INTERNATIONAL ADVANCED LEVEL

Mathematics, Further Mathematics and Pure Mathematics

SPECIFICATION

Pearson Edexcel International Advanced Subsidiary in Mathematics (XMA01)
Pearson Edexcel International Advanced Level in Mathematics (YMA01)
Pearson Edexcel International Advanced Subsidiary in Further Mathematics (XFM01)
Pearson Edexcel International Advanced Level in Further Mathematics (YFM01)
Pearson Edexcel International Advanced Subsidiary in Pure Mathematics (XPM01)
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For first teaching in September 2013
First examination January 2014

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About this specification

Pearson Edexcel International Advanced Level in Mathematics is designed for use in schools and colleges outside the United Kingdom. It is part of a suite of International Advanced Level qualifications offered by Pearson.

This qualification has been approved by Pearson Education Limited as meeting the criteria for Pearson's Self Regulated Framework.

Pearson's Self Regulated Framework is designed for qualifications that have been customised to meet the needs of a particular range of learners and stakeholders. These qualifications are not accredited or regulated by any UK regulatory body.

The Pearson Edexcel International Advanced Level in Mathematics has:

- 12 units tested fully by written examination
- a variety of units allowing many different combinations resulting in flexible delivery options
- pathways leading to full International Advanced Subsidiary and International Advanced Level in Mathematics, Further Mathematics and Pure Mathematics
- an updated Further Pure Mathematics 1 unit with extra depth added to some topics for teaching in the first year of study
- units Further Pure Mathematics 2 and Further Pure Mathematics 3 to allow a coherent curriculum in further mathematics
- bigger blocks of learning for the combined Core Mathematics units, providing more synoptic assessment.

Specification updates

This specification is Issue 1 and is valid for the Pearson Edexcel International Advanced Subsidiary and International Advanced Level examination from 2014. If there are any significant changes to the specification Pearson will write to centres to let them know. Changes will also be posted on our website.

For more information please visit www.edexcel.com/ial

Using this specification

This specification has been designed to give guidance to teachers and encourage effective delivery of the qualification. The following information will help you get the most out of the content and guidance.

Examples: throughout the unit content, we have included examples of what could be covered or what might support teaching and learning. It is important to note that examples are for illustrative purposes only and centres can use other examples. We have included examples that are easily understood and recognised by international centres.

Unit assessments use a range of material and are not limited to the examples given. Teachers should deliver the qualification using a good range of examples to support the assessment of the unit content.

Depth and breadth of content: teachers should prepare students to respond to assessment questions. Teachers should use the full range of content and all the assessment objectives given in *Section B: Specification overview.*

Qualification abbreviations

International Advanced Level – IAL

International Advanced Subsidiary – IAS

International Advanced Level 2 (the additional content required for an IAL) – IA2

Contents

Contents

A Specification at a glance

Summary of unit content

Core Mathematics

Unit	Summary of unit content
C12	Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; exponentials and logarithms; trigonometry; differentiation; integration.
C34	Algebra and functions; sequences and series; trigonometry; exponentials and logarithms; coordinate geometry in the (x, y) plane; differentiation; integration; numerical methods; vectors.

Further Pure Mathematics

Unit	Summary of unit content
F1	Complex numbers; roots of quadratic equations; numerical solution of equations; coordinate systems; matrix algebra; transformations using matrices; series; proof.
F2	Inequalities; series; further complex numbers; first order differential equations; second order differential equations; Maclaurin and Taylor series; Polar coordinates.
F3	Hyperbolic functions; further coordinate systems; differentiation; integration; vectors; further matrix algebra.

Mechanics

Unit	Summary of unit content
M1	Mathematical models in mechanics; vectors in mechanics; kinematics of a particle moving in a straight line; dynamics of a particle moving in a straight line or plane; statics of a particle; moments.
M2	Kinematics of a particle moving in a straight line or plane; centres of mass; work and energy; collisions; statics of rigid bodies.
M3	Further kinematics; elastic strings and springs; further dynamics; motion in a circle; statics of rigid bodies.

Statistics

Unit	Summary of unit content
S1	Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.
S2	The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.
S3	Combinations of random variables; sampling; estimation, confidence intervals and tests; goodness of fit and contingency tables; regression and correlation.

Decision Mathematics

Unit	Summary of unit content
D1	Algorithms; algorithms on graphs; the route inspection problem; critical path analysis; linear programming; matchings.

B Specification overview

Summary of assessment requirements

Unit number	Unit title	Unit code*	Level	Method of assessment	Availability	First assessment	IAS weighting	IAL weighting
C12	Core Mathematics 12	WMA01	IAS	1 written paper	January and June	January 2014	66.6% of IAS	33.3% of IAL
C34	Core Mathematics 34	WMA02	IA2	1 written paper	January and June	January 2014	Not available for IAS award	33.3% of IAL
F1	Further Pure Mathematics 1	WFM01	IAS	1 written paper	January and June	January 2014	33.3% of IAS	16.7% of IAL
F2	Further Pure Mathematics 2	WFM02	IA2	1 written paper	June	June 2014	33.3% of IAS	16.7% of IAL
F3	Further Pure Mathematics 3	WFM03	IA2	1 written paper	June	June 2014	33.3% of IAS	16.7% of IAL
M1	Mechanics 1	WME01	IAS	1 written paper	January and June	January 2014	33.3% of IAS	16.7% of IAL
M2	Mechanics 2	WME02	IA2	1 written paper	January and June	January 2014	33.3% of IAS	16.7% of IAL
M3	Mechanics 3	WME03	IA2	1 written paper	January and June	January 2014	33.3% of IAS	16.7% of IAL
S1	Statistics 1	WST01	IAS	1 written paper	January and June	January 2014	33.3% of IAS	16.7% of IAL
S2	Statistics 2	WST02	IA2	1 written paper	January and June	January 2014	33.3% of IAS	16.7% of IAL
S3	Statistics 3	WST03	IA2	1 written paper	June	June 2014	33.3% of IAS	16.7% of IAL
D1	Decision Mathematics 1	WDM01	IAS	1 written paper	January and June	January 2014	33.3% of IAS	116.7% of IAL

*See *Appendix 2* for description of this code and all other codes relevant to this qualification.

Assessment objectives and weightings

		Minumum weighting in IAS	Minumum weighting in IA2	Minumum weighting in IAL
AO1	Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%	30%	30%
AO2	Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%	30%	30%
AO3	Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.	10%	10%	10%
AO4	Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.	5%	5%	5%
AO5	Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%	5%	5%

Relationship of assessment objectives to units

All figures in the following table are expressed as marks out of 125.

Unit number	Assessment objective				
	AO1	AO2	AO3	AO4	AO5
Unit C12	46–54	42–50	8–21	8–17	5–13
Unit C34	42–50	42–50	8–17	8–17	8–17

All figures in the following table are expressed as marks out of 75.

Unit number	Assessment objective				
	AO1	AO2	AO3	AO4	AO5
Unit F1	25–30	25–30	0–5	5–10	5–10
Unit F2	25–30	25–30	0–5	7–12	5–10
Unit F3	25–30	25–30	0–5	7–12	5–10
Unit M1	20–25	20–25	15–20	6–11	4–9
Unit M2	20–25	20–25	10–15	7–12	5–10
Unit M3	20–25	25–30	10–15	5–10	5–10
Unit S1	20–25	20–25	15–20	5–10	5–10
Unit S2	25–30	20–25	10–15	5–10	5–10
Unit S3	25–30	20–25	10–15	5–10	5–10
Unit D1	20–25	20–25	15–20	5–10	5–10

Qualification summary

Aims

The 12 units have been designed for schools and colleges to produce courses which will encourage students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment
- develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs
- extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected
- recognise how a situation may be represented mathematically and understand the relationship between 'real-world' problems and standard and other mathematical models and how these can be refined and improved
- use mathematics as an effective means of communication
- read and comprehend mathematical arguments and articles concerning applications of mathematics
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations
- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

IAS/IA2 knowledge and understanding and skills

The knowledge, understanding and skills required for all Mathematics specifications are contained in the subject core. The units C12 and C34 comprise this core material.

C Mathematics, Further Mathematics and Pure Mathematics unit content

Course structure

Students study a variety of units, following pathways to their desired qualification.

