pure 4

Vectors and Three-Dimensional Coordinate Geometry
Matrix Algebra
Solution of Linear Equations
Determinants
Linear Independence

11.3 Statistics

AS MODULE – Statistics 1

Numerical Measures
Probability
Binomial Distribution
Normal Distribution
Estimation
Correlation and Regression

A2 MODULE – Statistics 2

Discrete Random Variables
Poisson Distribution
Continuous Random Variables
Estimation
Hypothesis Testing
Chi-Square (χ^2) Contingency Table Tests

A2 MODULE – Statistics 3

Further Probability
Linear Combinations of Random Variables
Distributional Approximations
Estimation
Hypothesis Testing

A2 MODULE – Statistics 4

Geometric and Exponenential Distributions
Estimators
Estimation
Hypothesis Testing
Chi-Squared (χ^2) Goodness of Fit Tests

11.4 Mechanics

AS MODULE – Mechanics 1

Mathematical Modelling
Kinematics in One and Two Dimensions
Statics and Forces
Momentum
Newton's Laws of Motion
Connected Particles
Projectiles

A2 MODULE – Mechanics 2

Mathematical Modelling
Moments and Centres of Mass
Kinematics
Newton's Laws of Motion
Application of Differential Equations
Uniform Circular Motion
Work and Energy
Vertical Circular Motion

A2 MODULE – Mechanics 3

Relative Motion
Dimensional Analysis
Collisions in one dimension
Collisions in two dimensions
Further Projectiles
Projectiles on Inclined Planes

A2 MODULE – Mechanics 4

Moments
Frameworks
Vector Product and Moments
Centres of mass by Integration for Uniform Bodies
Moments of Inertia
Motion of a Rigid Body about a Fixed Axis

A2 MODULE – Mechanics 5

Simple Harmonic Motion
Forced and Damped Harmonic Motion
Stability
Variable Mass Problems
Motion in a Plane using Polar Coordinates

11.5 Decision

AS MODULE – Decision 1

Simple Ideas of Algorithms
Graphs and Networks
Spanning Tree Problems
Matchings
Shortest Paths in Networks
Route Inspection Problem
Travelling Salesperson Problem
Linear Programming
Mathematical Modelling

A2 MODULE – Decision 2

Critical Path Analysis
Allocation
Dynamic Programming
Network Flows
Linear Programming
Game Theory for Zero Sum Games
Mathematical Modelling

12 AS Module Core 1

Candidates will be required to demonstrate:

a. construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language;

b. correct understanding and use of mathematical language and grammar in respect of terms such as ‘equals’, ‘identically equals’, ‘therefore’, ‘because’, ‘implies’, ‘is implied by’, ‘necessary’, ‘sufficient’ and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$.

Candidates are **not** allowed to use a calculator in the assessment unit for this module.

Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Quadratic Equations $ax^2+bx+c=0$ has roots $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Circles A circle, centre (a,b) and radius r, has equation

$$(x-a)^2+(y-b)^2=r^2$$

Differentiation

function	derivative	
ax^n	anx^{n-1}	n is a whole number
$\mathrm{f}(x)+\mathrm{g}(x)$	$\mathrm{f}'(x)+\mathrm{g}'(x)$	

Integration

function	integral	
ax^n	$\dfrac{a}{n+1}x^{n+1}+c$	n is a whole number
$\mathrm{f}'(x)+\mathrm{g}'(x)$	$\mathrm{f}(x)+\mathrm{g}(x)+c$	

Area Area under a curve $=\int_a^b y\,\mathrm{d}x\,(y\geqslant 0)$

12.1 Algebra

Use and manipulation of surds.

To include simplification and rationalisation of the denominator of a fraction.

Eg $\sqrt{12}+2\sqrt{27}=8\sqrt{3}$; $\dfrac{1}{\sqrt{2}-1}=\sqrt{2}+1$; $\dfrac{2\sqrt{3}+\sqrt{2}}{3\sqrt{2}+\sqrt{3}}=\dfrac{\sqrt{6}}{3}$

Quadratic functions and their graphs.	To include reference to the vertex and line of symmetry of the graph.
The discriminant of a quadratic function.	To include the conditions for equal roots, for distinct real roots and for no real roots
Factorisation of quadratic polynomials.	Eg factorisation of $2x^2 + x - 6$
Completing the square.	Eg $x^2 + 6x - 1 = (x+3)^2 - 10$; $2x^2 - 6x + 2 = 2(x-1.5)^2 - 2.5$
Solution of quadratic equations.	Use of any of factorisation, $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or completing the square will be accepted.
Simultaneous equations, e.g. one linear and one quadratic, analytical solution by substitution.	
Solution of linear and quadratic inequalities.	Eg $2x^2 + x \geqslant 6$
Algebraic manipulation of polynomials, including expanding brackets and collecting like terms.	
Simple algebraic division.	Applied to a quadratic or a cubic polynomial divided by a linear term of the form $(x+a)$ or $(x-a)$ where a is a small whole number. Any method will be accepted, e.g. by inspection, by equating coefficients or by formal division e.g. $\dfrac{x^3 - x^2 - 5x + 2}{x+2}$.
Use of the Remainder Theorem.	Knowledge that when a quadratic or cubic polynomial $\mathrm{f}(x)$ is divided by $(x-a)$ the remainder is $\mathrm{f}(a)$ and, that when $\mathrm{f}(a) = 0$, then $(x-a)$ is a factor and vice versa.
Use of the Factor Theorem.	Greatest level of difficulty as indicated by $x^3 - 5x^2 + 7x - 3$, i.e. a cubic always with a factor $(x+a)$ or $(x-a)$ where a is a small whole number but including the cases of three distinct linear factors, repeated linear factors or a quadratic factor which cannot be factorized in the real numbers.
Graphs of functions; sketching curves defined by simple equations.	Linear, quadratic and cubic functions. The $\mathrm{f}(x)$ notation may be used but only a very general idea of the concept of a function is required. Domain and range are **not** included. Graphs of circles are included.
Geometrical interpretation of algebraic solution of equations and use of intersection points of graphs of functions to solve equations.	Interpreting the solutions of equations as the intersection points of graphs and vice versa.

Knowledge of the effect of translations on graphs and their equations.	Applied to quadratic graphs and circles, i.e. $y=(x-a)^2+b$ as a translation of $y=x^2$ and $(x-a)^2+(y-b)^2=r^2$ as a translation of $x^2+y^2=r^2$.

12.2 Coordinate Geometry

Equation of a straight line, including the forms $y-y_1=m(x-x_1)$ and $ax+by+c=0$.	To include problems using gradients, mid-points and the distance between two points. The form $y=mx+c$ is also included.
Conditions for two straight lines to be parallel or perpendicular to each other.	Knowledge that the product of the gradients of two perpendicular lines is –1.
Coordinate geometry of the circle.	Candidates will be expected to complete the square to find the centre and radius of a circle where the equation of the circle is for example given as $x^2+4x+y^2-6y-12=0$.
The equation of a circle in the form $(x-a)^2+(y-b)^2=r^2$.	The use of the following circle properties is required: (i) the angle in a semicircle is a right angle; (ii) the perpendicular from the centre to a chord bisects the chord; (iii) the tangent to a circle is perpendicular to the radius at its point of contact.
The equation of the tangent and normal at a given point to a circle.	Implicit differentiation is **not** required. Candidates will be expected to use the coordinates of the centre and a point on the circle or of other appropriate points to find relevant gradients.
The intersection of a straight line and a curve.	Using algebraic methods. Candidates will be expected to interpret the geometrical implication of equal roots, distinct real roots or no real roots. Applications will be to either circles or graphs of quadratic functions.

12.3 Differentiation

The derivative of $f(x)$ as the gradient of the tangent to the graph of $y=f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change.	The notations $f'(x)$ or $\frac{dy}{dx}$ will be used. A general appreciation only of the derivative when interpreting it is required. Differentiation from first principles will **not** be tested.
Differentiation of polynomials.	
Applications of differentiation to gradients, tangents and normals, maxima and minima and stationary points, increasing and decreasing functions.	Questions will not be set requiring the determination of or knowledge of points of inflection. Questions may be set in the form of a practical problem where a function of a single variable has to be optimised.
Second order derivatives.	Application to determining maxima and minima.

12.4	Integration Indefinite integration as the reverse of differentiation Integration of polynomials.	
	Evaluation of definite integrals. Interpretation of the definite integral as the area under a curve.	Integration to determine the area of a region between a curve and the *x*-axis. To include regions wholly below the *x*-axis, i.e. knowledge that the integral will give a negative value. Questions involving regions partially above and below the *x*-axis will not be set. Questions may involve finding the area of a region bounded by a straight line and a curve, or by two curves.

13

AS Module
Core 2

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the module Core 1.
Candidates will be required to demonstrate:

a. Construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language;

b. correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient' and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$.

Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Trigonometry

In the triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

area $= \tfrac{1}{2} ab \sin C$

arc length of a circle, $l = r\theta$

area of a sector of a circle, $A = \tfrac{1}{2} r^2 \theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Laws of Logarithms

$$\log_a x + \log_a y = \log_a (xy)$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

$$k \log_a x = \log_a \left(x^k\right)$$

Differentiation

Function	derivative	
ax^n	nax^{n-1},	n is a rational number

Integration

Function	integral	
ax^n	$\dfrac{a}{n+1}x^{n+1}$,	n is a rational number, $n \neq -1$

13.1 Algebra and Functions

Laws of indices for all rational exponents.	
Knowledge of the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	Candidates are expected to use the terms reflection, translation and stretch in the x or y direction in their descriptions of these transformations. Eg graphs of $y = \sin 2x$; $y = \cos\left(x + 30^\circ\right)$; $y = 2^{x+3}$; $y = 2^{-x}$ Descriptions involving combinations of more than one transformation will not be tested.

13.2 Sequences and Series

Sequences, including those given by a formula for the nth term.	To include Σ notation for sums of series.
Sequences generated by a simple relation of the form $x_{n+1} = \mathrm{f}(x_n)$.	To include their use in finding of a limit L as $n \to \infty$ by putting $L = \mathrm{f}(L)$.
Arithmetic series, including the formula for the sum of the first n natural numbers.	
The sum of a finite geometric series.	
The sum to infinity of a convergent $(-1 < r < 1)$ geometric series.	Candidates should be familiar with the notation $\lvert r\rvert<1$ in this context.
The binomial expansion of $(1 + x)^n$ for positive integer n.	To include the notations $n!$ and $\binom{n}{r}$. Use of Pascal's triangle or formulae to expand $(a+b)^n$ will be accepted.

13.3 Trigonometry

The sine and cosine rules.	
The area of a triangle in the form $\frac{1}{2}ab\sin C$.	
Degree and radian measure.	
Arc length, area of a sector of a circle.	Knowledge of the formulae $l = r\theta$, $A = \frac{1}{2}r^2\theta$.

	Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.	The concepts of odd and even functions are not required.
	Knowledge and use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$, and $\sin^2\theta + \cos^2\theta = 1$.	
	Solution of simple trigonometric equations in a given interval of degrees or radians.	Maximum level of difficulty as indicated by $\sin 2\theta = -0.4$, $\sin(\theta - 20^\circ) = 0.2$, $2\sin\theta - \cos\theta = 0$ and $2\sin^2\theta + 5\cos\theta = 4$.
13.4	**Exponentials and logarithms**	
	$y = a^x$ and its graph.	Using the laws of indices where appropriate.
	Logarithms and the laws of logarithms.	$\log_a x + \log_a y = \log_a(xy)$; $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$; $k\log_a x = \log_a(x^k)$. The equivalence of $y = a^x$ and $x = \log_a y$.
	The solution of equations of the form $a^x = b$.	Use of a calculator logarithm function to solve for example $3^{2x} = 2$.
13.5	**Differentiation**	
	Differentiation of x^n, where n is a rational number, and related sums and differences.	i.e. expressions such as $x^{\frac{3}{2}} + \frac{3}{x^2}$, including terms which can be expressed as a single power such as $x\sqrt{x}$. Applications to techniques included in module Core 1.
13.6	**Integration**	
	Integration of x^n, $n \neq -1$, and related sums and differences.	i.e. expressions such as $x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$ or $\frac{x+2}{\sqrt{x}} = x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$. Applications to techniques included in module Core 1.
	Approximation of the area under a curve using the trapezium rule.	The term 'ordinate' will be used. To include a graphical determination of whether the rule over- or under- estimates the area and improvement of an estimate by increasing the number of steps.

14 A2 Module
Core 3

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Core 1 and Core 2. Candidates will be required to demonstrate:

a. construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language;

b. correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient' and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$;

c. methods of proof, including proof by contradiction and disproof by counter-example.

Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Trigonometry

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Differentiation

function	derivative
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$\sin kx$	$k \cos kx$
$\cos kx$	$-k \sin kx$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Volumes

Volume of solid of revolution:

About the x-axis: $V = \int_a^b \pi y^2 \, dx$

About the y-axis: $V = \int_c^d \pi x^2 \, dy$

Integration	Function	integral
	$\cos kx$	$\frac{1}{k}\sin kx + c$
	$\sin kx$	$-\frac{1}{k}\cos kx + c$
	e^{kx}	$\frac{1}{k}e^{kx} + c$
	$\frac{1}{x}$	$\ln\lvert x\rvert + c \quad (x \neq 0)$
	$f'(g(x))g'(x)$	$f(g(x)) + c$

14.1	Algebra and Functions	
	Definition of a function. Domain and range of a function.	Notation such as $f(x) = x^2 - 4$ may be used. Domain may be expressed as $x > 1$ for example and range may be expressed as $f(x) > -3$ for example.
	Composition of functions.	$fg(x) = f(g(x))$
	Inverse functions and their graphs.	The notation f^{-1} will be used for the inverse of f. To include reflection in $y = x$.
	The modulus function.	To include related graphs and the solution from them of inequalities such as $\lvert x+2\rvert < 3\lvert x\rvert$ using solutions of $\lvert x+2\rvert = 3\lvert x\rvert$.
	Combinations of the transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x+a)$, $y = f(ax)$.	For example the transformations of: e^x leading to $e^{2x} - 1$; $\ln x$ leading to $2\ln(x-1)$; $\sec x$ leading to $3\sec 2x$. Transformations on the graphs of functions included in modules Core 1 and Core 2.
14.2	Trigonometry	
	Knowledge of $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ functions. Understanding of their domains and graphs.	Knowledge that $-\frac{\pi}{2} \leqslant \sin^{-1} x \leqslant \frac{\pi}{2}$; $0 \leqslant \cos^{-1} x \leqslant \pi$; $-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$. The graphs of these functions as reflections of the relevant parts of trigonometric graphs in $y = x$ are included. The addition formulae for inverse functions are not required.
	Knowledge of secant, cosecant and cotangent. Their relationships to cosine, sine and tangent functions. Understanding of their domains and graphs.	
	Knowledge and use of $1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \operatorname{cosec}^2 x$.	Use in simple identities. Solution of trigonometric equations in a given interval, using these identities.