Students may study units leading to the following awards:

- International Advanced Subsidiary in Mathematics
- International Advanced Subsidiary in Further Mathematics
- International Advanced Subsidiary in Pure Mathematics
- International Advanced Level in Mathematics
- International Advanced Level in Further Mathematics
- International Advanced Level in Pure Mathematics.

Summary of awards:

International Advanced Subsidiary

Award	Compulsory units	Optional units
International Advanced Subsidiary in Mathematics	C12	M1, S1, D1
International Advanced Subsidiary in Further Mathematics	F1	Any*
International Advanced Subsidiary in Pure Mathematics	C12, F1	

*For International Advanced Subsidiary in Further Mathematics, excluded units are C12, C34.

International Advanced Level

Award	Compulsory units	Optional units
International Advanced Level in Mathematics	C12, C34	M1 and S1 or M1 and D1 or M1 and M2 or S1 and D1 or S1 and S2
International Advanced Level in Further Mathematics	F1 and either F2 or F3	Any*
International Advanced Level in Pure Mathematics	C12, C34, F1	F2 or F3

*For International Advanced Level in Further Mathematics, excluded units are C12, C34.

IAS/IAL combinations

Pearson Edexcel International Advanced Level in Mathematics

- The Pearson Edexcel International Advanced Level in Mathematics comprises four units.
- The International Advanced Subsidiary is the first half of the IAL course and comprises two units; Core Mathematics unit C12 plus one of the Applications units M1, S1 or D1.
- The full International Advanced Level award comprises four units; Core Mathematics units C12 and C34 plus two Applications units from the following five combinations: M1 and S1; M1 and D1; M1 and M2; S1 and D1; S1 and S2.
- The structure of this qualification allows teachers to construct a course of study which can be taught and assessed either as:
    - distinct modules of teaching and learning with related units of assessment taken at appropriate stages during the course; or
    - a linear course which is assessed in its entirety at the end.

Pearson Edexcel International Advanced Level in Further Mathematics

- The Pearson Edexcel International Advanced Level in Further Mathematics comprises six units.
- The International Advanced Subsidiary is the first half of the IAL course and comprises three units; Further Pure Mathematics unit F1 plus two other units (excluding C12, C34).
- The full International Advanced Level award comprises six units; Further Pure Mathematics units F1, F2, F3 and a further three Applications units (excluding C12, C34) to make a total of six units; **or** F1, either F2 or F3 and a further four Applications units (excluding C12, C34) to make a total of six units. Students who are awarded certificates in both International Advanced Level Mathematics and International Advanced Level Further Mathematics must use unit results from 10 different teaching modules.
- The structure of this qualification allows teachers to construct a course of study which can be taught and assessed either as:
    - distinct modules of teaching and learning with related units of assessment taken at appropriate stages during the course; or
    - a linear course which is assessed in its entirety at the end.

Pearson Edexcel International Advanced Level in Pure Mathematics

- The Pearson Edexcel International Advanced Level in Pure Mathematics comprises four units.
- The International Advanced Subsidiary is the first half of the IAL course and comprises two units; Core Mathematics unit C12 and F1.
- The full International Advanced Level award comprises four units; C12, C34, F1 and either F2 or F3.
- The structure of this qualification allows teachers to construct a course of study which can be taught and assessed either as:
    - distinct modules of teaching and learning with related units of assessment taken at appropriate stages during the course; or
    - a linear course which is assessed in its entirety at the end.

C12.1 Unit description

Algebra and functions; coordinate geometry in the (x, y) plane; sequences and series; exponentials and logarithms; trigonometry; differentiation; integration.

C12.2 Assessment information

Preamble

Construction and presentation of rigorous mathematical arguments through appropriate use of precise statements and logical deduction, involving correct use of symbols and appropriate connecting language is required. Students are expected to exhibit correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient', and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$.

Examination

The examination will consist of one $2\frac{1}{2}$ hour paper. It will contain 13 to 19 questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys: $+$, $-$, $\times$, $\div$, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, $\ln x$, e^x, $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Quadratic equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of logarithms

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a (x^k)$

Trigonometry

In the triangle ABC

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

area $= \frac{1}{2}ab \sin C$

Differentiation

function	derivative
x^n	nx^{n-1}

Integration

function	integral
x^n	$\dfrac{1}{n+1}x^{n+1} + c,\ n \neq -1$

Area

area under a curve $= \int_a^b y \, dx \ (y \geqslant 0)$

1. Algebra and functions

What students need to learn:

Laws of indices for all rational exponents.	The equivalence of $a^{m/n}$ and $\sqrt[n]{a^m}$ should be known.
Use and manipulation of surds.	Students should be able to rationalise denominators.
Quadratic functions and their graphs.	
The discriminant of a quadratic function.	
Completing the square. Solution of quadratic equations.	Solution of quadratic equations by factorisation, use of the formula and completing the square.
Simultaneous equations: analytical solution by substitution.	For example, where one equation is linear and one equation is quadratic.
Solution of linear and quadratic inequalities.	For example, $ax + b > cx + d$, $px^2 + qx + r \geqslant 0$, $px^2 + qx + r < ax + b$.
Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation.	Students should be able to use brackets. Factorisation of polynomials of degree n, $n \leqslant 3$, e.g. $x^3 + 4x^2 + 3x$. The notation $\mathrm{f}(x)$ may be used.
Simple algebraic division; use of the Factor Theorem and the Remainder Theorem.	Only division by $(x + a)$ or $(x - a)$ will be required. Students should know that if $\mathrm{f}(x) = 0$ when $x = a$, then $(x - a)$ is a factor of $\mathrm{f}(x)$. Students may be required to factorise cubic expressions such as $x^3 + 3x^2 - 4$ and $6x^3 + 11x^2 - x - 6$. Students should be familiar with the terms 'quotient' and 'remainder' and be able to determine the remainder when the polynomial $\mathrm{f}(x)$ is divided by $(ax + b)$.

Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations.	Functions to include simple cubic functions and the reciprocal function $y = \frac{k}{x}$ with $x \neq 0$. Knowledge of the term asymptote is expected. Also $y = a^x$ and its graph and trigonometric graphs – see section 4 and 5.
Knowledge of the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	Students should be able to apply one of these transformations to any of the above functions (quadratics, cubics, reciprocal, sine, cosine, tangent and power functions of the type $y = a^x$) and sketch the resulting graph. Given the graph of any function $y = \mathrm{f}(x)$ students should be able to sketch the graph resulting from one of these transformations.

2. Coordinate geometry in the (x, y) plane

What students need to learn:

Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$.	To include: (i) the equation of a line through two given points (ii) the equation of a line parallel (or perpendicular) to a given line through a given point. For example, the line perpendicular to the line $3x + 4y = 18$ through the point $(2, 3)$ has equation $y - 3 = \frac{4}{3}(x - 2)$.
Conditions for two straight lines to be parallel or perpendicular to each other.	
Coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$ and including use of the following circle properties: (i) the angle in a semicircle is a right angle; (ii) the perpendicular from the centre to a chord bisects the chord; (iii) the perpendicularity of radius and tangent.	Students should be able to find the radius and the coordinates of the centre of the circle given the equation of the circle, and vice versa.

3. Sequences and series

What students need to learn:

Sequences, including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = \mathrm{f}(x_n)$.	
Arithmetic series, including the formula for the sum of the first n natural numbers.	The general term and the sum to n terms of the series are required. The proof of the sum formula should be known. Understanding of Σ notation will be expected.
The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\lvert r \rvert < 1$.	The general term and the sum to n terms are required.
	The proof of the sum formula should be known.
Binomial expansion of $(1 + x)^n$ for positive integer n. The notations $n!$ and $\binom{n}{r}$.	Expansion of $(a + bx)^n$ may be required.

4. Exponentials and logarithms

What students need to learn:

$y = a^x$ and its graph.	
Laws of logarithms.	To include $\log_a xy \equiv \log_a x + \log_a y$, $\log_a \frac{x}{y} \equiv \log_a x - \log_a y$, $\log_a x^k \equiv k \log_a x$, $\log_a \frac{1}{x} \equiv -\log_a x$, $\log_a a = 1$
The solution of equations of the form $a^x = b$.	Students may use the change of base formula.