14.3	**Exponentials and Logarithms**	
	The function e^x and its graph.	
	The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.	
14.4	**Differentiation**	
	Differentiation of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, and linear combinations of these functions.	
	Differentiation using the product rule, the quotient rule, the chain rule and by the use of $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$.	Eg $x^2 \ln x$; $e^{3x} \sin x$; $\frac{e^{2x}-1}{e^{2x}+1}$; $\frac{2x+1}{3x-2}$ Eg A curve has equation $x = y^2 - 4y + 1$. Find $\frac{dy}{dx}$ when $y = 1$.
14.5	**Integration**	
	Integration of e^x, $\frac{1}{x}$, $\sin x$, $\cos x$.	
	Simple cases of integration: by inspection or substitution;	Eg $\int e^{-3x} dx$; $\int \sin 4x dx$; $\int x\sqrt{1+x^2} dx$
	by substitution;	Eg $\int x(2+x)^6 dx$; $\int x\sqrt{2x-3} dx$
	and integration by parts.	Eg $\int xe^{2x} dx$; $\int x \sin 3x dx$; $\int x \ln x dx$
	These methods as the reverse processes of the chain and product rules respectively.	Including the use of $\int \frac{f'(x)}{f(x)} dx = \ln\lvert f(x) \rvert + c$ by inspection or substitution.
	Evaluation of a volume of revolution.	The axes of revolution will be restricted to the x-axis and y-axis.

14.6 Numerical Methods

Location of roots of $f(x)=0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.	
Approximate solutions of equations using simple iterative methods, including recurrence relations of the form $x_{n+1}=f(x_n)$.	Rearrangement of equations to the form $x=g(x)$. Staircase and cobweb diagrams to illustrate the iteration and their use in considerations of convergence.
Numerical integration of functions using the mid-ordinate rule and Simpson's Rule.	To include improvement of an estimate by increasing the number of steps.

15 A2 Module
Core 4

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Core 1, Core 2 and Core 3.

Candidates will be required to demonstrate:

a. construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language;

b. correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient' and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$;

c. methods of proof, including proof by contradiction and disproof by counter-example.

Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Trigonometry

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 2\cos^2 A - 1 \\ 1 - 2\sin^2 A \end{cases}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$a\cos\theta + b\sin\theta = R\sin(\theta+\alpha)\text{, where } R=\sqrt{a^2+b^2} \text{ and } \tan\alpha = \frac{a}{b}$$

$$a\cos\theta - b\sin\theta = R\cos(\theta+\alpha)\text{, where } R=\sqrt{a^2+b^2} \text{ and } \tan\alpha = \frac{b}{a}$$

Vectors

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \end{bmatrix} = xa + yb + zc = \left(\sqrt{x^2+y^2+z^2}\right)\left(\sqrt{a^2+b^2+c^2}\right)\cos\theta$$

15.1 Algebra and Functions

Including use of the Factor and Remainder Theorem for divisors of the form $(ax+b)$.

Rational functions. Simplification of rational expressions including factorising and cancelling.

Expressions of the type $\dfrac{x^2-4x}{x^2-5x+4}=\dfrac{x(x-4)}{(x-4)(x-1)}=\dfrac{x}{x-1}$

Algebraic division.

Any method will be accepted, e.g. by inspection, by equating coefficients or by formal division.

$\dfrac{3x+4}{x-1}=3+\dfrac{7}{x-1}$; $\dfrac{2x^3-3x^2-2x+2}{x-2}=2x^2+x+\dfrac{2}{x-2}$;

$\dfrac{2x^2}{(x+5)(x-3)}=2-\dfrac{4x-30}{(x+5)(x-3)}$ by using the given identity

$\dfrac{2x^2}{(x+5)(x-3)}=\mathrm{A}+\dfrac{\mathrm{B}x+\mathrm{C}}{(x+5)(x-3)}$

Partial fractions (denominators not more complicated than repeated linear terms).

Greatest level of difficulty $\dfrac{3+2x^2}{(2x+1)(x-3)^2}$

Irreducible quadratic factors will not be tested.

15.2 Coordinate Geometry in the (x, y) plane

Cartesian and parametric equations of curves and conversion between the two forms.

Eg $x=t^2,\ y=2t$; $x=a\cos\theta, y=b\sin\theta$;

$x=\dfrac{1}{t},\ y=3t$; $x=t+\dfrac{1}{t}, y=t-\dfrac{1}{t}\Rightarrow(x+y)(x-y)=4$.

15.3 Sequences and Series

Binomial series for any rational n.

Expansion of $(1+x)^n$, $|x|<1$.

Greatest level of difficulty $(2+3x)^{-2}=\dfrac{1}{4}\left(1+\dfrac{3x}{2}\right)^{-2}$, expansion valid for $|x|<\dfrac{2}{3}$

Series expansion of rational functions including the use of partial fractions

Greatest level of difficulty $\dfrac{3+2x^2}{(2x+1)(x-3)^2}$.

15.4	**Trigonometry**	
	Use of formulae for $\sin(A \pm B), \cos(A \pm B)$ and $\tan(A \pm B)$	Use in simple identities.
	and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$.	Solution of trigonometric equations in a given interval Eg $2\sin x + 3\cos x = 1.5$, $-180^\circ < x \le 180^\circ$
	Knowledge and use of double angle formulae.	Knowledge that $\sin 2x = 2\sin x\cos x$ $\cos 2x = \cos^2 x - \sin^2 x$ $= 2\cos^2 x - 1$ $= 1 - 2\sin^2 x$ $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ is expected. Use in simple identities. For example, $\sin 3x = \sin(2x + x) = \sin x(3 - 4\sin^2 x)$ Solution of trigonometric equations in a given interval. For example, solve $3\sin 2x = \cos x$, $0 \le x \le 4\pi$. Use in integration. For example $\int \cos^2 x\,\mathrm{d}x$
15.5	**Exponentials and Logarithms**	
	Exponential growth and decay.	The use of exponential functions as models.
15.6	**Differentiation and Integration**	
	Formation of simple differential equations.	To include the context of growth and decay.
	Analytical solution of simple first order differential equations with separable variables.	To include applications to practical problems.
	Differentiation of simple functions defined implicitly or parametrically.	The second derivative of curves defined implicitly or parametrically is not required.
	Equations of tangents and normals for curves specified implicitly or in parametric form.	
	Simple cases of integration using partial fractions.	Greatest level of difficulty $\int \dfrac{(1-4x)}{(3x-4)(x+3)^2}\,\mathrm{d}x$; $\int \dfrac{x^2}{(x+5)(x-3)}\,\mathrm{d}x$.

15.7 Vectors

Vectors in two and three dimensions.	Column vectors will be used in questions but candidates may use **i, j, k** notation if they wish.
Magnitude of a vector.	
Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.	The result $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ Parallel vectors
Position vectors.	
The distance between two points.	
Vector equations of lines.	Equations of lines in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. Eg $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + t\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ To include the intersection of two straight lines in two and three dimensions. Parallel lines. Skew lines in three dimensions.
The scalar product. Its use for calculating the angle between two lines.	To include finding the coordinates of the foot of the perpendicular from a point to a line and hence the perpendicular distance from a point to a line.

16 AS Module *Further Pure 1*

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Core 1 and Core 2. Candidates will also be expected to know for section 16.6 that the roots of an equation f(x) = 0 can be located by considering changes of sign of f(x) in an interval of x in which f(x) is continuous.

Candidates may use relevant formulae included in the formulae booklet without proof.

16.1 Algebra and Graphs

Graphs of rational functions of the form $\frac{ax+b}{cx+d}$, $\frac{ax+b}{cx^2+dx+e}$ or $\frac{x^2+ax+b}{x^2+cx+d}$.	Sketching the graphs. Finding the equations of the asymptotes which will always be parallel to the coordinate axes. Finding points of intersection with the coordinate axes or other straight lines. Solving associated inequalities. Using quadratic theory (not calculus) to find the possible values of the function and the coordinates of the maximum or minimum points on the graph. Eg for $y=\frac{x^2+2}{x^2-4x}$, $y=k \Rightarrow x^2+2=kx^2-4kx$, which has real roots if $16k^2+8k-8 \geq -0$, i.e. if $k \leqslant -1$ or $k \geqslant \frac{1}{2}$; stationary points are $(1, -1)$ and $(-2, \frac{1}{2})$.
Graphs of parabolas, ellipses and hyperbolas with equations $y^2=4ax$, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $xy=c^2$.	Sketching the graphs. Finding points of intersection with the coordinate axes or other straight lines. Candidates will be expected to interpret the geometrical implication of equal roots, distinct real roots or no real roots. Knowledge of the effects on these equations of single transformations of these graphs involving translations, stretches parallel to the x-axis or y-axis, and reflections in the line $y=x$. Including the use of the equations of the asymptotes of the hyperbolas given in the formulae booklet..

16.2	**Complex Numbers**	
	Non-real roots of quadratic equations.	Complex conjugates – awareness that non-real roots of quadratic equations with real coefficients occur in conjugate pairs.
	Sum, difference and product of complex numbers in the form $x+\mathrm{i}\,y$.	
	Comparing real and imaginary parts.	Including solving equations eg $2z+z^{*}=1+\mathrm{i}$ where z^{*} is the conjugate of z.
16.3	**Roots and coefficients of a quadratic equation**	
	Manipulating expressions involving $\alpha+\beta$ and $\alpha\beta$.	Eg $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ Forming an equation with roots α^3,β^3 or $\frac{1}{\alpha},\frac{1}{\beta}$, $\alpha+\frac{2}{\beta},\beta+\frac{2}{\alpha}$ etc.
16.4	**Series**	
	Use of formulae for the sum of the squares and the sum of the cubes of the natural numbers.	Eg to find a polynomial expression for $\sum_{r=1}^{n} r^2(r+2)$ or $\sum_{r=1}^{n}\left(r^2-r+1\right)$.
16.5	**Calculus**	
	Finding the gradient of the tangent to a curve at a point, by taking the limit as h tends to zero of the gradient of a chord joining two points whose x-coordinates differ by h.	The equation will be given as $y=\mathrm{f}(x)$, where $\mathrm{f}(x)$ is a simple polynomial such as x^2-2x or x^4+3.
	Evaluation of simple improper integrals.	E.g. $\int_1^4 \frac{1}{\sqrt{x}}\,\mathrm{d}x$, $\int_4^{\infty} x^{-\frac{3}{2}}\,\mathrm{d}x$.
16.6	**Numerical Methods**	
	Finding roots of equations by interval bisection, linear interpolation and the Newton-Raphson method.	Graphical illustration of these methods.
	Solving differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{f}(x)$	Using a step-by-step method based on the linear approximations $y_{n+1}\approx y_n+h\,\mathrm{f}(x_n)$; $x_{n+1}=x_n+h$, with given values for x_0, y_0 and h.
	Reducing a relation to a linear law.	E.g. $\frac{1}{x}+\frac{1}{y}=k$; $y^2=ax^3+b$; $y=a\,x^n$; $y=ab^x$ Use of logarithms to base 10 where appropriate. Given numerical values of (x,y), drawing a linear graph and using it to estimate the values of the unknown constants.

16.7	**Trigonometry** General solutions of trigonometric equations including use of exact values for the sine, cosine and tangent of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$.	Eg $\sin 2x = \frac{\sqrt{3}}{2}$, $\cos\left(x+\frac{\pi}{6}\right) = -\frac{1}{\sqrt{2}}$, $\tan\left(\frac{\pi}{3}-2x\right)=1$, $\sin 2x = 0.3$, $\cos(3x-1) = -0.2$
16.8	**Matrices and Transformations** 2 × 2 and 2 × 1 matrices; addition and subtraction, multiplication by a scalar. Multiplying a 2 × 2 matrix by a 2 × 2 matrix or by a 2 × 1 matrix. The identity matrix **I** for a 2 × 2 matrix.	
	Transformations of points in the $x-y$ plane represented by 2 × 2 matrices.	Transformations will be restricted to rotations about the origin, reflections in a line through the origin, stretches parallel to the x-axis and y-axis, and enlargements with centre the origin. Use of the standard transformation matrices given in the formulae booklet. Combinations of these transformations e.g. $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

17 A2 Module *Further Pure 2*

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Core 1, Core 2, Core 3, Core 4 and Further Pure 1.

Candidates may use relevant formulae included in the formulae booklet without proof except where proof is required in this module and requested in a question.

17.1 Roots of Polynomials

The relations between the roots and the coefficients of a polynomial equation; the occurrence of the non-real roots in conjugate pairs when the coefficients of the polynomial are real.	

17.2 Complex Numbers

The Cartesian and polar co-ordinate forms of a complex number, its modulus, argument and conjugate.	$x+\mathrm{i}y$ and $r(\cos\theta+\mathrm{i}\sin\theta)$.
The sum, difference, product and quotient of two complex numbers.	The parts of this topic also included in module Further Pure 1 will be examined only in the context of the content of this module.
The representation of a complex number by a point on an Argand diagram; geometrical illustrations.	
Simple loci in the complex plane.	For example, $\lvert z-2-\mathrm{i}\rvert \leqslant 5$, $\arg(z-2)=\frac{\pi}{3}$ Maximum level of difficulty $\lvert z-a\rvert=\lvert z-b\rvert$ where a and b are complex numbers.

17.3 De Moivre's Theorem

De Moivre's theorem for integral n.	Use of $z+\frac{1}{z}=2\cos\theta$ and $z-\frac{1}{z}=2\,\mathrm{i}\sin\theta$, leading to, for example, expressing $\sin^5\theta$ in terms of multiple angles and $\tan 5\theta$ in term of powers of $\tan\theta$. Applications in evaluating integrals, for example, $\int\sin^5\theta\,\mathrm{d}\theta$.