5. Trigonometry

What students need to learn:

The sine and cosine rules, and the area of a triangle in the form $\frac{1}{2}ab \sin C$.	
Radian measure, including use for arc length and area of sector.	Use of the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ for a circle.
Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.	Knowledge of graphs of curves with equations such as $y = 3 \sin x$, $y = \sin \left(x + \frac{\pi}{6}\right)$, $y = \sin 2x$ is expected.
Knowledge and use of $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$, and $\sin^2 \theta + \cos^2 \theta = 1$.	
Solution of simple trigonometric equations in a given interval.	Students should be able to solve equations such as $\sin\left(x + \frac{\pi}{2}\right) = \frac{3}{4}$ for $0 < x < 2\pi$, $\cos (x + 30°) = \frac{1}{2}$ for $-180° < x < 180°$, $\tan 2x = 1$ for $90° < x < 270°$, $6 \cos^2 x + \sin x - 5 = 0$, $0° \leqslant x < 360°$, $\sin^2\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$ for $-\pi \leqslant x < \pi$.

6. Differentiation

What students need to learn:

The derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.	For example, knowledge that $\frac{\mathrm{d}y}{\mathrm{d}x}$ is the rate of change of y with respect to x. Knowledge of the chain rule is not required. The notation $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$ may be used.
Differentiation of x^n, and related sums and differences.	For example, for $n \neq 1$, the ability to differentiate expressions such as $(2x + 5)(x - 1)$ and $\frac{x^2 + 5x - 3}{3x^{\frac{1}{2}}}$ is expected.
Applications of differentiation to gradients, tangents and normals.	Use of differentiation to find equations of tangents and normals at specific points on a curve.
Applications of differentiation to maxima and minima and stationary points, increasing and decreasing functions.	To include applications to curve sketching. Maxima and minima problems may be set in the context of a practical problem.

7. Integration

What students need to learn:

Indefinite integration as the reverse of differentiation.	Students should know that a constant of integration is required.
Integration of x^n.	For example, the ability to integrate expressions such as $\frac{1}{2}x^2 - 3x^{-\frac{1}{2}}$ and $\dfrac{(x+2)^2}{x^{\frac{1}{2}}}$ is expected. Given $f'(x)$ and a point on the curve, students should be able to find an equation of the curve in the form $y = f(x)$.
Evaluation of definite integrals.	
Interpretation of the definite integral as the area under a curve.	Students will be expected to be able to evaluate the area of a region bounded by a curve and given straight lines. E.g. find the finite area bounded by the curve $y = 6x - x^2$ and the line $y = 2x$. $\int x \, dy$ will not be required.
Approximation of area under a curve using the trapezium rule.	For example, evaluate $\int_0^1 \sqrt{(2x+1)}\, dx$ using the values of $\sqrt{(2x+1)}$ at $x = 0, 0.25, 0.5, 0.75$ and 1.

C34.1 Unit description

Algebra and functions; sequences and series; trigonometry; exponentials and logarithms; coordinate geometry in the (x, y) plane; differentiation; integration; numerical methods; vectors.

C34.2 Assessment information

Prerequisites and preamble

Prerequisites

A knowledge of the specification for C12, the preamble, prerequisites and associated formulae, is assumed and may be tested.

Preamble

Methods of proof, including proof by contradiction and disproof by counter-example, are required. At least one question on the paper will require the use of proof.

Examination

The examination will consist of one $2\frac{1}{2}$ hour paper. It will contain 10 to 15 questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Trigonometry

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\text{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$

Differentiation

function	derivative
$\sin kx$	$k \cos kx$
$\cos kx$	$-k \sin kx$
e^{kx}	$k\mathrm{e}^{kx}$
$\ln x$	$\dfrac{1}{x}$
$\mathrm{f}(x) + \mathrm{g}(x)$	$\mathrm{f}'(x) + \mathrm{g}'(x)$
$\mathrm{f}(x)\mathrm{g}(x)$	$\mathrm{f}'(x)\mathrm{g}(x) + \mathrm{f}(x)\mathrm{g}'(x)$
$\mathrm{f}(\mathrm{g}(x))$	$\mathrm{f}'(\mathrm{g}(x))\mathrm{g}'(x)$
a^x	$a^x \ln a$

Integration

function	integral
$\cos kx$	$\dfrac{1}{k} \sin kx + c$
$\sin kx$	$-\dfrac{1}{k} \cos kx + c$
e^{kx}	$\dfrac{1}{k} \mathrm{e}^{kx} + c$
$\dfrac{1}{x}$	$\ln \lvert x \rvert + c,\ x \neq 0$
$\mathrm{f}'(x) + \mathrm{g}'(x)$	$\mathrm{f}(x) + \mathrm{g}(x) + c$
$\mathrm{f}'(\mathrm{g}(x))\mathrm{g}'(x)$	$\mathrm{f}(\mathrm{g}(x)) + c$
a^x	$\dfrac{a^x}{\ln \mathrm{a}}$

Vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$

1. Algebra and functions

What students need to learn:

Simplification of rational expressions including factorising and cancelling, and algebraic division.	Denominators of rational expressions will be linear or quadratic, e.g. $\dfrac{1}{ax+b}$, $\dfrac{ax+b}{px^2+qx+r}$, $\dfrac{x^3+1}{x^2-1}$.
Rational functions. Partial fractions (denominators not more complicated than repeated linear terms).	Partial fractions to include denominators such as $(ax+b)(cx+d)(ex+f)$ and $(ax+b)(cx+d)^2$. The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions. Quadratic factors in the denominator such as (x^2+a), $a>0$, are *not* required.
Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.	The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $\mathrm{f}: x \mapsto$ and $\mathrm{f}(x)$ will be used. Students should know that fg will mean 'do g first, then f'. Students should know that if f^{-1} exists, then $\mathrm{f}^{-1}\mathrm{f}(x)=\mathrm{ff}^{-1}(x)=x$.
The modulus function.	Students should be able to sketch the graphs of $y=\lvert ax+b\rvert$ and the graphs of $y=\lvert\mathrm{f}(x)\rvert$ and $y=\mathrm{f}(\lvert x\rvert)$, given the graph of $y=\mathrm{f}(x)$.
Combinations of the transformations $y=\mathrm{f}(x)$ as represented by $y=a\mathrm{f}(x)$, $y=\mathrm{f}(x)+a$, $y=\mathrm{f}(x+a)$, $y=\mathrm{f}(ax)$.	Students should be able to sketch the graph of, for example, $y=2\mathrm{f}(3x)$, $y=\mathrm{f}(-x)+1$, given the graph of $y=\mathrm{f}(x)$ or the graph of, for example, $y=3+\sin 2x$, $y=-\cos\left(x+\dfrac{\pi}{4}\right)$. The graph of $y=\mathrm{f}(ax+b)$ will *not* be required.

2. Sequences and series

What students need to learn:

Binomial series for any rational n.	For $\lvert x\rvert < \dfrac{b}{a}$, students should be able to obtain the expansion of $(ax + b)^n$, and the expansion of rational functions by decomposition into partial fractions.

3. Trigonometry

What students need to learn:

Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.	Angles measured in both degrees and radians.
Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$.	
Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm a)$ or $r\sin(\theta \pm a)$.	To include application to half angles. Knowledge of the $t\ (\tan\frac{1}{2}\theta)$ formulae will *not* be required. Students should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval,and to prove simple identities such as $\cos x\cos 2x + \sin x\sin 2x \equiv \cos x$.

4. Exponentials and logarithms

What students need to learn:

The function e^x and its graph.	To include the graph of $y = \mathrm{e}^{ax+b} + c$.
The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.	Solution of equations of the form $\mathrm{e}^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.

5. Coordinate geometry in the (x, y) plane

What students need to learn:

Parametric equations of curves and conversion between cartesian and parametric forms.	Students should be able to find the area under a curve given its parametric equations. Students will not be expected to sketch a curve from its parametric equations.

6. Differentiation

What students need to learn:

Differentiation of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$ and their sums and differences.	
Differentiation using the product rule, the quotient rule and the chain rule.	Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard forms using products, quotients and composition, such as $2x^4 \sin x$, $\frac{e^{3x}}{x}$, $\cos x^2$ and $\tan^2 2x$.
The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.	E.g. finding $\frac{dy}{dx}$ for $x = \sin 3y$.
Differentiation of simple functions defined implicitly or parametrically.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.
Exponential growth and decay.	Knowledge and use of the result $\frac{d}{dx}(a^x) = a^x \ln a$ is expected.
Formation of simple differential equations.	Questions involving connected rates of change may be set.

7. Integration

What students need to learn:

Integration of e^x, $\frac{1}{x}$, $\sin x$, $\cos x$.	To include integration of standard functions such as $\sin 3x$, $\sec^2 2x$, $\tan x$, e^{5x}, $\frac{1}{2x}$. Students should recognise integrals of the form $\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c$. Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.
Evaluation of volume of revolution.	$\pi\int y^2\,dx$ is required, but *not* $\pi\int x^2\,dy$. Students should be able to find a volume of revolution, given parametric equations.
Simple cases of integration by substitution and integration by parts. These methods as the reverse processes of the chain and product rules respectively.	Except in the simplest of cases the substitution will be given. The integral $\int \ln x\,dx$ is required. More than one application of integration by parts may be required, for example $\int x^2 e^x\,dx$.
Simple cases of integration using partial fractions.	Integration of rational expressions such as those arising from partial fractions, e.g. $\frac{2}{3x+5}$, $\frac{3}{(x-1)^2}$. Note that the integration of other rational expressions, such as $\frac{x}{x^2+5}$ and $\frac{2}{(2x-1)^4}$ is also required (see above paragraphs).
Analytical solution of simple first order differential equations with separable variables.	General and particular solutions will be required.

8. Numerical methods

What students need to learn:

Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.	
Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$.	Solution of equations by use of iterative procedures for which leads will be given.
Numerical integration of functions.	Application of the trapezium rule to functions covered in C34. Use of increasing number of trapezia to improve accuracy and estimate error will be required. Questions will not require more than three iterations. Simpson's Rule is *not* required.

9. Vectors

What students need to learn:

Vectors in two and three dimensions.	
Magnitude of a vector.	Students should be able to find a unit vector in the direction of $\mathbf{a}$, and be familiar with $\lvert a \rvert$.
Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.	
Position vectors.	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.
The distance between two points.	The distance d between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$.
Vector equations of lines.	To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} - \mathbf{c})$. Intersection, or otherwise, of two lines.
The scalar product. Its use for calculating the angle between two lines.	Students should know that for $\overrightarrow{OA} = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a}\,.\,\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\cos \angle AOB = \dfrac{\mathbf{a}\,.\,\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$. Students should know that if $\mathbf{a}\,.\,\mathbf{b} = 0$, and $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors, then $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

Unit F1 Further Pure Mathematics 1

IAS compulsory unit for Further Mathematics and Pure Mathematics

Externally assessed

F1.1 Unit description

Complex numbers; roots of quadratic equations; numerical solution of equations; coordinate systems; matrix algebra; transformations using matrices; series; proof.

F1.2 Assessment information

Prerequisites

A knowledge of the specification for C12, its prerequisites, preambles and associated formulae, is assumed and may be tested.

It is also necessary for students:

- to have a knowledge of location of roots of $\mathrm{f}(x) = 0$ by considering changes of sign of $\mathrm{f}(x)$ in an interval in which $\mathrm{f}(x)$ is continuous
- to have a knowledge of rotating shapes through any angle about (0, 0)
- to be able to divide a cubic polynomial by a quadratic polynomial
- to be able to divide a quartic polynomial by a quadratic polynomial.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about nine questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, \mathrm{e}^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Roots of quadratic equations

For $ax^2 + bx + c = 0$: $\alpha + \beta = -\dfrac{b}{a}$, $\alpha\beta = \dfrac{c}{a}$

Series

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

1. Complex numbers

What students need to learn:

Definition of complex numbers in the form $a + \mathrm{i}b$ and $r\cos\theta + \mathrm{i}\,r\sin\theta$.	The meaning of conjugate, modulus, argument, real part, imaginary part and equality of complex numbers should be known.
Sum, product and quotient of complex numbers.	$\lvert z_1z_2\rvert = \lvert z_1\rvert\,\lvert z_2\rvert$ Knowledge of the result $\arg(z_1z_2) = \arg z_1 + \arg z_2$ is not required.
Geometrical representation of complex numbers in the Argand diagram.	
Geometrical representation of sums, products and quotients of complex numbers.	
Complex solutions of quadratic equations with real coefficients.	
Finding conjugate complex roots and a real root of a cubic equation with integer coefficients.	Knowledge that if z_1 is a root of $\mathrm{f}(z) = 0$ then z_1^* is also a root.

Finding conjugate complex roots and/or real roots of a quartic equation with real coefficients.	For example, (i) $\mathrm{f}(x) = x^4 - x^3 - 5x^2 + 7x + 10$ Given that $x = 2 + \mathrm{i}$ is a root of $\mathrm{f}(x) = 0$, use algebra to find the three other roots of $\mathrm{f}(x) = 0$. (ii) $\mathrm{g}(x) = x^4 - x^3 + 6x^2 + 14x - 20$ Given $\mathrm{g}(1) = 0$ and $\mathrm{g}(-2) = 0$, use algebra to solve $\mathrm{g}(x) = 0$ completely.

2. Roots of quadratic equations

What students need to learn:

Sum of roots and product of roots of a quadratic equation.	For the equation $ax^2 + bx + c = 0$, whose roots are α and β then $\alpha + \beta = -\dfrac{b}{a}$, $\alpha\beta = \dfrac{c}{a}$.
Manipulation of expressions involving the sum of roots and product of roots.	Knowledge of the identity $\alpha^3 + \beta^3 \equiv (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.
Forming quadratic equations with new roots.	For example, with roots α^3, β^3; $\dfrac{1}{\alpha}$, $\dfrac{1}{\beta}$; $\dfrac{1}{\alpha^2}$, $\dfrac{1}{\beta^2}$; $\alpha + \dfrac{2}{\beta}$, $\beta + \dfrac{2}{\alpha}$; etc.

3. Numerical solution of equations

What students need to learn:

Equations of the form $f(x) = 0$ solved numerically by: (i) interval bisection, (ii) linear interpolation, (iii) the Newton–Raphson process.	$f(x)$ will only involve functions used in C12. For the Newton-Raphson process, the only differentiation required will be as defined in unit C12.

4. Coordinate systems

What students need to learn:

Cartesian equations for the parabola and rectangular hyperbola.	Students should be familiar with the equations: $y^2 = 4ax$ or $x = at^2$, $y = 2at$ and $xy = c^2$ or $x = ct$, $y = \frac{c}{t}$.
Idea of parametric equation for parabola and rectangular hyperbola.	The idea of $(at^2, 2at)$ as a general point on the parabola is all that is required.
The focus-directrix property of the parabola.	Concept of focus and directrix and parabola as locus of points equidistant from focus and directrix.
Tangents and normals to these curves.	Differentiation of $y = 2a^{\frac{1}{2}}x^{\frac{1}{2}}$, $\quad y = \frac{c^2}{x}$. Parametric differentiation is not required.

5. Matrix algebra

What students need to learn:

Addition and subtraction of matrices.	
Multiplication of a matrix by a scalar.	
Products of matrices.	
Evaluation of 2×2 determinants.	Evaluation of 2×2 determinants.
Inverse of 2×2 matrices.	Use of the relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.

6. Transformations using matrices

What students need to learn:

Linear transformations of column vectors in two dimensions and their matrix representation.	The transformation represented by $\mathbf{AB}$ is the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$.
Applications of 2×2 matrices to represent geometrical transformations.	Identification and use of the matrix representation of single transformations from: reflection in coordinate axes and lines $y = \pm x$, rotation through any angle about $(0, 0)$, stretches parallel to the x-axis and y-axis, and enlargement about centre $(0, 0)$, with scale factor k, $(k \neq 0)$, where $k \in \mathbb{R}$.
Combinations of transformations.	Identification and use of the matrix representation of combined transformations.
The inverse (when it exists) of a given transformation or combination of transformations.	Idea of the determinant as an area scale factor in transformations.

7. Series

What students need to learn:

Summation of simple finite series.	Students should be able to sum series such as $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$, $\sum_{r=1}^{n} r(r^2+2)$. The method of differences is not required.

8. Proof

What students need to learn:

Proof by mathematical induction.