	De Moivre's theorem; the *n*th roots of unity, the exponential form of a complex number.	The use, without justification, of the identity $e^{ix} = \cos x + i \sin x$.
	Solutions of equations of the form $z^n = a + ib$.	To include geometric interpretation and use, for example, in expressing $\cos \frac{5\pi}{12}$ in surd form.
17.4	**Proof by Induction** Applications to sequences and series, and other problems.	Eg proving that $7^n + 4^n + 1$ is divisible by 6, or $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ where *n* is a positive integer.
17.5	**Finite Series** Summation of a finite series by any method such as induction, partial fractions or differencing.	Eg $\sum_{r=1}^{n} r.r! = \sum_{r=1}^{n} [(r+1)! - r!]$
17.6	**The calculus of inverse trigonometrical functions**	Use of the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ as given in the formulae booklet. To include the use of the standard integrals $\int \frac{1}{a^2 + x^2} dx$; $\int \frac{1}{\sqrt{a^2 - x^2}} dx$ given in the formulae booklet.
17.7	**Hyperbolic Functions** Hyperbolic and inverse hyperbolic functions and their derivatives; applications to integration.	The proofs mentioned below require expressing hyperbolic functions in terms of exponential functions. To include solution of equations of the form $a \sinh x + b \cosh x = c$. Use of basic definitions in proving simple identities. Maximum level of difficulty: $\sinh(x+y) \equiv \sinh x \cosh y + \cosh x \sinh y$. The logarithmic forms of the inverse functions, given in the formulae booklet, may be required. Proofs of these results may also be required. Proofs of the results of differentiation of the hyperbolic functions, given in the formula booklet, are included. Knowledge, proof and use of: $\cosh^2 x - \sinh^2 x = 1$ $1 - \tanh^2 x = \text{sech}^2 x$ $\coth^2 x - 1 = \text{cosech}^2 x$ Familiarity with the graphs of $\sinh x$, $\cosh x$, $\tanh x$, $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$.

17.8 Arc length and Area of surface of revolution about the x–axis

Calculation of the arc length of a curve and the area of a surface of revolution using Cartesian or parametric coordinates.

Use of the following formulae will be expected:

$$s = \int_{x_1}^{x_2} \left[1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{\frac{1}{2}} \mathrm{d}x \quad = \int_{t_1}^{t_2} \left[\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right]^{\frac{1}{2}} \mathrm{d}t$$

$$S = 2\pi \int_{x_1}^{x_2} y \left[1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{\frac{1}{2}} \mathrm{d}x = 2\pi \int_{t_1}^{t_2} y \left[\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right]^{\frac{1}{2}} \mathrm{d}t$$

18 A2 Module *Further Pure 3*

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Core 1, Core 2, Core 3, Core 4 and Further Pure 1.

Candidates may use relevant formulae included in the formulae booklet without proof.

18.1 Series and Limits

Maclaurin series

Expansions of e^x, $\ln(1+x)$, $\cos x$ and $\sin x$, and $(1+x)^n$ for rational values of n.

Use of the range of values of x for which these expansions are valid, as given in the formulae booklet, is expected to determine the range of values for which expansions of related functions are valid;

e.g. $\ln\left(\frac{1+x}{1-x}\right)$; $(1-2x)^{\frac{1}{2}}e^x$.

Knowledge and use, for $k>0$, of $\lim x^k e^{-x}$ as x tends to infinity and $\lim x^k \ln x$ as x tends to zero.

Improper integrals.

E.g. $\int_0^e x \ln x \, dx$, $\int_0^\infty x e^{-x} \, dx$.

Candidates will be expected to show the limiting processes used.

Use of series expansion to find limits.

E.g. $\lim\limits_{x\to 0}\frac{e^x-1}{x}$; $\lim\limits_{x\to 0}\frac{\sin 3x}{x}$; $\lim\limits_{x\to 0}\frac{x^2 e^x}{\cos 2x - 1}$; $\lim\limits_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}$

18.2 Polar Coordinates

Relationship between polar and Cartesian coordinates.

The convention $r>0$ will be used. The sketching of curves given by equations of the form $r=f(\theta)$ may be required. Knowledge of the formula $\tan\phi = r\frac{d\theta}{dr}$ is not required.

Use of the formula $\text{area} = \int_\alpha^\beta \frac{1}{2}r^2 d\theta$.

18.3 Differential Equations

The concept of a differential equation and its order.

The relationship of order to the number of arbitrary constants in the general solution will be expected.

Boundary values and initial conditions, general solutions and particular solutions.

18.4	**Differential Equations – First Order**	
	Analytical solution of first order linear differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}y = \mathrm{Q}$ where P and Q are functions of x.	To include use of an integrating factor and solution by complementary function and particular integral.
	Numerical methods for the solution of differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$.	
	Euler's formula and extensions to second order methods for this first order differential equation.	Formulae to be used will be stated explicitly in questions, but candidates should be familiar with standard notation such as used in Euler's formula $y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$, the formula $y_{r+1} = y_{r-1} + 2h\,\mathrm{f}(x_r, y_r)$, and the formula $y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$ where $k_1 = h\,\mathrm{f}(x_r, y_r)$ and $k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1)$.
18.5	**Differential Equations – Second Order**	
	Solution of differential equations of the form $a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\frac{\mathrm{d}y}{\mathrm{d}x} + cy = 0$, where a, b and c are integers, by using an auxiliary equation whose roots may be real or complex.	Including repeated roots.
	Solution of equations of the form $a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\frac{\mathrm{d}y}{\mathrm{d}x} + cy = \mathrm{f}(x)$ where a, b and c are integers by finding the complementary function and a particular integral	Finding particular integrals will be restricted to cases where $\mathrm{f}(x)$ is of the form e^{kx}, $\cos kx$, $\sin kx$ or a polynomial of degree at most 4, or a linear combination of any of the above.

Solution of differential equations of the form:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}+P\frac{\mathrm{d}y}{\mathrm{d}x}+Qy=R$$

where P, Q, and R are functions of x. A substitution will always be given which reduces the differential equation to a form which can be directly solved using the other analytical methods in 18.4 and 18.5 of this specification or by separating variables.

Level of difficulty as indicated by:

(a) Given $x^2\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}-2y=x$ use the substitution $x=\mathrm{e}^t$

to show that $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}-\frac{\mathrm{d}y}{\mathrm{d}t}-2y=\mathrm{e}^t$.

Hence find y in terms of t
Hence find y in terms of x

(b) $(1-x^2)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}-2x\frac{\mathrm{d}y}{\mathrm{d}x}=0$ use the substitution $u=\frac{\mathrm{d}y}{\mathrm{d}x}$

to show that $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{2xu}{1-x^2}$

and hence that $u=\frac{A}{1-x^2}$, where A is an arbitrary constant.

Hence find y in terms of x.

19 A2 Module *Further Pure 4*

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Core 1, Core 2, Core 3, Core 4 and Further Pure 1.

Candidates may use relevant formulae included in the formulae booklet without proof.

19.1 Vectors and Three–Dimensional Coordinate Geometry

Definition and properties of the vector product. Calculation of vector products.	Including the use of vector products in the calculation of the area of a triangle or parallelogram.
Calculation of scalar triple products.	Including the use of the scalar triple product in the calculation of the volume of a parallelepiped and in identifying coplanar vectors. Proof of the distributive law and knowledge of particular formulae is not required.
Applications of vectors to two- and three-dimensional geometry, involving points, lines and planes.	Including the equation of a line in the form $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=0$. Vector equation of a plane in the form $\mathbf{r.\,n} = d$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. Intersection of a line and a plane. Angle between a line and a plane and between two planes.
Cartesian coordinate geometry of lines and planes. Direction ratios and direction cosines.	To include finding the equation of the line of intersection of two non-parallel planes. Including the use of $l^2+m^2+n^2=1$ where l, m, n are the direction cosines. Knowledge of formulae other than those in the formulae booklet will not be expected.

19.2 Matrix Algebra

Matrix algebra of up to 3 x 3 matrices, including the inverse of a 2 x 2 or 3 x 3 matrix. The identity matrix **I** for 2 x 2 and 3 x 3 matrices.	Including non-square matrices and use of the results $(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$ and $(\mathbf{AB})^{\mathrm{T}}=\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$. Singular and non-singular matrices.
Matrix transformations in two dimensions: shears.	Candidates will be expected to recognise the matrix for a shear parallel to the x or y axis. Where the line of invariant points is not the x or y axis candidates will be informed that the matrix represents a shear. The combination of a shear with a matrix transformation from MFP1 is included.

	Rotations, reflections and enlargements in three dimensions, and combinations of these.	Rotations about the coordinate axes only. Reflections in the planes $x=0,\ y=0,\ z=0,\ x=y,\ x=z,\ y=z$ only.
	Invariant points and invariant lines.	
	Eigenvalues and eigenvectors of 2×2 and 3×3 matrices.	Characteristic equations. Real eigenvalues only. Repeated eigenvalues may be included.
	Diagonalisation of 2×2 and 3×3 matrices.	$\mathbf{M}=\mathbf{UDU}^{-1}$ where $\mathbf{D}$ is a diagonal matrix featuring the eigenvalues and $\mathbf{U}$ is a matrix whose columns are the eigenvectors. Use of the result $\mathbf{M}^n=\mathbf{UD}^n\mathbf{U}^{-1}$
19.3	Solution of Linear Equations	
	Consideration of up to three linear equations in up to three unknowns. Their geometrical interpretation and solution.	Any method of solution is acceptable.
19.4	Determinants	
	Second order and third order determinants, and their manipulation.	Including the use of the result $\det(\mathbf{AB})=\det\mathbf{A}\det\mathbf{B}$, but a general treatment of products is not required.
	Factorisation of determinants.	Using row and/or column operations or other suitable methods.
	Calculation of area and volume scale factors for transformation representing enlargements in two and three dimensions.	
19.5	Linear Independence	
	Linear independence and dependence of vectors.	

20 AS Module Statistics 1

Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formula, which is **not** included in the formulae booklet, but which may be required to answer questions.

$$(\text{residual})_i = y_i - a - bx_i$$

20.1 Numerical Measures

Standard deviation and variance calculated on ungrouped and grouped data.	Where raw data are given, candidates will be expected to be able to obtain standard deviation and mean values directly from calculators. Where summarised data are given, candidates may be required to use the formula from the booklet provided for the examination. It is advisable for candidates to know whether to divide by n or $(n-1)$ when calculating the variance; either divisor will be accepted unless a question specifically requests an unbiased estimate of a population variance.
Linear scaling.	Artificial questions requiring linear scaling will not be set, but candidates should be aware of the effect of linear scaling on numerical measures.
Choice of numerical measures.	Candidates will be expected to be able to choose numerical measures, including mean, median, mode, range and interquartile range, appropriate to given contexts. Linear interpolation will not be required.

20.2 Probability

Elementary probability; the concept of a random event and its probability.	Assigning probabilities to events using relative frequencies or equally likely outcomes. Candidates will be expected to understand set notation but its use will not be essential.
Addition law of probability. Mutually exclusive events.	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$; two events only. $P(A \cup B) = P(A) + P(B)$; two or more events. $P(A') = 1 - P(A)$.
Multiplication law of probability and conditional probability.	$P(A \cap B) = P(A) \times P(B \mid A) = P(B) \times P(A \mid B)$; two or more events.
Independent events.	$P(A \cap B) = P(A) \times P(B)$; two or more events.
Application of probability laws.	Only simple problems will be set that can be solved by direct application of the probability laws, by counting equally likely outcomes and/or the construction and the use of frequency tables or relative frequency (probability) tables. Questions requiring the use of tree diagrams or Venn diagrams will not be set, but their use will be permitted.

20.3	**Binomial Distribution**	
	Discrete random variables.	Only an understanding of the concepts; not examined beyond binomial distributions.
	Conditions for application of a binomial distribution.	
	Calculation of probabilities using formula.	Use of $\binom{n}{x}$ notation.
	Use of tables.	
	Mean, variance and standard deviation of a binomial distribution.	Knowledge, but not derivations, will be required.
20.4	**Normal Distribution**	
	Continuous random variables.	Only an understanding of the concepts; not examined beyond normal distributions.
	Properties of normal distributions.	Shape, symmetry and area properties. Knowledge that approximately $\frac{2}{3}$ of observations lie within $\mu \pm \sigma$, and equivalent results.
	Calculation of probabilities.	Transformation to the standardised normal distribution and use of the supplied tables. Interpolation will not be essential; rounding $z-$values to two decimal places will be accepted.
	Mean, variance and standard deviation of a normal distribution.	To include finding unknown mean and/or standard deviation by making use of the table of percentage points. (Candidates may be required to solve two simultaneous equations.)
20.5	**Estimation**	
	Population and sample.	To include the terms 'parameter' and 'statistic'. Candidates will be expected to understand the concept of a simple random sample. Methods for obtaining simple random samples will not be tested directly in the written examination.
	Unbiased estimators of a population mean and variance.	$\bar{X}$ and S^2 respectively.
	The sampling distribution of the mean of a random sample from a normal distribution.	To include the standard error of the sample mean, $\frac{\sigma}{\sqrt{n}}$, and its estimator, $\frac{S}{\sqrt{n}}$.
	A normal distribution as an approximation to the sampling distribution of the mean of a large sample from any distribution.	Knowledge and application of the Central Limit Theorem.

Confidence intervals for the mean of a normal distribution with known variance.	Only confidence intervals symmetrical about the mean will be required.
Confidence intervals for the mean of a distribution using a normal approximation.	Large samples only. Known and unknown variance.
Inferences from confidence intervals.	Based on whether a calculated confidence interval includes or does not include a 'hypothesised' mean value.

20.6 Correlation and Regression

Calculation and interpretation of the product moment correlation coefficient.	Where raw data are given, candidates should be encouraged to obtain correlation coefficient values directly from calculators. Where summarised data are given, candidates may be required to use a formula from the booklet provided for the examination. Calculations from grouped data are excluded. Importance of checking for approximate linear relationship but no hypothesis tests. Understanding that association does not necessarily imply cause and effect.
Identification of response (dependent) and explanatory (independent) variables in regression.	
Calculation of least squares regression lines with one explanatory variable. Scatter diagrams and drawing a regression line thereon.	Where raw data are given, candidates should be encouraged to obtain gradient and intercept values directly from calculators. Where summarised data are given, candidates may be required to use formulae from the booklet provided for the examination. Practical interpretation of values for the gradient and intercept. Use of line for prediction within range of observed values of explanatory variable. Appreciation of the dangers of extrapolation.
Calculation of residuals.	Use of $(\text{residual})_i = y_i - a - bx_i$. Examination of residuals to check plausibility of model and to identify outliers. Appreciation of the possible large influence of outliers on the fitted line.
Linear scaling.	Artificial questions requiring linear scaling will not be set, but candidates should be aware of the effect of linear scaling in correlation and regression.

21 A2 Module *Statistics 2*

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the module Statistics 1 and Core 1.

Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ and $\mathrm{Var}(aX) = a^2\mathrm{Var}(X)$

$\mathrm{P}(a < X < b) = \int_a^b \mathrm{f}(x)\,\mathrm{d}x$

P(Type I error) = P(reject H_0 | H_0 true) and
P(Type II error) = P(accept H_0 | H_0 false)

$E_{ij} = \dfrac{R_i \times C_j}{T}$ and $\nu =$ (rows – 1)(columns – 1)

Yates' correction (for 2×2 table) is $\chi^2 = \sum \dfrac{\left(|O_i - E_i| - 0.5\right)^2}{E_i}$

21.1 Discrete Random Variables

Discrete random variables and their associated probability distributions.	The number of possible outcomes will be finite. Distributions will be given or easily determined in the form of a table or simple function.
Mean, variance and standard deviation.	Knowledge of the formulae $\mathrm{E}(X) = \sum x_i p_i$, $\mathrm{E}(\mathrm{g}(X)) = \sum \mathrm{g}(x_i)p_i$, $\mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$, $\mathrm{E}(aX + b) = a\,\mathrm{E}(X) + b$ and $\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$ will be expected.
Mean, variance and standard deviation of a simple function of a discrete random variable.	Eg $\mathrm{E}(2X + 3)$, $\mathrm{E}(5X^2)$, $\mathrm{E}(10X^{-1})$, $\mathrm{E}(100X^{-2})$ Eg $\mathrm{Var}(3X)$, $\mathrm{Var}(4X - 5)$, $\mathrm{Var}(6X^{-1})$.

21.2 Poisson Distribution

Conditions for application of a Poisson distribution.	
Calculation of probabilities using formula.	To include calculation of values of $\mathrm{e}^{-\lambda}$ from a calculator.

	Use of Tables.	
	Mean, variance and standard deviation of a Poisson distribution.	Knowledge, but not derivations, will be required.
	Distribution of sum of independent Poisson distributions.	Result, not proof.
21.3	**Continuous Random Variables**	
	Differences from discrete random variables.	
	Probability density functions, cumulative distribution functions and their relationship.	$\mathrm{F}(x)=\int_{-\infty}^{x}\mathrm{f}(t)\mathrm{d}t$ and $\mathrm{f}(x)=\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{F}(x)$. Polynomial integration only.
	The probability of an observation lying in a specified interval.	$\mathrm{P}(a<X<b)=\int_{a}^{b}\mathrm{f}(x)\mathrm{d}x$ and $\mathrm{P}(X=a)=0$.
	Median, quartiles and percentiles.	
	Mean, variance and standard deviation.	Knowledge of the formulae $\mathrm{E}(X)=\int x\,\mathrm{f}(x)\mathrm{d}x$, $\mathrm{E}(\mathrm{g}(X))=\int \mathrm{g}(x)\mathrm{f}(x)\mathrm{d}x$, $\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$, $\mathrm{E}(aX+b)=aE(X)+b$ and $\mathrm{Var}(aX+b)=a^2\mathrm{Var}(X)$ will be expected.
	Mean, variance and standard deviation of a simple function of a continuous random variable.	E.g. $\mathrm{E}(2X+3)$, $\mathrm{E}(5X^2)$, $\mathrm{E}(10X^{-1})$, $\mathrm{E}(100X^{-2})$. E.g. $\mathrm{Var}(3X)$, $\mathrm{Var}(4X-5)$, $\mathrm{Var}(6X^{-1})$.
	Rectangular distribution.	Calculation of probabilities, proofs of mean, variance and standard deviation.
21.4	**Estimation**	
	Confidence intervals for the mean of a normal distribution with unknown variance.	Using a t distribution. Only confidence intervals symmetrical about the mean will be required. Questions may involve a knowledge of confidence intervals from the module Statistics 1.

21.5 Hypothesis Testing

Null and alternative hypotheses.	The null hypothesis to be of the form that a parameter takes a specified value.
One tailed and two tailed tests, significance level, critical value, critical region, acceptance region, test statistic, Type I and Type II errors.	The concepts of Type I errors (reject $H_0 \mid H_0$ true) and Type II errors (accept $H_0 \mid H_0$ false) should be understood but questions which require the calculation of the risk of a Type II error will not be set. The significance level to be used in a hypothesis test will usually be given.
Tests for the mean of a normal distribution with known variance.	Using a z-statistic.
Tests for the mean of a normal distribution with unknown variance.	Using a t-statistic.
Tests for the mean of a distribution using a normal approximation.	Large samples only. Known and unknown variance.

21.6 Chi-Squared (χ^2) Contingency Table Tests

Introduction to χ^2 distribution.	To include use of the supplied tables.
Use of $\sum \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2-statistic.	
Conditions for approximation to be valid.	The convention that all E_i should be greater than 5 will be expected.
Test for independence in contingency tables.	Use of Yates' correction for 2×2 tables will be required.

22

A2 Module
Statistics 3

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Statistics 1 and 2 and Core 1 and 2.

Candidate may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

For X_i independently distributed (μ_i, σ_i^2), then $\sum a_i X_i$ is distributed $\left(\sum a_i \mu_i, \sum a_i^2 \sigma_i^2\right)$

Power = 1 – P(Type II error)

22.1	Further Probability	
	Bayes' Theorem.	Knowledge and application to at most three events. The construction and use of tree diagrams
22.2	Linear Combinations of Random Variables	
	Mean, variance and standard deviation of a linear combination of two (discrete or continuous) random variables.	To include covariance and correlation. Implications of independence. Applications, rather than proofs, will be required.
	Mean, variance and standard deviation of a linear combination of independent (discrete or continuous) random variables.	Use of these, rather than proofs, will be required.
	Linear combinations of independent normal random variables.	Use of these only.

22.3 Distributional Approximations

Mean, variance and standard deviation of binomial and Poisson distributions.	Proofs using E(X) and E(X(X – 1)) together with $\sum p_i = 1$.
A Poisson distribution as an approximation to a binomial distribution.	Conditions for use.
A normal distribution as an approximation to a binomial distribution.	Conditions for use. Knowledge and use of continuity corrections.
A normal distribution as an approximation to a Poisson distribution.	Conditions for use. Knowledge and use of continuity corrections.

22.4 Estimation

Estimation of sample sizes necessary to achieve confidence intervals of a required width with a given level of confidence.	Questions may be set based on a knowledge of confidence intervals from the module Statistics 1.
Confidence intervals for the difference between the means of two independent normal distributions with known variances.	Symmetric intervals only. Using a normal distribution.
Confidence intervals for the difference between the means of two independent distributions using normal approximations.	Large samples only. Known and unknown variances.
The mean, variance and standard deviation of a sample proportion.	
Unbiased estimator of a population proportion.	$\hat{P}$
A normal distribution as an approximation to the sampling distribution of a sample proportion based on a large sample.	$\mathrm{N}\left(p, \dfrac{p(1-p)}{n}\right)$

	Approximate confidence intervals for a population proportion and for the mean of a Poisson distribution.	Using normal approximations. The use of a continuity correction will not be required in these cases.
	Approximate confidence intervals for the difference between two population proportions and for the difference between the means of two Poisson distributions.	Using normal approximations. The use of continuity corrections will not be required in these cases.
22.5	Hypothesis Testing	
	The notion of the power of a test.	Candidates may be asked to calculate the probability of a Type II error or the power for a simple alternative hypothesis of a specific test, but they will not be asked to derive a power function. Questions may be set which require the calculation of a z-statistic using knowledge from the module Statistics 1. The significance level to be used in a hypothesis test will usually be given.
	Tests for the difference between the means of two independent normal distributions with known variances.	Using a *z*-statistic.
	Tests for the difference between the means of two independent distributions using normal approximations.	Large samples only. Known and unknown variances.
	Tests for a population proportion and for the mean of a Poisson distribution.	Using exact probabilities or, when appropriate, normal approximations where a continuity correction will not be required.
	Tests for the difference between two population proportions and for the difference between the means of two Poisson distributions.	Using normal approximations where continuity corrections will not be required. In cases where the null hypothesis is testing an equality, a pooling of variances will be expected.
	Use of the supplied tables to test $H_0 : \rho = 0$ for a bivariate normal population.	Where ρ denotes the population product moment correlation coefficient.

23 A2 Module *Statistics 4*

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Statistics 1 and Statistics 2 and Core 1, Core 2 and Core 3.

Those candidates who have not studied the module Statistics 3 will also require knowledge of the mean, variance and standard deviation of a difference between two independent normal random variables.

Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

For an exponential distribution, $\mathrm{F}(x) = 1 - \mathrm{e}^{-\lambda x}$

Efficiency of *Estimator A* relative to *Estimator B* =

$$\frac{1/\mathrm{Var}(\textit{Estimator } A)}{1/\mathrm{Var}(\textit{Estimator } B)}$$

23.1 Geometric and Exponential Distributions

Conditions for application of a geometric distribution.	
Calculation of probabilities for a geometric distribution using formula.	
Mean, variance and standard deviation of a geometric distribution.	Knowledge and derivations will be expected.
Conditions for application of an exponential distribution.	Knowledge that lengths of intervals between Poisson events have an exponential distribution.
Calculation of probabilities for an exponential distribution.	Using cumulative distribution function or integration of probability density function.
Mean, variance and standard deviation of an exponential distribution.	Knowledge and derivations will be expected.

23.2	**Estimators**	
	Review of the concepts of a sample statistic and its sampling distribution, and of a population parameter.	
	Estimators and estimates.	
	Properties of estimators.	Unbiasedness, consistency, relative efficiency. Mean and variance of pooled estimators of means and proportions. Proof that $\mathrm{E}\left(S^2\right)=\sigma^2$.
23.3	**Estimation**	
	Confidence intervals for the difference between the means of two normal distributions with unknown variances.	Independent and paired samples. For independent samples, only when the population variances may be assumed equal so that a pooled estimate of variance may be calculated. Small samples only. Using a t distribution.
	Confidence intervals for a normal population variance (or standard deviation) based on a random sample.	Using a χ^2 distribution.
	Confidence intervals for the ratio of two normal population variances (or standard deviations) based on independent random samples.	Introduction to F distribution. To include use of the supplied tables. Using an F distribution.
23.4	**Hypothesis Testing**	
		The significance level to be used in a hypothesis test will usually be given.
	Tests for the difference between the means of two normal distributions with unknown variances.	Independent and paired samples. For independent samples, only when the population variances may be assumed equal so that a pooled estimate of variance may be calculated. Small samples only. Using a t-statistic.
	Tests for a normal population variance (or standard deviation) based on a random sample.	Using a χ^2-statistic.
	Tests for the ratio of two normal population variances (or standard deviations) based on independent random samples.	Using an F-statistic.

23.5	**Chi-Squared (χ^2) Goodness of Fit Tests**	
	Use of $\sum \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2-statistic.	
	Conditions for approximation to be valid.	The convention that all E_i should be greater than 5 will be expected.
	Goodness of fit tests.	Discrete probabilities based on either a discrete or a continuous distribution. Questions may be set based on a knowledge of discrete or continuous random variables from the module Statistics 2. Integration may be required for continuous random variables.

24 AS Module Mechanics 1

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the modules Core 1 and Core 2.
Candidates may use relevant formulae included in the formulae booklet without proof.
Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Constant Acceleration Formulae	$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
	$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$
	$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
	$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
	$v^2 = u^2 + 2as$	
Weight	$W = mg$	
Momentum	Momentum $= mv$	
Newton's Second Law	$F = ma$ or	Force = rate of change of momentum
Friction, dynamic	$F = \mu R$	
Friction, static	$F \leqslant \mu R$	

24.1 Mathematical Modelling

Use of assumptions in simplifying reality. Mathematical analysis of models.	Candidates are expected to use mathematical models to solve problems. Modelling will include the appreciation that: it is appropriate at times to treat relatively large moving bodies as point masses; the friction law $F \leqslant \mu R$ is experimental.
Interpretation and validity of models.	Candidates should be able to comment on the modelling assumptions made when using terms such as particle, light, inextensible string, smooth surface and motion under gravity.
Refinement and extension of models.	

24.2	Kinematics in One and Two Dimensions	
	Displacement, speed, velocity, acceleration.	Understanding the difference between displacement and distance.
	Sketching and interpreting kinematics graphs.	Use of gradients and area under graphs to solve problems.
	Use of constant acceleration equations.	$s = ut + \frac{1}{2}at^2$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ $s = vt - \frac{1}{2}at^2$ $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $v = u + at$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ $s = \frac{1}{2}(u + v)t$ $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$ $v^2 = u^2 + 2as$
	Vertical motion under gravity.	
	Average speed and average velocity.	
	Application of vectors in two dimensions to represent position, velocity or acceleration.	Resolving quantities into two perpendicular components.
	Use of unit vectors **i** and **j**.	Candidates may work with column vectors.
	Magnitude and direction of quantities represented by a vector.	
	Finding position, velocity, speed and acceleration of a particle moving in two dimensions with constant acceleration.	The solution of problems such as when a particle is at a specified position or velocity, or finding position, velocity or acceleration at a specified time. Use of constant acceleration equations in vector form, for example, $\mathbf{v} = \mathbf{u} + \mathbf{a}t$.
	Problems involving resultant velocities.	To include solutions using either vectors or vector triangles.
24.3	Statics and Forces	
	Drawing force diagrams, identifying forces present and clearly labelling diagrams.	When drawing diagrams, candidates should distinguish clearly between forces and other quantities such as velocity.
	Force of gravity (Newton's Universal Law not required).	The acceleration due to gravity, g, will be taken as 9.8 m s^{-2}.
	Friction, limiting friction, coefficient of friction and the relationship of $F \leqslant \mu R$	Candidates should be able to derive and work with inequalities from the relationship $F \leqslant \mu R$.