To include induction proofs for

(i) summation of series

e.g. show $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ or

$$\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$$

(ii) divisibility

e.g. show $3^{2n} + 11$ is divisible by 4.

(iii) finding general terms in a sequence

e.g. if $u_{n+1} = 3u_n + 4$ with $u_1 = 1$, prove that $u_n = 3n - 2$.

(iv) matrix products

e.g. show $\begin{pmatrix} -2 & -1 \\ 9 & -4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & -n \\ 9n & 3n+1 \end{pmatrix}$.

Unit F2 Further Pure Mathematics 2

IA2 optional unit for Further Mathematics and Pure Mathematics — Externally assessed

F2.1 Unit description

Inequalities; series; further complex numbers; first order differential equations; second order differential equations; Maclaurin and Taylor series; Polar coordinates.

F2.2 Assessment information

Prerequisites

A knowledge of the specifications for C12, C34 and F1, their prerequisites, preambles and associated formulae, is assumed and may be tested.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about eight questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Questions will be set in SI units and other units in common usage.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, \mathrm{e}^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

1. Inequalities

What students need to learn:

The manipulation and solution of algebraic inequalities and inequations, including those involving the modulus sign.	The solution of inequalities such as $\frac{1}{x-a} > \frac{x}{x-b}$, $\lvert x^2 - 1\rvert > 2(x+1)$.

2. Series

What students need to learn:

Summation of simple finite series using the method of differences.	Students should be able to sum series such as $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ by using partial fractions such as $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$.

3. Further complex numbers

What students need to learn:

Euler's relation $e^{i\theta} = \cos\theta + i\sin\theta$.	Students should be familiar with $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ and $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$.
De Moivre's theorem and its application to trigonometric identities and to roots of a complex number.	To include finding $\cos n\theta$ and $\sin m\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$ and also powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles. Students should be able to prove De Moivre's theorem for any integer n.
Loci and regions in the Argand diagram.	Loci such as $\lvert z-a\rvert = b$, $\lvert z-a\rvert = k\lvert z-b\rvert$, $\arg(z-a) = \beta$, $\arg\frac{z-a}{z-b} = \beta$ and regions such as $\lvert z-a\rvert \leqslant \lvert z-b\rvert$, $\lvert z-a\rvert \leqslant b$.
Elementary transformations from the z-plane to the w-plane.	Transformations such as $w = z^2$ and $w = \frac{az+b}{cz+d}$, where $a, b, c, d \in \mathbb{C}$, may be set.

4. First order differential equations

What students need to learn:

Further solution of first order differential equations with separable variables.	The formation of the differential equation may be required. Students will be expected to obtain particular solutions and also sketch members of the family of solution curves.
First order linear differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} + Py = Q$ where P and Q are functions of x.	The integrating factor $\mathrm{e}^{\int P\,\mathrm{d}x}$ may be quoted without proof.
Differential equations reducible to the above types by means of a given substitution.	

5. Second order differential equations

What students need to learn:

The linear second order differential equation $a\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\frac{\mathrm{d}y}{\mathrm{d}x} + cy = \mathrm{f}(x)$. where $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are real constants and the particular integral can be found by inspection or trial.	The auxiliary equation may have real distinct, equal or complex roots. $\mathrm{f}(x)$ will have one of the forms $k\mathrm{e}^{px}$, $A + Bx$, $p + qx + cx^2$ or $m \cos \omega x + n \sin \omega x$. Students should be familiar with the terms 'complementary function' and 'particular integral'. Students should be able to solve equations of the form $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = \sin 2x$.
Differential equations reducible to the above types by means of a given substitution.	

6. Maclaurin and Taylor series

What students need to learn:

Third and higher order derivatives.	
Derivation and use of Maclaurin series.	The derivation of the series expansion of e^x, $\sin x$, $\cos x$, $\ln(1 + x)$ and other simple functions may be required.
Derivation and use of Taylor series.	The derivation, for example, of the expansion of $\sin x$ in ascending powers of $(x - \pi)$ up to and including the term in $(x - \pi)^3$.
Use of Taylor series method for series solutions of differential equations.	Students may, for example, be required to find the solution in powers of x as far as the term in x^4,of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, such that $y = 1$, $\frac{dy}{dx} = 0$ at $x = 0$.

7. Polar coordinates

What students need to learn:

Polar coordinates (r, θ), $r \geqslant 0$.	The sketching of curves such as $\theta = \alpha$, $r = p \sec(\alpha - \theta)$, $r = a$, $r = 2a \cos\theta$, $r = k\theta$, $r = a(1 \pm \cos\theta)$, $r = a(3 + 2\cos\theta)$, $r = a\cos 2\theta$ and $r^2 = a^2 \cos 2\theta$ may be set.
Use of the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$ for area.	The ability to find tangents parallel to, or at right angles to, the initial line is expected.

Unit F3 Further Pure Mathematics 3

IA2 optional unit for Further Mathematics and Pure Mathematics — Externally assessed

F3.1 Unit description

Hyperbolic functions; further coordinate systems; differentiation; integration; vectors; further matrix algebra.

F3.2 Assessment information

Prerequisites

A knowledge of the specifications for C12, C34 and F1, their prerequisites, preambles and associated formulae, is assumed and may be tested.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about eight questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Questions will be set in SI units and other units in common usage.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, \mathrm{e}^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

1. Hyberbolic functions

What students need to learn:

Definition of the six hyperbolic functions in terms of exponentials. Graphs and properties of the hyperbolic functions.	For example, $\cosh x = \frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x})$, $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{\mathrm{e}^x + \mathrm{e}^{-x}}$. Students should be able to derive and use simple identities such as $\cosh^2 x - \sinh^2 x \equiv 1$ and $\cosh^2 x + \sinh^2 x \equiv \cosh 2x$. and to solve equations such as $a \cosh x + b \sinh x = c$.
Inverse hyperbolic functions, their graphs, properties and logarithmic equivalents.	E.g. $\operatorname{arsinh} x = \ln[x + \sqrt{(1 + x^2)}]$. Students may be required to prove this and similar results.

2. Further coordinate systems

What students need to learn:

Cartesian and parametric equations for the ellipse and hyperbola.	Extension of work from F1. Students should be familiar with the equations: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $x = a\cos t$, $y = b\sin t$. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $x = a\sec t$, $y = b\tan t$; $x = a\cosh t$, $y = b\sinh t$.
The focus-directrix properties of the ellipse and hyperbola, including the eccentricity.	For example, students should know that, for the ellipse, $b^2 = a^2(1 - e^2)$, the foci are $(ae, 0)$ and $(-ae, 0)$ and the equations of the directrices are $x = +\frac{a}{e}$ and $x = -\frac{a}{e}$.
Tangents and normals to these curves.	The condition for $y = mx + c$ to be a tangent to these curves is expected to be known.
Simple loci problems.	

3. Differentiation

What students need to learn:

Differentiation of hyperbolic functions and expressions involving them.	For example, $\tanh 3x$, $x\sinh^2 x$, $\frac{\cosh 2x}{\sqrt{(x+1)}}$.
Differentiation of inverse functions, including trigonometric and hyperbolic functions.	For example, $\arcsin x + x\sqrt{(1 - x^2)}$, $\frac{1}{2}\operatorname{artanh} x^2$.

4. Integration

What students need to learn:

Integration of hyperbolic functions and expressions involving them.	
Integration of inverse trigonometric and hyperbolic functions.	For example, $\int \text{arsinh}\, x\, \mathrm{d}x$, $\int \arctan x\, \mathrm{d}x$.
Integration using hyperbolic and trigonometric substitutions.	To include the integrals of $1/(a^2 + x^2)$, $1/\sqrt{(a^2 - x^2)}$, $1/\sqrt{(a^2 + x^2)}$, $1/\sqrt{(x^2 - a^2)}$.
Use of substitution for integrals involving quadratic surds.	In more complicated cases, substitutions will be given.
The derivation and use of simple reduction formulae.	Students should be able to derive formulae such as $nI_n = (n - 1)I_{n-2}$, $n \geqslant 2$, for $I_n = \int_0^{\frac{\pi}{2}} \sin^n x\, \mathrm{d}x$, $I_{n+2} = \dfrac{2\sin(n+1)x}{n+1} + I_n$ for $I_n = \int \dfrac{\sin nx}{\sin x}\, \mathrm{d}x$, $n > 0$.
The calculation of arc length and the area of a surface of revolution.	The equation of the curve may be given in cartesian or parametric form. Equations in polar form will not be set.

5. Vectors

What students need to learn:

The vector product $\mathbf{a} \times \mathbf{b}$ and the triple scalar product $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$.	The interpretation of $\lvert\mathbf{a} \times \mathbf{b}\rvert$ as an area and $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ as a volume.
Use of vectors in problems involving points, lines and planes.	Students may be required to use equivalent cartesian forms also.
The equation of a line in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$.	Applications to include (i) distance from a point to a plane, (ii) line of intersection of two planes, (iii) shortest distance between two skew lines.
The equation of a plane in the forms $\mathbf{r} \cdot \mathbf{n} = p$, $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$.	Students may be required to use equivalent cartesian forms also.

6. Further matrix algebra

What students need to learn:

Linear transformations of column vectors in two and three dimensions and their matrix representation.	Extension of work from F1 to 3 dimensions.
Combination of transformations. Products of matrices.	The transformation represented by $\mathbf{AB}$ is the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$.
Transpose of a matrix.	Use of the relation $(\mathbf{AB})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$.
Evaluation of $\mathbf{3} \times \mathbf{3}$ determinants.	Singular and non-singular matrices.
Inverse of $\mathbf{3} \times \mathbf{3}$ matrices.	Use of the relation $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.
The inverse (when it exists) of a given transformation or combination of transformations.	
Eigenvalues and eigenvectors of 2×2 and 3×3 matrices.	Normalised vectors may be required.
Reduction of symmetric matrices to diagonal form.	Students should be able to find an orthogonal matrix $\mathbf{P}$ such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP}$ is diagonal.

M1.1 Unit description

Mathematical models in mechanics; vectors in mechanics; kinematics of a particle moving in a straight line; dynamics of a particle moving in a straight line or plane; statics of a particle; moments.

M1.2 Assessment information

Prerequisites

A knowledge of C12, its preambles and associated formulae and of vectors in two dimensions.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about seven questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \dfrac{1}{x}, x^y, \ln x, e^x$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Students will be expected to know and be able to recall and use the following formulae:

Momentum $= mv$

Impulse $= mv - mu$

For constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

1. Mathematical models in mechanics

What students need to learn:

The basic ideas of mathematical modelling as applied in Mechanics.	Students should be familiar with the terms: particle, lamina, rigid body, rod (light, uniform, non-uniform), inextensible string, smooth and rough surface, light smooth pulley, bead, wire, peg. Students should be familiar with the assumptions made in using these models.

2. Vectors in mechanics

What students need to learn:

Magnitude and direction of a vector. Resultant of vectors may also be required.	Students may be required to resolve a vector into two components or use a vector diagram. Questions may be set involving the unit vectors $\mathbf{i}$ and $\mathbf{j}$.
Application of vectors to displacements, velocities, accelerations and forces in a plane.	Use of $\text{velocity} = \dfrac{\text{change of displacement}}{\text{time}}$ in the case of constant velocity, and of $\text{acceleration} = \dfrac{\text{change of velocity}}{\text{time}}$ in the case of constant acceleration, will be required.

3. Kinematics of a particle moving in a straight line

What students need to learn:

Motion in a straight line with constant acceleration.	Graphical solutions may be required, including displacement-time, velocity-time, speed-time and acceleration-time graphs. Knowledge and use of formulae for constant acceleration will be required.

4. Dynamics of a partivle moving in a straight line or plane

What students need to learn:

The concept of a force. Newton's laws of motion.	Simple problems involving constant acceleration in scalar form or as a vector of the form $a\mathbf{i} + b\mathbf{j}$.
Simple applications including the motion of two connected particles.	Problems may include (i) the motion of two connected particles moving in a straight line or under gravity when the forces on each particle are constant; problems involving smooth fixed pulleys and/or pegs may be set; (ii) motion under a force which changes from one fixed value to another, e.g. a particle hitting the ground; (iii) motion directly up or down a smooth or rough inclined plane.
Momentum and impulse. The impulse-momentum principle. The principle of conservation of momentum applied to two particles colliding directly.	Knowledge of Newton's law of restitution is not required. Problems will be confined to those of a one-dimensional nature.
Coefficient of friction.	An understanding of $\mathrm{F} = \mu R$ when a particle is moving.

5. Statics of a particle

What students need to learn:

Forces treated as vectors. Resolution of forces.	
Equilibrium of a particle under coplanar forces. Weight, normal reaction, tension and thrust, friction.	Only simple cases of the application of the conditions for equilibrium to uncomplicated systems will be required.
Coefficient of friction.	An understanding of $\mathrm{F} \leqslant \mu R$ in a situation of equilibrium.

6. Moments

What students need to learn:

Moment of a force.	Simple problems involving coplanar parallel forces acting on a body and conditions for equilibrium in such situations.

M2.1 Unit description

Kinematics of a particle moving in a straight line or plane; centres of mass; work and energy; collisions; statics of rigid bodies.

M2.2 Assessment information

Prerequisites

A knowledge of the specifications for C12, C34 and M1, and their prerequisites and associated formulae, is assumed and may be tested.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about seven questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Students will be expected to know and be able to recall and use the following formulae:

Kinetic energy $= \frac{1}{2}mv^2$

Potential energy $= mgh$

1. Kinematics of a particle moving in a straight line or plane

What students need to learn:

Motion in a vertical plane with constant acceleration, e.g. under gravity.	
Simple cases of motion of a projectile.	
Velocity and acceleration when the displacement is a function of time.	The setting up and solution of equations of the form $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{f}(t)$ or $\frac{\mathrm{d}v}{\mathrm{d}t} = g(t)$ will be consistent with the level of calculus in C12 and C34.
Differentiation and integration of a vector with respect to time.	For example, given that $\mathbf{r} = t^2\mathbf{i} + t^{3/2}\mathbf{j}$, find $\dot{\mathbf{r}}$ and $\ddot{\mathbf{r}}$ at a given time.

2. Centres of mass

What students need to learn:

Centre of mass of a discrete mass distribution in one and two dimensions.	
Centre of mass of uniform plane figures, and simple cases of composite plane figures.	The use of an axis of symmetry will be acceptable where appropriate. Use of integration is not required. Figures may include the shapes referred to in the formulae book. Results given in the formulae book may be quoted without proof.
Simple cases of equilibrium of a plane lamina.	The lamina may (i) be suspended from a fixed point; (ii) free to rotate about a fixed horizontal axis; (iii) be put on an inclined plane.

3. Work and energy

What students need to learn:

Kinetic and potential energy, work and power. The work-energy principle. The principle of conservation of mechanical energy.	Problems involving motion under a constant resistance and/or up and down an inclined plane may be set.

4. Collisions

What students need to learn:

Momentum as a vector. The impulse-momentum principle in vector form. Conservation of linear momentum.	
Direct impact of elastic particles. Newton's law of restitution. Loss of mechanical energy due to impact.	Students will be expected to know and use the inequalities $0 \leqslant e \leqslant 1$ (where e is the coefficient of restitution).
Successive impacts of up to three particles or two particles and a smooth plane surface.	Collision with a plane surface will not involve oblique impact.

5. Statics of rigid bodies

What students need to learn:

Moment of a force.	
Equilibrium of rigid bodies.	Problems involving parallel and non-parallel coplanar forces. Problems may include rods or ladders resting against smooth or rough vertical walls and on smooth or rough ground.

M3.1 Unit description

Further kinematics; elastic strings and springs; further dynamics; motion in a circle; statics of rigid bodies.

M3.2 Assessment information

Prerequisites

A knowledge of the specifications for C12, C34, M1 and M2, and their prerequisites and associated formulae, is assumed and may be tested.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about seven questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Students will be expected to know and be able to recall and use the following formulae:

The tension in an elastic string $= \dfrac{\lambda x}{l}$.

The energy stored in an elastic string $= \dfrac{\lambda x^2}{2l}$.