	Normal reaction forces.	
	Tensions in strings and rods, thrusts in rods.	
	Modelling forces as vectors.	Candidates will be required to resolve forces only in two dimensions.
	Finding the resultant of a number of forces acting at a point Finding the resultant force acting on a particle.	Candidates will be expected to express the resultant using components of a vector and to find the magnitude and direction of the resultant.
	Knowledge that the resultant force is zero if a body is in equilibrium	Find unknown forces on bodies that are at rest.
24.4	Momentum	
	Concept of momentum	Momentum as a vector in one or two dimensions. (Resolving velocities is not required.) Momentum $= mv$
	The principle of conservation of momentum applied to two particles.	Knowledge of Newton's law of restitution is not required.
24.5	Newton's Laws of Motion.	
	Newton's three laws of motion.	Problems may be set in one or two dimensions
	Simple applications of the above to the linear motion of a particle of constant mass.	Including a particle moving up or down an inclined plane.
	Use of $F = \mu R$ as a model for dynamic friction.	
24.6	Connected Particles	
	Connected particle problems.	To include the motion of two particles connected by a light inextensible string passing over a smooth fixed peg or a smooth light pulley, when the forces on each particle are constant. Also includes other connected particle problems, such as a car and trailer.
24.7	Projectiles	
	Motion of a particle under gravity in two dimensions.	Candidates will be expected to state and use equations of the form $x = V \cos \alpha t$ and $y = V \sin \alpha t - \frac{1}{2} g t^2$. Candidates should be aware of any assumptions they are making.
	Calculate range, time of flight and maximum height.	Formulae for the range, time of flight and maximum height should not be quoted in examinations. Inclined plane and problems involving resistance will not be set. The use of the identity $\sin 2\theta = 2 \sin\theta \cos\theta$ will not be required. Candidates may be expected to find initial speeds or angles of projection.
	Modification of equations to take account of the height of release.	

25 A2 Module *Mechanics 2*

A knowledge of the topics and associated formulae from Modules Core 1 – Core 4, and Mechanics 1 is required.
Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Centres of Mass	$\bar{X}\sum m_i = \sum m_i x_i$ and $\bar{Y}\sum m_i = \sum m_i y_i$
Circular Motion	$v = r\omega$, $a = r\omega^2$ and $a = \frac{v^2}{r}$
Work and Energy	Work done, constant force: Work $= Fd\cos\theta$ Work done, variable force in direction of motion in a straight line: Work $= \int \mathrm{F}\,\mathrm{d}x$ Gravitational Potential Energy $= mgh$ Kinetic Energy $= \frac{1}{2}mv^2$ Elastic potential energy $= \frac{\lambda}{2l}e^2$
Hooke's Law	$T = \frac{\lambda}{l}e$

25.1 Mathematical Modelling

The application of mathematical modelling to situations that relate to the topics covered in this module.

25.2 Moments and Centres of Mass

Finding the moment of a force about a given point.	Knowledge that when a rigid body is in equilibrium, the resultant force and the resultant moment are both zero.
Determining the forces acting on a rigid body when in equilibrium.	This will include situations where all the forces are parallel, as on a horizontal beam or where the forces act in two dimensions, as on a ladder leaning against a wall.

	Centres of Mass.	Integration methods are not required.
	Finding centres of mass by symmetry (e.g. for circle, rectangle).	
	Finding the centre of mass of a system of particles.	Centre of mass of a system of particles is given by $(\bar{X}, \bar{Y})$ where $\bar{X}\sum m_i = \sum m_i x_i$ and $\bar{Y}\sum m_i = \sum m_i y_i$
	Finding the centre of mass of a composite body.	
	Finding the position of a body when suspended from a given point and in equilibrium.	
25.3	Kinematics	
	Relationship between position, velocity and acceleration in one, two or three dimensions, involving variable acceleration.	Application of calculus techniques will be required to solve problems.
	Finding position, velocity and acceleration vectors, by the differentiation or integration of $\mathrm{f}(t)\mathbf{i}+\mathrm{g}(t)\mathbf{j}+\mathrm{h}(t)\mathbf{k}$, with respect to *t*.	If $\mathbf{r}=\mathrm{f}(t)\mathbf{i}+\mathrm{g}(t)\mathbf{j}+\mathrm{h}(t)\mathbf{k}$ then $\mathbf{v}=\mathrm{f}'(t)\mathbf{i}+\mathrm{g}'(t)\mathbf{j}+\mathrm{h}'(t)\mathbf{k}$ and $\mathbf{a}=\mathrm{f}''(t)\mathbf{i}+\mathrm{g}''(t)\mathbf{j}+\mathrm{h}''(t)\mathbf{k}$ Vectors may be expressed in the form a**i**+b**j** +c**k** or as column vectors. Candidates may use either notation.
25.4	Newton's Laws of Motion	
	Application of Newton's laws to situations, with variable acceleration.	Problems will be posed in one, two or three dimensions and may require the use of integration or differentiation.
25.5	Application of Differential Equations	
	One-dimensional problems where simple differential equations are formed as a result of the application of Newton's second law.	Use of $\frac{\mathrm{d}v}{\mathrm{d}t}$ for acceleration, to form simple differential equations, for example, $m\frac{\mathrm{d}v}{\mathrm{d}t}=-\frac{k}{\sqrt{v}}$ or $m\frac{\mathrm{d}v}{\mathrm{d}t}=k(v-2)$. Use of $a=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{\mathrm{d}v}{\mathrm{d}t}$; $v=\frac{\mathrm{d}x}{\mathrm{d}t}$. The use of $\frac{\mathrm{d}v}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}x}$ is not required. Problems will require the use of the method of separation of variables.

25.6	**Uniform Circular Motion**	
	Motion of a particle in a circle with constant speed.	Problems will involve either horizontal circles or situations, such as a satellite describing a circular orbit, where the gravitational force is towards the centre of the circle.
	Knowledge and use of the relationships $v = r\omega$, $a = r\omega^2$ and $a = \frac{v^2}{r}$.	
	Angular speed in radians s^{-1} converted from other units such as revolutions per minute or time for one revolution.	Use of the term angular speed.
	Position, velocity and acceleration vectors in relation to circular motion in terms of $\mathbf{i}$ and $\mathbf{j}$.	Candidates may be required to show that motion is circular by showing that the body is at a constant distance from a given point
	Conical pendulum.	
25.7	**Work and Energy**	
	Work done by a constant force.	Forces may or may not act in the direction of motion. Work done $= Fd\cos\theta$
	Gravitational potential energy.	Universal law of gravitation will not be required. Gravitational Potential Energy $= mgh$
	Kinetic energy.	Kinetic Energy $= \frac{1}{2}mv^2$
	The work-energy principle.	Use of Work Done = Change in Kinetic Energy.
	Conservation of mechanical energy.	Solution of problems using conservation of energy. One-dimensional problems only for variable forces.
	Work done by a variable force.	Use of $\int F \, dx$ will only be used for elastic strings and springs.
	Hooke's law.	$T = \frac{\lambda}{l}e$.
	Elastic potential energy for strings and springs.	Candidates will be expected to quote the formula for elastic potential energy unless explicitly asked to derive it.
	Power, as the rate at which a force does work, and the relationship $P = Fv$.	
25.8	**Vertical Circular Motion**	
	Circular motion in a vertical plane.	Includes conditions to complete vertical circles.

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A2 Module
Mechanics 3

A knowledge of the topics and associated formulae from Modules Mechanics 1, Core 1 and Core 2 is required. A knowledge of the trigonometric identity $\sec^2 x = 1 + \tan^2 x$ is also required. Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Momentum and Collision	$I = mv - mu$	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$
	$I = Ft$	$\mathbf{I} = \mathbf{F}t$
	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$	
	$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$	
	$v = eu$	
	$v_1 - v_2 = -e(u_1 - u_2)$	

26.1 Relative Motion

Relative velocity.	Velocities may be expressed in the form $a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$ or as column vectors.
Use of relative velocity and initial conditions to find relative displacement.	
Interception and closest approach.	Use of calculus or completing the square. Geometric approaches may be required.

26.2 Dimensional Analysis

Finding dimensions of quantities. Prediction of formulae. Checks on working, using dimensional consistency.	Finding the dimensions of quantities in terms of M, L and T. Using this method to predict the indices in proposed formulae, for example, for the period of a simple pendulum. Use dimensional analysis to find units, and as a check on working.

26.3	Collisions in one dimension	
	Momentum.	
	Impulse as change of momentum.	Knowledge and use of the equation $I = mv - mu$.
	Impulse as Force × Time.	$I = Ft$
	Impulse as $\int F\,dt$	Applied to explosions as well as collisions.
	Conservation of momentum.	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
	Newton's Experimental Law.	$v = eu$
	Coefficient of restitution.	$v_1 - v_2 = -e(u_1 - u_2)$
26.4	Collisions in two dimensions	
	Momentum as a vector.	
	Impulse as a vector.	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ and $\mathbf{I}=\mathbf{F}t$ will be required.
	Conservation of momentum in two dimensions.	$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$
	Coefficient of restitution and Newton's experimental law.	
	Impacts with a fixed surface.	The impact may be at any angle to the surface. Candidates may be asked to find the impulse on the body. Questions that require the use of trigonometric identities will not be set.
	Oblique Collisions	Collisions between two smooth spheres. Candidates will be expected to consider components of velocities parallel and perpendicular to the line of centres.
26.5	Further Projectiles	
	Elimination of time from equations to derive the equation of the trajectory of a projectile	Candidates will **not** be required to know the formula $y = x\tan\alpha - \frac{gx^2}{2v^2}(1+\tan^2\alpha)$, but should be able to derive it when needed. The identity $1+\tan^2\theta = \sec^2\theta$ will be required.
26.6	Projectiles on Inclined Planes	
	Projectiles launched onto inclined planes.	Problems will be set on projectiles that are launched and land on an inclined plane. Candidates may approach these problems by resolving the acceleration parallel and perpendicular to the plane. Questions may be set which require the use of trigonometric identities, but any identities which are needed, apart from $\tan^2+1=\sec^2 x$ and those in Core 2, will be given in the examination paper. Candidates will be expected to find the maximum range for a given slope and speed of projection. Candidates may be expected to determine whether a projectile lands at a higher or lower point on the plane after a bounce.

27 A2 Module Mechanics 4

A knowledge of the topics and associated formulae from Modules Core 1 – Core 4, Mechanics 1 and Mechanics 2 is required. Candidates may use relevant formulae included in the formulae booklet without proof.

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Rotations

Moment of a Force $= \mathbf{r} \times \mathbf{F}$

Moment of Inertia $= \sum_{i=1}^{n} m_i x_i^2$

Rotational Kinetic Energy $= \frac{1}{2} I\omega^2$ or $\frac{1}{2} I\dot{\theta}^2$

Resultant Moment $= I\ddot{\theta}$

Centre of Mass

For a Uniform Lamina

$$\bar{x} = \frac{\int_a^b xy\,\mathrm{d}x}{\int_a^b y\,\mathrm{d}x} \qquad \bar{y} = \frac{\int_a^b \frac{1}{2} y^2\,\mathrm{d}x}{\int_a^b y\,\mathrm{d}x}$$

For a Solid of Revolution (about the x-axis)

$$\bar{x} = \frac{\int_a^b \pi x y^2\,\mathrm{d}x}{\int_a^b \pi y^2\,\mathrm{d}x}$$

27.1 Moments

Couples.

Understanding of the concept of a couple.

Reduction of systems of coplanar forces.

Reduction to a single force, a single couple or to a couple and a force acting at a point. The line of action of a resultant force may be required.

Conditions for sliding and toppling.

Determining how equilibrium will be broken in situations, such as a force applied to a solid on a horizontal surface or on an inclined plane with an increasing slope. Derivation of inequalities that must be satisfied for equilibrium.

27.2	Frameworks	
	Finding unknown forces acting on a framework. Finding the forces in the members of a light, smoothly jointed framework. Determining whether rods are in tension or compression.	Awareness of assumptions made when solving framework problems.
27.3	Vector Product and Moments	
	The vector product	
	$\mathbf{i}\times\mathbf{i}=\mathbf{0},\ \mathbf{i}\times\mathbf{j}=\mathbf{k},\ \mathbf{j}\times\mathbf{i}=-\mathbf{k}$, etc	Candidates may use determinants to find vector products.
	The result $\lvert\mathbf{a}\times\mathbf{b}\rvert=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta$	
	The moment of a force as $\mathbf{r}\times\mathbf{F}$.	
	Vector methods for resultant force and moment. Application to simple problems.	Finding condition for equilibrium, unknown forces or points of application.
27.4	Centres of mass by Integration for Uniform Bodies	
	Centre of mass of a uniform lamina by integration. Centre of mass of a uniform solid formed by rotating a region about the x-axis.	Finding x and y coordinates of the centre of mass.
27.5	Moments of Inertia	
	Moments of inertia for a system of particles.	About any axis $I=\sum_{i=1}^{n} m_i x_i^2$
	Moments of inertia for uniform bodies by integration.	Candidates should be able to derive standard results, i.e. rod, rectangular lamina, hollow or solid sphere and cylinder.
	Moments of inertia of composite bodies.	Bodies formed from simple shapes.
	Parallel and perpendicular axis theorems.	Application to finding moments of inertia about different axes.

27.6	**Motion of a rigid body about a smooth fixed axis.**	
	Angular velocity and acceleration of a rigid body.	To exclude small oscillations of a compound pendulum.
	Motion of a rigid body about a fixed horizontal or vertical axis.	$I\ddot{\theta} =$ Resultant Moment Including motion under the action of a couple.
	Rotational kinetic energy and the principle of conservation of energy.	Rotational Kinetic Energy $= \frac{1}{2}I\omega^2$ or $\frac{1}{2}I\dot{\theta}^2$ To include problems such as the motion of a mass falling under gravity while fixed to the end of a light inextensible string wound round a pulley of given moment of inertia.
	Moment of momentum (angular momentum).	
	The principle of conservation of angular momentum.	To include simple collision problems, e.g. a particle colliding with a rod rotating about a fixed axis.
	Forces acting on the axis of rotation	

28 A2 Module *Mechanics 5*

A knowledge of the topics and associated formulae from Modules Core 1 – Core 4, Mechanics 1 and Mechanics 2 is required.
Candidates may use relevant formulae included in the formulae booklet without proof.
Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Energy Formulae	$\text{PE} = mgh$ Gravitational Potential Energy
	$\text{EPE} = \frac{1}{2}k e^2$ or $\frac{\lambda}{2l}e^2$ Elastic Potential Energy
Simple Harmonic Motion	$v^2 = \omega^2(a^2 - x^2)$
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x$

28.1 Simple Harmonic Motion

Knowledge of the definition of simple harmonic motion.	
Finding frequency, period and amplitude	
Knowledge and use of the formula $v^2 = \omega^2\left(a^2 - x^2\right)$.	
Formation of simple second order differential equations to show that simple harmonic motion takes place.	Problems will be set involving elastic strings and springs. Candidates will be required to be familiar with both modulus of elasticity and stiffness. They should be aware of and understand the relationship $k = \frac{\lambda}{l}$.
Solution of second order differential equations of the form $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x$	State solutions in the form $x = A\cos(\omega t + \alpha)$ or $x = A\cos(\omega t) + B\sin(\omega t)$ and use these in problems.
Simple Pendulum.	Formation and solution of the differential equation, including the use of a small angle approximation. Finding the period.