For SHM:

$\ddot{x} = -\omega^2 x$,

$x = a\cos\omega t$ or $x = a\sin\omega t$,

$v^2 = \omega^2(a^2 - x^2)$,

$T = \dfrac{2\pi}{\omega}$

1. Further kinematics

What students need to learn:

Kinematics of a particle moving in a straight line when the acceleration is a function of the displacement (x), or time (t).	The setting up and solution of equations where $\dfrac{dv}{dt} = f(t)$, $v\dfrac{dv}{dx} = f(x)$, $\dfrac{dx}{dt} = f(x)$ or $\dfrac{dx}{dt} = f(t)$ will be consistent with the level of calculus required in units C12 and C34.

2. Elastic strings and springs

What students need to learn:

Elastic strings and springs. Hooke's law.	
Energy stored in an elastic string or spring.	Simple problems using the work-energy principle involving kinetic energy, potential energy and elastic energy.

3. Further dynamics

What students need to learn:

Newton's laws of motion, for a particle moving in one dimension, when the applied force is variable.	The solution of the resulting equations will be consistent with the level of calculus in units C12 and C34. Problems may involve the law of gravitation, i.e. the inverse square law.
Simple harmonic motion.	Proof that a particle moves with simple harmonic motion in a given situation may be required (i.e. showing that $\ddot{x} = -\omega^2 x$). Geometric or calculus methods of solution will be acceptable. Students will be expected to be familiar with standard formulae, which may be quoted without proof.
Oscillations of a particle attached to the end of an elastic string or spring.	Oscillations will be in the direction of the string or spring only.

4. Motion in a circle

What students need to learn:

Angular speed.	
Radial acceleration in circular motion. The forms $r\omega^2$ and $\frac{v^2}{r}$ are required.	
Uniform motion of a particle moving in a horizontal circle.	Problems involving the 'conical pendulum', an elastic string, motion on a banked surface, as well as other contexts, may be set.
Motion of a particle in a vertical circle.	

5. Statics of rigid bodies

What students need to learn:

Centre of mass of uniform rigid bodies and simple composite bodies.	The use of integration and/or symmetry to determine the centre of mass of a uniform body will be required.
Simple cases of equilibrium of rigid bodies.	To include (i) suspension of a body from a fixed point, (ii) a rigid body placed on a horizontal or inclined plane.

Unit S1 Statistics 1

IAS optional unit for Mathematics and Further Mathematics Externally assessed

S1.1 Unit description

Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.

S1.2 Assessment information

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about seven questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y, \ln x, e^x, x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Students will be expected to know and be able to recall and use the following formulae:

Mean $= \bar{x} = \dfrac{\Sigma x}{n}$ or $\dfrac{\Sigma fx}{\Sigma f}$

Standard deviation $= \sqrt{(\text{Variance})}$

Interquartile range $= \text{IQR} = \text{Q}_3 - \text{Q}_1$

$\text{P}(A') = 1 - \text{P}(A)$

For independent events A and B,

$\text{P}(B \mid A) = \text{P}(B), \text{P}(A \mid B) = \text{P}(A),$

$\text{P}(A \cap B) = \text{P}(A)\,\text{P}(B)$

$\text{E}(aX + b) = a\text{E}(X) + b$

$\text{Var}\,(aX + b) = a^2\,\text{Var}\,(X)$

Cumulative distribution function for a discrete random variable:

$$\text{F}(x_0) = \text{P}(X \leqslant x0) = \sum_{x \leqslant x0} \text{p}(x)$$

Standardised Normal Random Variable $Z = \dfrac{X - \mu}{\sigma}$

where $X \sim \text{N}\,(\mu, \sigma^2)$

1. Mathematical models in probability and statistics

What students need to learn:

The basic ideas of mathematical modelling as applied in probability and statistics.

2. Representation and summary of data

What students need to learn:

Histograms, stem and leaf diagrams, box plots.	Using histograms, stem and leaf diagrams and box plots to compare distributions. Back-to-back stem and leaf diagrams may be required. Drawing of histograms, stem and leaf diagrams or box plots will not be the direct focus of examination questions.
Measures of location — mean, median, mode.	Calculation of mean, mode and median, range and interquartile range will not be the direct focus of examination questions. Students will be expected to draw simple inferences and give interpretations to measures of location and dispersion. Significance tests will not be expected. Data may be discrete, continuous, grouped or ungrouped. Understanding and use of coding.
Measures of dispersion — variance, standard deviation, range and interpercentile ranges.	Simple interpolation may be required. Interpretation of measures of location and dispersion.
Skewness. Concepts of outliers.	Students may be asked to illustrate the location of outliers on a box plot. Any rule to identify outliers will be specified in the question.

3. Probability

What students need to learn:

Elementary probability.	
Sample space. Exclusive and complementary events. Conditional probability.	Understanding and use of $P(A') = 1 - P(A)$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A \cap B) = P(A)\,P(B \mid A)$.
Independence of two events.	$P(B \mid A) = P(B)$, $P(A \mid B) = P(A)$, $P(A \cap B) = P(A)\,P(B)$.
Sum and product laws.	Use of tree diagrams and Venn diagrams. Sampling with and without replacement.

4 Correlation and regression

What students need to learn:

Scatter diagrams. Linear regression.	Calculation of the equation of a linear regression line using the method of least squares. Students may be required to draw this regression line on a scatter diagram.
Explanatory (independent) and response (dependent) variables. Applications and interpretations.	Use to make predictions within the range of values of the explanatory variable and the dangers of extrapolation. Derivations will not be required. Variables other than x and y may be used. Linear change of variable may be required.
The product moment correlation coefficient, its use, interpretation and limitations.	Derivations and tests of significance will not be required.

5. Discrete random variables

What students need to learn:

The concept of a discrete random variable.	
The probability function and the cumulative distribution function for a discrete random variable.	Simple uses of the probability function $\mathrm{p}(x)$ where $\mathrm{p}(x) = \mathrm{P}(X = x)$. Use of the cumulative distribution function: $\mathrm{F}(x_0) = \mathrm{P}(X \leqslant x_0) = \sum_{x \leqslant x_0} \mathrm{p}(x)$.
Mean and variance of a discrete random variable.	Use of $\mathrm{E}(X)$, $\mathrm{E}(X^2)$ for calculating the variance of X. Knowledge and use of $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$, $\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$.
The discrete uniform distribution.	The mean and variance of this distribution.

6. The Normal distribution

What students need to learn:

The Normal distribution including the mean, variance and use of tables of the cumulative distribution function.	Knowledge of the shape and the symmetry of the distribution is required. Knowledge of the probability density function is not required. Derivation of the mean, variance and cumulative distribution function is not required. Interpolation is not necessary. Questions may involve the solution of simultaneous equations.

S2.1 Unit description

The Binomial and Poisson distributions; continuous random variables; continuous distributions; samples; hypothesis tests.

S2.2 Assessment information

Prerequisites

A knowledge of the specification for S1 and its prerequisites and associated formulae, together with a knowledge of differentiation and integration of polynomials, binomial coefficients in connection with the binomial distribution and the evaluation of the exponential function, is assumed and may be tested.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about seven questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+$, $-$, $\times$, $\div$, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, $\ln x$, e^x, $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Students will be expected to know and be able to recall and use the following formulae:

For the continuous random variable X having probability density function $\mathrm{f}(x)$,

$$\mathrm{P}(a < X \leqslant b) = \int_a^b \mathrm{f}(x)\,\mathrm{d}x.$$

$$\mathrm{f}(x) = \frac{\mathrm{dF}(x)}{\mathrm{d}x}.$$

1. The Binomial and Poisson distributions

What students need to learn:

The binomial and Poisson distributions.	Students will be expected to use these distributions to model a real-world situation and to comment critically on their appropriateness. Cumulative probabilities by calculation or by reference to tables. Students will be expected to use the additive property of the Poisson distribution – e.g. if the number of events per minute $\sim \mathrm{Po}(\lambda)$ then the number of events per 5 minutes $\sim \mathrm{Po}(5\lambda)$.
The mean and variance of the binomial and Poisson distributions.	No derivations will be required.
The use of the Poisson distribution as an approximation to the binomial distribution.	