28.2	**Forced and Damped Harmonic Motion**	
	Understanding the terms *forcing* and *damping* and solution of problems involving them. Candidates should be able to set up and solve differential equations in situations involving damping and forcing.	Damping will be proportional only to velocity. Forcing forces will be simple polynomials or of the form $a\sin(\omega t+\alpha)$, $\omega a\sin t\ + b\cos\omega t$ or ae^{bt} .
	Light, critical and heavy damping.	Candidates should be able to determine which of these will take place.
	Resonance.	Solutions may be required for the case where the forcing frequency is equal to the natural frequency.
	Application to spring/mass systems.	
28.3	**Stability**	
	Finding and determining whether positions of equilibrium are stable or unstable.	Use of potential energy methods. Problems will involve gravitational and elastic potential energy.
28.4	**Variable Mass Problems**	
	Equation of motion for variable mass.	Derive equations of motion for variable mass problems, for example, a rocket burning fuel, or a falling raindrop. Rocket problems will be set in zero or constant gravitational fields.
28.5	**Motion in a Plane using Polar Coordinates**	
	Polar coordinates	
	Transverse and radial components of velocity in polar form.	These results may be stated. No proof will be required.
	Transverse and radial components of acceleration in polar form.	
	Application of polar form of velocity and acceleration.	
	Application to simple central forces.	No specific knowledge of planetary motion will be required.

29 AS Module *Decision 1*

29.1	**Simple Ideas of Algorithms** Correctness, finiteness and generality. Stopping conditions.	Candidates should appreciate that for a given input an algorithm produces a unique output. Candidates will not be required to write algorithms in examinations, but may be required to trace, correct, complete or amend a given algorithm, and compare or comment on the number of iterations required. The algorithm may be presented as a flow diagram.
	Bubble, shuttle, shell, quicksort algorithms.	Candidates should appreciate the relative advantages of the different algorithms in terms of the number of iterations required. When using the quicksort algorithm, the first number in each list will be taken as the pivot.
29.2	**Graphs and Networks**	
	Vertices, edges, edge weights, paths, cycles, simple graphs.	
	Adjacency/distance matrices.	For storage of graphs.
	Connectedness.	
	Directed and undirected graphs	
	Degree of a vertex, odd and even vertices, Eulerian trails and Hamiltonian cycles.	
	Trees.	
	Bipartite and complete graph.	Use of the notations K_n and $K_{m,n}$
29.3	**Spanning Tree Problems** Prim's and Kruskal's algorithms to find minimum spanning trees. Relative advantage of the two algorithms. Greediness.	Minimum length spanning trees are also called minimum connectors. Candidates will be expected to apply these algorithms in graphical, and for Prim's algorithm also in tabular, form. Candidates may be required to comment on the appropriateness of their solution in its context.
29.4	**Matchings**	
	Use of bipartite graphs.	
	Improvement of matching using an algorithm.	Use of an alternating path.
29.5	**Shortest Paths in Networks** Dijkstra's algorithm.	Problems involving shortest and quickest routes and paths of minimum cost. Including a labelling technique to identify the shortest path. Candidates may be required to comment on the appropriateness of their solution in its context.

29.6	**Route Inspection Problem** Chinese Postman problem.	Candidates should appreciate the significance of the odd vertices. Although problems with more than four odd vertices will not be set, candidates must be able to calculate the number of possible pairings for n odd vertices. Candidates may be required to comment on the appropriateness of their solution in its context.
29.7	**Travelling Salesperson Problem** Conversion of a practical problem into the classical problem of finding a Hamiltonian cycle. Determination of upper bounds by nearest neighbour algorithm. Determination of lower bounds on route lengths using minimum spanning trees.	By deleting a node, then adding the two shortest distances to the node and the length of the minimum spanning tree for the remaining graph. Candidates may be required to comment on the appropriateness of their solution in its context.
29.8	**Linear Programming** Graphical solution of two-variable problems.	Candidates will be expected to formulate a variety of problems as linear programmes. They may be required to use up to a maximum of 3 variables, which may reduce to two variable requiring a graphical solution. In the case of two decision variables candidates may be expected to plot a feasible region and objective line. Candidates may be required to comment on the appropriateness of their solution in its context.
29.9	**Mathematical modelling** The application of mathematical modelling to situations that relate to the topics covered in this module.	Including the interpretation of results in context.

30 A2 Module *Decision 2*

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in the linear programming section of Decision 1.

30.1	**Critical Path Analysis** Representation of compound projects by activity networks, algorithm to find the critical path(s); cascade (or Gantt) diagrams; resource histograms and resource levelling.	Activity-on-node representation will be used for project networks. Heuristic procedures only are required for resource levelling. Candidates may be required to comment on the appropriateness of their solution in its context
30.2	**Allocation** The Hungarian algorithm.	Including the use of a dummy row or column for unbalanced problems. The use of an algorithm to establish a maximal matching may be required.
30.3	**Dynamic Programming** The ability to cope with negative edge lengths. Application to production planning. Finding minimum or maximum path through a network. Solving maximin and minimax problems.	A stage and state approach may be required in dynamic programming problems.
30.4	**Network Flows** Maximum flow/minimum cut theorem. Labelling procedure.	Problems may require super-sources and sinks, may have upper and lower capacities and may have vertex restrictions. For flow augmentation.
30.5	**Linear Programming** The Simplex method and the Simplex tableau.	Candidates will be expected to introduce slack variables, iterate using a tableau and interpret the outcome at each stage.

30.6	**Game Theory for Zero Sum Games** Pay-off matrix, play-safe strategies and saddle points. Optimal mixed strategies for the graphical method.	Reduction of pay-off matrix using dominance arguments. Candidates may be required to comment on the appropriateness of their solution in its context
30.7	**Mathematical modelling** The application of mathematical modelling to situations that relate to the topics covered in this module.	Including the interpretation of results in context.

Key Skills and Other Issues

31 Key Skills – Teaching, Developing and Providing Opportunities for Generating Evidence

31.1 Introduction

The Key Skills Qualification requires candidates to demonstrate levels of achievement in the Key Skills of *Application of Number*, *Communication* and *Information Technology*.

The units for the 'wider' Key Skills of *Improving Own Learning and Performance*, *Working with Others* and *Problem Solving* are also available. The acquisition and demonstration of ability in these 'wider' Key Skills are deemed highly desirable for all candidates, but they do not form part of the Key Skills Qualification.

Copies of the Key Skills Units may be downloaded from the QCA Website (www.qca.org.uk/keyskills).

The units for each Key Skill comprise three sections:

A. What you need to know.

B. What you must do.

C. Guidance.

Candidates following a course of study based on this specification for Mathematics, Pure Mathematics, and/or Further Mathematics can be offered opportunities to develop and generate evidence of attainment in aspects of the Key Skills of *Communication, Application of Number, Information Technology, Working with Others,* and *Improving Own Learning and Performance*. Areas of study and learning that can be used to encourage the acquisition and use of Key Skills, and to provide opportunities to generate evidence for Part B of the units, are signposted below. The study of Mathematics does not easily lend itself to developing the Key Skill of Problem Solving. Therefore, this Key Skill is not signposted.

31.2 Key Skills Opportunities in Mathematics

The matrices below signpost the opportunities in the teaching and learning modules of this specification for the acquisition, development and production of evidence for Part B of the Key Skills units of *Communication, Application of Number, Information Technology, Working with Others,* and *Improving Own Learning and Performance* at *Level 3* in the teaching and learning modules of this specification. The degree of opportunity in any one module will depend on a number of centre-specific factors, including teaching strategies and level of resources.

Communication

What you must do ...	**Signposting of Opportunities for Generating Evidence in Modules**					
	MPC1	**MPC2**	**MPC3**	**MPC4**	**MFP1**	**MFP2**
C3.1a Contribute to discussions	✓	✓	✓	✓	✓	✓
C3.1b Make a presentation	✓	✓	✓	✓	✓	✓
C3.2 Read and synthesise information						
C3.3 Write different types of document						

Communication

What you must do ...	**Signposting of Opportunities for Generating Evidence in Modules**					
	MFP3	**MFP4**	**MS1A**	**MS1B**	**MS2B**	
C3.1a Contribute to discussions	✓	✓	✓	✓	✓	
C3.1b Make a presentation	✓	✓	✓	✓	✓	
C3.2 Read and synthesise information			✓			
C3.3 Write different types of document			✓			

Communication

What you must do ...	**Signposting of Opportunities for Generating Evidence in Modules**					
	MS03	**MS04**	**MM1B**	**MM2B**		
C3.1a Contribute to discussions	✓	✓	✓	✓		
C3.1b Make a presentation	✓	✓	✓	✓		
C3.2 Read and synthesise information						
C3.3 Write different types of document						

Communication

What you must do ...	**Signposting of Opportunities for Generating Evidence in Modules**					
	MM03	**MM04**	**MM05**	**MD01**	**MD02**	
C3.1a Contribute to discussions	✓	✓	✓	✓	✓	
C3.1b Make a presentation	✓	✓	✓	✓	✓	
C3.2 Read and synthesise information						
C3.3 Write different types of document						

Application of Number

What you must do ...	**Signposting of Opportunities for Generating Evidence in Modules**					
	MPC1	**MPC2**	**MPC3**	**MPC4**	**MFP1**	**MFP2**
N3.1 Plan and interpret information from different sources	✓	✓	✓	✓	✓	✓
N3.2 Carry out multi-stage calculations	✓	✓	✓	✓	✓	✓
N3.3 Present findings, explain results and justify choice of methods	✓	✓	✓	✓	✓	✓

Application of Number

What you must do ...	**Signposting of Opportunities for Generating Evidence in Modules**					
	MFP3	**MFP4**	**MS1A**	**MS1B**	**MS2B**	
N3.1 Plan and interpret information from different sources	✓	✓	✓	✓	✓	
N3.2 Carry out multi-stage calculations	✓	✓	✓	✓	✓	
N3.3 Present findings, explain results and justify choice of methods	✓	✓	✓	✓	✓	

Application of Number

What you must do ...	**Signposting of Opportunities for Generating Evidence in Modules**					
	MS03	**MS04**	**MM1B**	**MM2B**		
N3.1 Plan and interpret information from different sources	✓	✓	✓	✓		
N3.2 Carry out multi-stage calculations	✓	✓	✓	✓		
N3.3 Present findings, explain results and justify choice of methods	✓	✓	✓	✓		

Application of Number

What you must do ...	**Signposting of Opportunities for Generating Evidence in Modules**					
	MM03	**MM04**	**MM05**	**MD01**	**MD02**	
N3.1 Plan and interpret information from different sources	✓	✓	✓	✓	✓	
N3.2 Carry out multi-stage calculations	✓	✓	✓	✓	✓	
N3.3 Present findings, explain results and justify choice of methods	✓	✓	✓	✓	✓	

Information Technology

What you must do ...	Signposting of Opportunities for Generating Evidence in Modules					
	MPC1	MPC2	MPC3	MPC4	MFP1	MFP2
IT3.1 Plan and use different sources to search for and select information						
IT3.2 Explore, develop and exchange information, and derive new information						
IT3.3 Present information including text, numbers and images						

Information Technology

What you must do ...	Signposting of Opportunities for Generating Evidence in Modules					
	MFP3	MFP4	MS1A	MS1B	MS2B	
IT3.1 Plan and use different sources to search for and select information			✓	✓	✓	
IT3.2 Explore, develop and exchange information, and derive new information			✓	✓	✓	
IT3.3 Present information including text, numbers and images			✓	✓	✓	

Information Technology

What you must do ...	Signposting of Opportunities for Generating Evidence in Modules					
	MS03	MS04	MM1B	MM2B		
IT3.1 Plan and use different sources to search for and select information	✓	✓	✓	✓		
IT3.2 Explore, develop and exchange information, and derive new information	✓	✓	✓	✓		
IT3.3 Present information including text, numbers and images	✓	✓	✓	✓		

Information Technology

What you must do ...	Signposting of Opportunities for Generating Evidence in Modules					
	MM03	MM04	MM05	MD01	MD02	
IT3.1 Plan and use different sources to search for and select information	✓	✓	✓	✓	✓	
IT3.2 Explore, develop and exchange information, and derive new information	✓	✓	✓	✓	✓	
IT3.3 Present information including text, numbers and images	✓	✓	✓	✓	✓	

Working with Others

What you must do…	Signposting of Opportunities for Generating Evidence in Modules					
	MPC1	MPC2	MPC3	MPC4	MFP1	MFP2
WO3.1 Plan the activity						
WO3.2 Work towards agreed objectives						
WO3.3 Review the activity						

Working with Others

What you must do…	Signposting of Opportunities for Generating Evidence in Modules					
	MFP3	MFP4	MS1A	MS1B	MS2B	
WO3.1 Plan the activity			✓	✓	✓	
WO3.2 Work towards agreed objectives			✓	✓	✓	
WO3.3 Review the activity			✓	✓	✓	

Working with Others

What you must do…	Signposting of Opportunities for Generating Evidence in Modules					
	MS03	MS04	MM1B	MM2B		
WO3.1 Plan the activity	✓	✓	✓	✓		
WO3.2 Work towards agreed objectives	✓	✓	✓	✓		
WO3.3 Review the activity	✓	✓	✓	✓		

Working with Others

What you must do…	Signposting of Opportunities for Generating Evidence in Modules					
	MM03	MM04	MM05	MD01	MD02	
WO3.1 Plan the activity	✓	✓	✓	✓	✓	
WO3.2 Work towards agreed objectives	✓	✓	✓	✓	✓	
WO3.3 Review the activity	✓	✓	✓	✓	✓	

Improving Own Learning and Performance

What you must do…	Signposting of Opportunities for Generating Evidence in Modules					
	MPC1	MPC2	MPC3	MPC4	MFP1	MFP2
LP3.1 Agree and plan targets	✓	✓	✓	✓	✓	✓
LP3.2 Seek feedback and support	✓	✓	✓	✓	✓	✓
LP3.3 Review progress	✓	✓	✓	✓	✓	✓

Improving Own Learning and Performance

What you must do…	Signposting of Opportunities for Generating Evidence in Modules					
	MFP3	MFP4	MS1A	MS1B	MS2B	
LP3.1 Agree and plan targets	✓	✓	✓	✓	✓	
LP3.2 Seek feedback and support	✓	✓	✓	✓	✓	
LP3.3 Review progress	✓	✓	✓	✓	✓	

Improving Own Learning and Performance

What you must do…	Signposting of Opportunities for Generating Evidence in Modules					
	MS03	MS04	MM1B	MM2B		
LP3.1 Agree and plan targets	✓	✓	✓	✓		
LP3.2 Seek feedback and support	✓	✓	✓	✓		
LP3.3 Review progress	✓	✓	✓	✓		

Improving Own Learning and Performance

What you must do…	Signposting of Opportunities for Generating Evidence in Modules					
	MM03	MM04	MM05	MD01	MD02	
LP3.1 Agree and plan targets	✓	✓	✓	✓	✓	
LP3.2 Seek feedback and support	✓	✓	✓	✓	✓	
LP3.3 Review progress	✓	✓	✓	✓	✓	

Note: The signposting in the tables above represents the opportunities to acquire, and produce evidence of, the Key Skills which are possible through this specification. There may be other opportunities to achieve these and other aspects of Key Skills via this specification, but such opportunities are dependent on the detailed course of study delivered within centres.

31.3 Key Skills in the Assessment of Mathematics

The Key Skill of *Application of Number* must contribute to the study of Mathematics and Further Mathematics. Aspects of *Application of Number* form an intrinsic part of the Assessment Objectives, and hence will form part of the assessment requirements for all units.

31.4 Further Guidance

More specific guidance and examples of tasks that can provide evidence of single Key Skills or composite tasks that can provide evidence of more than one Key Skill are given in the Teachers' Guides published for AQA Mathematics Specification A (6300) and AQA Mathematics and Statistics Specification B (6320).

32 Spiritual, Moral, Ethical, Social, Cultural and Other Issues

32.1 Spiritual, Moral, Ethical, Social and Cultural Issues

Contexts used during the study of the modules may contribute to students' understanding of spiritual, moral and cultural issues.

32.2 European Dimension

AQA has taken account of the 1988 Resolution of the Council of the European Community in preparing this specification and associated specimen papers.

32.3 Environmental Education

AQA has taken account of the 1988 Resolution of the Council of the European Community and the Report *Environmental Responsibility: An Agenda for Further and Higher Education* 1993 in preparing this specification and associated specimen papers.

32.4 Avoidance of Bias

AQA has taken great care in the preparation of this specification and associated specimen papers to avoid bias of any kind.

Centre-assessed Component

33 Nature of Centre-assessed Component

33.1

Candidates will present one task, of approximately 8–10 hours' duration, for MS1A.

It is intended that coursework assessment should be an integral part of the teaching and learning process. As a consequence, candidates should feel that at least some of their ongoing work will contribute to the final result. Coursework thus provides an opportunity for candidates to conduct an extended piece of mathematical reasoning that will also enhance their understanding of an area of the specification content.

In coursework, candidates will use a reflective or creative approach to apply their knowledge to a real-life problem. Candidates will make sensible assumptions, formulate and test hypotheses, carry out appropriate mathematical analyses and produce reports in which they interpret their results in context and comment on the suitability of their results in terms of the original task.

Coursework also provides an appropriate method for generating evidence for five of the six Key Skills: *Communication*, *Application of Number*, *Information Technology*, *Improving Own Learning*, and *Working with Others*.

33.2 Early Notification

Centres must advise AQA of their intention to enter candidates, using the *Estimated Entries Form* supplied to Examinations Officers so that a Guidance Pack for teachers can be supplied and a Coursework Adviser allocated. This will also enable AQA to send out an order form in September for centres to request the *Candidate Record Forms* appropriate for their intended unit entries.

33.3 Relationship of Coursework to Assessment Objectives

All Assessment Objectives can be met in coursework. The following pages show, for the unit MS1A, the *Marking Grid* with the weightings of each Assessment Objective.

The *Marking Grid* for Statistics should be used for the coursework tasks submitted for unit Statistics 1.

34 Guidance on Setting Centre-assessed Component

A list of recommended coursework tasks is provided in the Guidance Pack. The pack provides guidance on appropriate coursework , exemplar materials for reference purposes together with marked pieces of work showing clearly how the criteria are to be applied. Where a centre or a candidate wishes to submit their own task the centre may submit the task to their allocated Coursework Adviser in order that guidance may be offered on the suitability of the task. Such tasks must be submitted at least six weeks prior to their use by the centre.

35 Assessment Criteria

35.1 Introduction

Coursework tasks for assessment will be marked internally by teachers making reference to the *Marking Grid* and *Mark Breakdown* on the *Candidate Record Forms*. The grid will be used to measure **positive** achievement, according to descriptors, within a number of categories.

35.2 Criteria

The *Marking Grid* and *Mark Breakdown* have four strands, each of which represents a different set of criteria.

The criteria within each strand on the *Marking Grid* are intended to indicate the essential characteristics that should be identified at various levels of performance to be expected from candidates within that unit.

Teachers must complete the *Marking Grid* section on the *Candidate Record Form* for each candidate.

The *Mark Breakdown* section of the *Candidate Record Form* gives a further detailed breakdown of marks for each strand. This section is optional. Teachers can use it as a guide in reaching the final mark for the assessment of coursework tasks. Completing this section will be useful for centres for internal standardisation procedures, and could be used by AQA as a basis for feedback to centres.

The final mark for the assessment of the coursework tasks is the sum of the marks for each strand.

If none of the criteria has been met in any strand then zero marks must be awarded in that strand.

35.3 Marking Grid for Statistics Coursework Tasks

Strand	Marks 0 → 8	Marks 9 → 15	Marks 16 → 20	Assessment Objective mark allocation
Design	Problem defined and understood. Aims and objectives discussed. Some discussion of how the sample was obtained.	The approach to the task is coherent. The strategies to be employed are appropriate. Clear explanation of how sample was obtained. Some discussion of the statistical theories or distributions used.	A well-balanced and coherent approach. Clear discussion and justification of the statistical theories or distributions used in relation to the task.	AO1 6 AO2 9 AO3 3 AO4 2 AO5 0
Data Collection and Statistical Analysis	Adequate data collected. Raw data clearly set out. Some relevant calculations are correct.	A range of relevant statistical calculations are used. Most calculations are correct, and quoted to an appropriate degree of accuracy.	A full range of calculations are used. The calculations are correct and appropriate to the task.	AO1 10 AO2 4 AO3 0 AO4 0 AO5 6
Interpretation / Validation	A reasoned attempt is made to interpret the results. Some discussion of how realistic the results are. Some discussion of possible modifications.	Results are interpreted. Attempt to relate the task to the original problem. Clear discussion of possible modifications/ improvements which could have been made.	Results are fully interpreted within a statistical context. Outcomes are clearly related to the original task. Clear discussion of the effects of the sampling and data collection methods used.	AO1 2 AO2 0 AO3 12 AO4 6 AO5 0
Communication	The report is presented clearly and organised with some explanation. Diagrams are effective and appropriate. Conclusions are stated.	The report is clear and well organised. Other areas of work which could have been investigated are discussed. The report is consistent with a piece of work of 8–10 hours.	Appropriate language and notations used throughout. The report is clear and concise and of sufficient depth and difficulty.	AO1 6 AO2 10 AO3 2 AO4 0 AO5 2

Total marks for each Assessment Objective:

AO1	24
AO2	23
AO3	17
AO4	8
AO5	8
	80

35.4 Evidence to Support the Award of Marks

The coursework task for each candidate must show clear, annotated evidence of having been marked under the four strands. Calculations must be checked for accuracy and annotated accordingly.

It is perfectly acceptable for parts of a candidate's coursework to be taken from other sources as long as all such cases are clearly identified in the text and fully acknowledged either on the *Candidate Record Form* or in the supporting evidence. Where phrases, sentences or longer passages are quoted directly from a source, candidates should use quotation marks.

Teachers should keep records of their assessments during the course, in a form which facilitates the complete and accurate submission of the final assessments at the end of the course.

When the assessments are complete, the final marks awarded under each of the strands must be entered on the *Candidate Record Form*. The *Marking Grid* section must be completed; the *Mark Breakdown* section is optional. Supporting information should also be recorded in the section provided on the last page of the form.

36 Supervision and Authentication

36.1 Supervision of Candidates' Work

Candidates' work for assessment must be undertaken under conditions which allow the teacher to supervise the work and enable the work to be authenticated. If it is necessary for some assessed work to be done outside the centre, sufficient work must take place under direct supervision to allow the teacher to authenticate each candidate's whole work with confidence.

36.2 Guidance by the Teacher

The work assessed must be solely that of the candidate concerned. Any assistance given to an individual candidate which is beyond that given to the group, as a whole must be recorded on the *Candidate Record Form*.

It is expected that candidates will start their coursework after consultation with their teacher. It is important that discussion should take place between the teacher and the candidate at all stages of the work involved; the coursework is not being carried out solely for the purpose of assessment; it is part of the teaching/learning process and the teacher will need to be involved in the work of the candidate if he or she is to be able to use this approach as part of the course of study.

When a candidate has need of assistance in completing a piece of work, such assistance should be given but the teacher must take the degree of assistance into account when making the assessment and, where necessary, should add appropriate comments on the *Candidate Record Form*. Assistance in learning a new area of mathematics for use in a problem is acceptable, and no deduction of marks should be made for such assistance.

It is accepted that candidates may wish to conduct initial data collection or experimental work in groups. Where candidates work as a group, it must be possible to identify the individual contribution of each candidate so that the requirements of the specification are met.

36.3 Unfair Practice

At the start of the course, the supervising teacher is responsible for informing candidates of the AQA Regulations concerning malpractice. Candidates must not take part in any unfair practice in the preparation of coursework to be submitted for assessment, and must understand that to present material copied directly from books or other sources without acknowledgement will be regarded as deliberate deception. Centres must report suspected malpractice to AQA. The penalties for malpractice are set out in the AQA Regulations.

36.4 Authentication of Candidates' Work

Both the candidate and the teacher are required to sign declarations on the *Candidate Record Form*, confirming that the work submitted for assessment is the candidate's own. The form declares that the work was conducted under the specified conditions, and requires the teacher to record details of any additional assistance.

37 Standardisation

37.1 Annual Standardising Meetings

Annual standardisation meetings will usually be held in the autumn term. Centres entering candidates for the first time must send a representative to the meetings. Attendance is also mandatory in the following cases:

- where there has been a serious misinterpretation of the specification requirements;
- where the nature of coursework tasks set by a centre has been inappropriate;
- where a significant adjustment has been made to a centre's marks in the previous year's examination.

Otherwise attendance is at the discretion of centres. At these meetings, support will be provided for centres in the development of appropriate coursework tasks and assessment procedures.

37.2 Internal Standardisation of Marking

The centre is required to standardise the assessments across different teachers and teaching groups to ensure that all candidates at the centre have been judged against the same standards. If two or more teachers are involved in marking a component, one teacher must be designated as responsible for internal standardisation. Common pieces of work must be marked on a trial basis and differences between assessments discussed at a training session in which all teachers involved must participate. The teacher responsible for standardising the marking must ensure that the training includes the use of reference and archive materials such as work from a previous year or examples provided by AQA. The centre is required to send to the moderator a signed form *Centre Declaration Sheet* confirming that the marking of coursework at the centre has been standardised. If only one teacher has undertaken the marking, that person must sign this form.

38 Administrative Procedures

38.1 Recording Assessments

A separate *Candidate Record Form* must be completed for the coursework entered by each candidate. The candidates' work must be marked according to the assessment criteria set out in Section 35.2, then the marks and supporting information must be recorded on the *Candidate Record Form* in accordance with the instructions in Section 35.4.

Details of any additional assistance must be given, and the teacher must sign the *Candidate Record Form*. The candidate must also complete and sign the first page of the form.

The completed *Candidate Record Form(s)* for each candidate must be attached to the work and made available to AQA on request.

38.2 Submitting Marks and Sample Work for Moderation

The total coursework mark for each candidate must be submitted to AQA on the mark sheets provided or by Electronic Data Interchange (EDI) by the specified date. Centres will be informed which candidates' work is required in the samples to be submitted to their moderator.

38.3 Factors Affecting Individual Candidates

Teachers should be able to accommodate the occasional absence of candidates by ensuring that the opportunity is given for them to make up missed assessments.

Special consideration should be requested for candidates whose work has been affected by illness or other exceptional circumstances. Information about the procedure is issued separately. Centres should ask for a copy of *Regulations and Guidance relating to Candidates with Particular Requirements*.

If work is lost, AQA should be notified immediately of the date of the loss, how it occurred and who was responsible for the loss. AQA will advise on the procedures to be followed in such cases.

Where special help which goes beyond normal learning support is given, AQA must be informed so that such help can be taken into account when assessment and moderation take place.

Candidates who move from one centre to another during the course sometimes present a problem for a scheme of internal assessment. Possible courses of action depend on the stage at which the move takes place. If the move occurs early in the course, the new centre should take responsibility for assessment. If it occurs late in the course, it may be possible to accept the assessments made at the previous centre. Centres should contact AQA at the earliest possible stage for advice about appropriate arrangements in individual cases.

38.4 Retaining Evidence and Re-using Marks

The centre must retain the work of all candidates, with *Candidate Record Forms* attached, under secure conditions from the time it is assessed; this is to allow for the possibility of an enquiry-about-results. The work may be returned to candidates after the issue of results provided that no enquiry-about-result is to be made which will include re-moderation of the coursework component. If an enquiry-about-result is to be made, the work must remain under secure conditions until requested by AQA.

Candidates wishing to improve the result of any unit containing coursework may carry forward their moderated coursework mark from a previous series.

39 Moderation

39.1 Moderation Procedures

Moderation of the coursework is by inspection of a sample of candidates' work, sent by post from the centre for scrutiny by a moderator appointed by AQA. The centre marks must be submitted to AQA by the specified date.

Following the re-marking of the sample work, the moderator's marks are compared with the centre's marks to determine whether any adjustment is needed in order to bring the centre's assessments into line with standards generally. In some cases, it may be necessary for the moderator to call for the work of other candidates. In order to meet this possible request, centres must have available the coursework and *Candidate Record Form* of every candidate entered for the examination and be prepared to submit it on demand. Mark adjustments will normally preserve the centre's order of merit, but where major discrepancies are found, AQA reserves the right to alter the order of merit.

39.2 Post-moderation Procedures

On publication of the GCE results, the centre is supplied with details of the final marks for the coursework component.

The candidates' work is returned to the centre after the examination. The centre receives a report form from their moderator giving feedback on the appropriateness of the tasks set, the accuracy of the assessments made, and the reasons for any adjustments to the marks.

Some candidates' work may be retained by AQA for archive purposes.

Awarding and Reporting

40 Grading, Shelf-life and Re-sits

40.1 Qualification Titles

The qualification based on these specifications have the following titles:

AQA Advanced Subsidiary GCE in Mathematics;
AQA Advanced GCE in Mathematics;
AQA Advanced Subsidiary GCE in Pure Mathematics;
AQA Advanced GCE in Pure Mathematics;
AQA Advanced Subsidiary GCE in Further Mathematics;
AQA Advanced GCE in Further Mathematics.

40.2 Grading System

The AS qualifications will be graded on a five-point scale: A, B, C, D and E. The full A level qualifications will be graded on a six-point scale: A*, A, B, C, D and E. To be awarded an A* in Mathematics, candidates will need to achieve a grade A on the full A level qualification and 90% of the maximum uniform mark on the aggregate of MPC3 and MPC4. To be awarded an A* in Pure Mathematics, candidates will need to achieve grade A on the full A level qualification and 90% of the maximum uniform mark on the aggregate of all three A2 units. To be awarded an A* in Further Mathematics, candidates will need to achieve grade A on the full A level qualification and 90% of the maximum uniform mark on the aggregate of the best three of the A2 units which contributed towards Further Mathematics. For all qualifications, candidates who fail to reach the minimum standard for grade E will be recorded as U (unclassified) and will not receive a qualification certificate.

Individual assessment unit results will be certificated.

40.3 Shelf-life of Unit Results

The shelf-life of individual unit results, prior to certification of the qualification, is limited only by the shelf-life of the specification.

40.4 Assessment Unit Re-Sits

Each assessment unit may be re-taken an unlimited number of times within the shelf-life of the specification. The best result will count towards the final award. Candidates who wish to repeat an award must enter for at least one of the contributing units and also enter for certification (cash-in). There is no facility to decline an award once it has been issued.

40.5 Carrying Forward of Coursework Marks

Candidates re-taking a unit with coursework may carry forward their moderated coursework marks. These marks have a shelf-life which is limited only by the shelf-life of the specification, and they may be carried forward an unlimited number of times within this shelf-life.

40.6 Minimum Requirements

Candidates will be graded on the basis of work submitted for the award of the qualification.

40.7 Awarding and Reporting

This specification complies with the grading, awarding and certification requirements of the current GCSE, GCE, Principal Learning and Project Code of Practice April 2011, and will be revised in the light of any subsequent changes for future years.

Appendices

A Grade Descriptions

The following grade descriptors indicate the level of attainment characteristic of the given grade at AS and A Level. They give a general indication of the required learning outcomes at each specific grade. The descriptors should be interpreted in relation to the content outlined in the specification; they are not designed to define that content.

The grade awarded will depend, in practice, on the extent to which the candidate has met the Assessment Objectives (as in Section 6) overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

Grade A

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments. When confronted with unstructured problems, they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical information from extended pieces of prose having mathematical content. They can comment meaningfully on the mathematical information.

Candidates make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Grade C Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems, they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Candidates recall or recognise most of the standard models that are needed and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation, they sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical context. They give some useful comments on this mathematical information.

Candidates usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Grade E Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Candidates recall or recognise some of the standard models that are needed and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation.

Candidates sometimes comprehend or understand the meaning of translations in mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical content. They attempt to comment on this mathematical information.

Candidates often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

B

Formulae for AS and A Level Mathematics Specifications

This appendix lists formulae which relate to the Core modules, MPC1 – MPC4, and which candidates are expected to remember. These formulae will **not** be included in the AQA formulae booklet.

Quadratic equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of logarithms

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a \left(x^k\right)$

Trigonometry

In the triangle ABC:

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\text{area} = \dfrac{1}{2} ab \sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\text{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k \cos kx$
$\cos kx$	$-k \sin kx$
e^{kx}	$k\, e^{kx}$
$\ln x$	$\dfrac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)\, g(x)$	$f'(x)\, g(x) + f(x)\, g'(x)$
$f(g(x))$	$f'(g(x))\, g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c, \quad n \neq -1$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx}+c$
$\frac{1}{x}$	$\ln \lvert x \rvert + c, \quad x \neq 0$
$f'(x)+g'(x)$	$f(x)+g(x)+c$
$f'(g(x))\,g'(x)$	$f(g(x))+c$

Area area under a curve $= \int_a^b y \, dx, \quad y \geq 0$

Vectors

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \end{bmatrix} = xa + yb + zc$$

C Mathematical Notation

Set notation	$\in$	is an element of
	$\notin$	is not an element of
	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
	$\{x : \ldots\}$	the set of all x such that …
	$\mathrm{n}(A)$	the number of elements in set A
	$\varnothing$	the empty set
	□	the universal set
	A'	the complement of the set A
	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
	$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{Z}^+\right\}$
	$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
	$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
	$\mathbb{R}$	the set of real numbers
	$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
	$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
	$\mathbb{C}$	the set of complex numbers
	(x, y)	the ordered pair x, y
	$A \times B$	the Cartesian product of sets A and B, i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
	$\subseteq$	is a subset of
	$\subset$	is a proper subset of
	$\cup$	union
	$\cap$	intersection
	$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
	$[a, b), [a, b$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
	$(a, b],]a, b$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
	$(a, b),]a, b$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
	$y\ R\ x$	y is related to x by the relation R
	$y \sim x$	y is equivalent to x, in the context of some equivalence relation

Miscellaneous symbols	$=$	is equal to
	$\neq$	is not equal to
	$\equiv$	is identical to or is congruent to
	$\approx$	is approximately equal to
	$\cong$	is isomorphic to
	$\propto$	is proportional to
	$<$	is less than
	$\leqslant$	is less than or equal to, is not greater than
	$>$	is greater than
	$\geqslant$	is greater than or equal to, is not less than
	∞	infinity
	$p \wedge q$	p and q
	$p \vee q$	p or q (or both)
	$\sim p$	not p
	$p \Rightarrow q$	p implies q (if p then q)
	$p \Leftarrow q$	p is implied by q (if q then p)
	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
	$\exists$	there exists
	$\forall$	for all
Operations	$a+b$	a plus b
	$a-b$	a minus b
	$a \times b,\ ab,\ a.b$	a multiplied by b
	$a \div b,\ \frac{a}{b},\ a/b$	a divided by b
	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
	$\sqrt{a}$	the positive square root of a
	$\lvert a \rvert$	the modulus of a
	$n!$	n factorial
	$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$ for $n \in \mathbb{Z}^+$
		$\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

Functions	$\mathrm{f}(x)$	the value of the function f at x
	$\mathrm{f}: A \to B$	f is a function under which each element of set A has an image in set B
	$\mathrm{f}: x \to y$	the function f maps the element x to the element y
	f^{-1}	the inverse function of the function f
	$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{g}\,\mathrm{f}(x) = \mathrm{g}(\mathrm{f}(x))$
	$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
	$\Delta x, \delta x$	an increment of x
	$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
	$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	first, second, ... , nth derivatives of $\mathrm{f}(x)$ with respect to x
	$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
	$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
	$\frac{\partial V}{\partial x}$	the partial derivative of V with respect to x
	$\dot{x}, \ddot{x}, \ldots$	the first, second, ... derivatives of x with respect to t
Exponential and logarithmic functions	e	base of natural logarithms
	e^x, $\exp x$	exponential function of x
	$\log_a x$	logarithm to the base a of x
	$\ln x$, $\log_\mathrm{e} x$	natural logarithm of x
	$\log_{10} x$	logarithm of x to base 10
Circular and hyperbolic functions	sin, cos, tan, cosec, sec, cot	the circular functions
	$\sin^{-1}, \cos^{-1}, \tan^{-1}, \mathrm{cosec}^{-1}, \sec^{-1}, \cot^{-1}$	the inverse circular functions
	sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions
	$\sinh^{-1}, \cosh^{-1}, \tanh^{-1}, \mathrm{cosech}^{-1}, \mathrm{sech}^{-1}, \coth^{-1}$	the inverse hyperbolic functions

Complex numbers	i, j	square root of -1
	z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta)$
	Re z	the real part of z, Re $z = x$
	Im z	the imaginary part of z, Im $z = y$
	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$
	arg z	the argument of z, arg $z = \theta$, $-\pi < \theta \le \pi$
	z^*	the complex conjugate of z, $x - \mathrm{i}y$
Matrices	**M**	a matrix M
	$\mathbf{M}^{-1}$	the inverse of the matrix M
	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix M
	det **M** or $\lvert\mathbf{M}\rvert$	the determinant of the square matrix M
Vectors	**a**	the vector **a**
	$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
	$\hat{\mathbf{a}}$	a unit vector in the direction of **a**
	i, **j**, **k**	unit vectors in the directions of the Cartesian coordinate axes
	$\lvert\mathbf{a}\rvert$, a	the magnitude of **a**
	$\lvert\overrightarrow{AB}\rvert$, AB	the magnitude of $\overrightarrow{AB}$
	a.b	the scalar product of **a** and **b**
	$\mathbf{a} \times \mathbf{b}$	the vector product of **a** and **b**
Probability and statistics	A, B, C, etc.	events
	$A \cup B$	union of the events A and B
	$A \cap B$	intersection of the events A and B
	$\mathrm{P}(A)$	probability of the event A
	A'	complement of the event A
	$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
	X, Y, R, etc.	random variables
	x, y, r, etc.	values of the random variables X, Y, R, etc.
	$x_1, x_2, \ldots$	observations
	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur

$p(x)$	probability function $P(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
$f(x), g(x), \ldots$	the value of the probability density function of the continuous random variable X
$F(x), G(x), \ldots$	the value of the (cumulative) distribution function $P(X \le x)$ of the continuous random variable X
$E(X)$	expectation of the random variable X
$E[g(X)]$	expectation of $g(X)$
$Var(X)$	variance of the random variable X
$Cov(X, Y)$	covariance of the random variables X and Y
$B(n, p)$	binomial distribution with parameters n and p
$Po(\lambda)$	Poisson distribution with parameter λ
$Geo(p)$	geometric distribution with parameter p
$N(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$	sample mean
s^2	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$
z	value of the standardised normal variable with distribution $N(0, 1)$
$\Phi(z)$	corresponding (cumulative) distribution function
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
a	intercept with the vertical axis in the linear regression equation
b	gradient in the linear regression equation

D Record Forms

Candidate Record Forms

Candidate Record Forms, Centre Declaration Sheets and GCE Mathematics specifics forms are available on the AQA website in the Administration Area. They can be accessed via the following link
http://www.aqa.org.uk/admin/p_course.php

E

Overlaps with other Qualifications

Subject awards in other AQA specifications, including the AQA GCE Statistics specification, are not prohibited combinations with subject awards in this AQA GCE Mathematics specification. However, there are overlaps in subject content between the Statistics units in this specification and the AQA GCE Statistics specification, and between the Mechanics units in this specification and the AQA GCE Physics specifications A and B. Qualifications from other awarding bodies with the same or similar titles can be expected to have a similar degree of overlap.

F

Relationship to other AQA GCE Mathematics and Statistics Specifications

Relationship to AQA GCE Mathematics A (6300)

This specification is a development from both the AQA GCE Mathematics A specificaion (6300) and the AQA GCE Mathematics and Statistics B specification (6320). Most units in this specification have a close equivalent in the previous specifications. The nearest equivalent modules/units are shown below for AQA GCE Mathematics A specification (6300).

New unit	Old unit	New unit	Old unit
-	MAME	MS1A	MAS1
-	MAP1	MS2B	MAS2
MPC1	-	-	MAS3
MPC2	-	-	MAS4
MPC3	MAP2	MS03	-
MPC4	MAP3	MS04	-
MFP1	-	MM1B	MAM1
MFP2	MAP4	MM2B	MAM2
MFP3	MAP5	MM03	-
MFP4	MAP6	MM04	MAM3
		MM05	MAM4
		MD01	MAD1
		MD02	MAD2

Relationship to AQA GCE Mathematics and Statistics B (6320)

This specification is a development from both the AQA GCE Mathematics and Statistics B specification (6320) and the AQA GCE Mathematics A specificaion (6300). Most units in this specification have a close equivalent in the previous specifications. The nearest equivalent modules/units are shown below for AQA GCE Mathematics and Statistics B specification (6320).

New unit	Old unit	New unit	Old unit
MPC1	-	MS1B	MBS1
MPC2	-	MS2B	MBS4/5
MPC3	MBP4	MS03	-
MPC4	MBP5	MS04	-
MPF1	MBP3	-	MBS6
MPF2	-	-	MBS7
MFP3	-	MM1B	MBM1
MFP4	-	MM2B	MBM2/3
		MM03	MBM4
		MM04	MBM5
		MM05	MBM6

Relationship to AQA GCE Statistics (6380)

The two Statistics 1 units in this specification are identical with the two Statistics 1 units in AQA GCE Statistics (6380). The subject content in the module and the assessment for the unit are the same for each of the pairs of units shown below.

MS1A is identical to SS1A
MS1B is identical to SS1B

This is to allow flexibility for candidates who are not sure whether they want to study AS Mathematics or AS Statistics. However, there are limitations on the entries that a candidate can make for these units. See section 3.4 for details.