2. Continuous random variables

What students need to learn:

The concept of a continuous random variable.	
The probability density function and the cumulative distribution function for a continuous random variable.	Use of the probability density function f(x), where $\mathrm{P}(a < X \leqslant b) = \int_a^b \mathrm{f}(x)\,\mathrm{d}x$. Use of the cumulative distribution function $\mathrm{F}(x_0) = \mathrm{P}(X \leqslant x_0) = \int_{-\infty}^{x_0} \mathrm{f}(x)\,\mathrm{d}x$. The formulae used in defining f(x) will be restricted to simple polynomials which may be expressed piecewise.
Relationship between density and distribution functions.	$\mathrm{f}(x) = \dfrac{\mathrm{dF}(x)}{\mathrm{d}x}$.
Mean and variance of continuous random variables.	
Mode, median and quartiles of continuous random variables.	

3. Continuous distributions

What students need to learn:

The continuous uniform (rectangular) distribution.	Including the derivation of the mean, variance and cumulative distribution function.
Use of the Normal distribution as an approximation to the binomial distribution and the Poisson distribution, with the application of the continuity correction.	

4. Hypothesis tests

What students need to learn:

Population, census and sample. Sampling unit, sampling frame.	Students will be expected to know the advantages and disadvantages associated with a census and a sample survey.
Concepts of a statistic and its sampling distribution.	
Concept and interpretation of a hypothesis test. Null and alternative hypotheses.	Use of hypothesis tests for refinement of mathematical models.
Critical region.	Use of a statistic as a test statistic.
One-tailed and two-tailed tests.	
Hypothesis tests for the parameter p of a binomial distribution and for the mean of a Poisson distribution.	Students are expected to know how to use tables to carry out these tests. Questions may also be set not involving tabular values. Tests on sample proportion involving the normal approximation will not be set.

S3.1 Unit description

Combinations of random variables; sampling; estimation, confidence intervals and tests; goodness of fit and contingency tables; regression and correlation.

S3.2 Assessment information

Prerequisites

A knowledge of the specifications for S1 and S2, and their prerequisites and associated formulae, is assumed and may be tested.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about seven questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+$, $-$, $\times$, $\div$, π, x^2, $\sqrt{x}$, $\frac{1}{x}$, x^y, $\ln x$, e^x, $x!$, sine, cosine and tangent and their inverses in degrees and decimals of a degree, and in radians; memory. Calculators with a facility for symbolic algebra, differentiation and/or integration are not permitted.

Formulae

Formulae which students are expected to know are given overleaf and these will not appear in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This section lists formulae that students are expected to remember and that will not be included in formulae booklets.

Students will be expected to know and be able to recall and use the following formulae:

$aX \pm bY \sim \mathrm{N}(a\mu_x \pm b\mu_y, a^2\sigma_x^{\ 2} + b^2\sigma_y^{\ 2})$ where X and Y are independent and $X \sim \mathrm{N}(\mu_x, \sigma_x^{\ 2})$ and $Y \sim \mathrm{N}(\mu_y, \sigma_y^{\ 2})$.

1. Combinations of random variables

What students need to learn:

Distribution of linear combinations of independent Normal random variables.	If $X \sim \mathrm{N}(\mu_x, \sigma_x^{\ 2})$ and $Y \sim \mathrm{N}(\mu_y, \sigma_y^{\ 2})$ independently, then $aX \pm bY \sim \mathrm{N}(a\mu_x \pm b\mu_y, a^2\sigma_x^{\ 2} + b^2\sigma_y^{\ 2})$. No proofs required.

2. Sampling

What students need to learn:

Methods for collecting data. Simple random sampling. Use of random numbers for sampling.	
Other methods of sampling: stratified, systematic, quota.	The circumstances in which they might be used. Their advantages and disadvantages.

3. Estimation, confidence intervals and tests

What students need to learn:

Concepts of standard error, estimator, bias.	The sample mean, $\bar{x}$, and the sample variance, $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, as unbiased estimates of the corresponding population parameters.

The distribution of the sample mean $\overline{X}$.	$\overline{X}$ has mean μ and variance $\dfrac{\sigma^2}{n}$. If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$. No proofs required.
Concept of a confidence interval and its interpretation.	Link with hypothesis tests.
Confidence limits for a Normal mean, with variance known.	Students will be expected to know how to apply the Normal distribution and use the standard error and obtain confidence intervals for the mean, rather than be concerned with any theoretical derivations.
Hypothesis tests for the mean of a Normal distribution with variance known.	Use of $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$.
Use of Central Limit theorem to extend hypothesis tests and confidence intervals to samples from non-Normal distributions. Use of large sample results to extend to the case in which the variance is unknown.	$\dfrac{\overline{X} - \mu}{S/\sqrt{n}}$ can be treated as $\mathrm{N}(0, 1)$ when n is large. A knowledge of the t-distribution is not required.
Hypothesis test for the difference between the means of two Normal distributions with variances known.	Use of $\dfrac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x} + \dfrac{\sigma_y^2}{n_y}}} \sim \mathrm{N}(0, 1)$.
Use of large sample results to extend to the case in which the population variances are unknown.	Use of $\dfrac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\dfrac{S_x^2}{n_x} + \dfrac{S_y^2}{n_y}}} \sim \mathrm{N}(0, 1)$. A knowledge of the t-distribution is not required.

4. Goodness of fit and contingency tables

What students need to learn:

The null and alternative hypotheses. The use of $\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ as an approximate χ^2 statistic.	Applications to include the discrete uniform, binomial, Normal, Poisson and continuous uniform (rectangular) distributions. Lengthy calculations will not be required.
Degrees of freedom.	Students will be expected to determine the degrees of freedom when one or more parameters are estimated from the data. Cells should be combined when $E_i < 5$. Yates' correction is not required.

5. Registration and correlation

What students need to learn:

Spearman's rank correlation coefficient, its use, interpretation and limitations.	Numerical questions involving ties will not be set. Some understanding of how to deal with ties will be expected.
Testing the hypothesis that a correlation is zero.	Use of tables for Spearman's and product moment correlation coefficients.

Unit D1 Decision Mathematics 1

IAS optional unit for Mathematics and Further Mathematics — Externally assessed

D1.1 Unit description

Algorithms; algorithms on graphs; the route inspection problem; critical path analysis; linear programming; matchings.

D1.2 Assessment information

Preamble

Students should be familiar with the terms defined in the glossary attached to this specification. Students should show clearly how an algorithm has been applied. Matrix representation will be required but matrix manipulation is not required. Students will be required to model and interpret situations, including cross-checking between models and reality.

Examination

The examination will consist of one $1\frac{1}{2}$ hour paper. It will contain about seven questions of varying length. The mark allocations per question will be stated on the paper. All questions should be attempted.

Calculators

Students are expected to have available a calculator with at least the following keys:

$+, -, \times, \div, \pi, x^2, \sqrt{x}, \frac{1}{x}, x^y$ and memory. Calculators with a facility for symbolic

algebra, differentiation and/or integration are not permitted.

Formulae

Students are expected to know any other formulae which might be required by the specification and which are not included in the booklet, *Mathematical Formulae including Statistical Formulae and Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

1. Algorithms

What students need to learn:

The general ideas of algorithms and the implementation of an algorithm given by a flow chart or text.	The order of an algorithm is not expected.
	Whenever finding the middle item of any list, the method defined in the glossary must be used.
Students should be familiar with bin packing, bubble sort, quick sort, binary search.	When using the quick sort algorithm, the pivot should be chosen as the middle item of the list.

2. Algorithms on graphs

What students need to learn:

The minimum spanning tree (minimum connector) problem. Prim's and Kruskal's (greedy) algorithm.	Matrix representation for Prim's algorithm is expected. Drawing a network from a given matrix and writing down the matrix associated with a network will be involved.
Dijkstra's algorithm for finding the shortest path.	

3. The route inspection problem

What students need to learn:

Algorithm for finding the shortest route around a network, travelling along every edge at least once and ending at the start vertex. The network will have up to four odd nodes.	Also known as the 'Chinese postman' problem. Students will be expected to use inspection to consider all possible pairings of odd nodes. (The application of Floyd's algorithm to the odd nodes is not required.)

4. Critcal path analysis

What students need to learn:

Modelling of a project by an activity network, from a precedence table.	Activity on arc will be used. The use of dummies is included. In a precedence network, precedence tables will only show immediate predecessors.
Completion of the precedence table for a given activity network.	
Algorithm for finding the critical path. Earliest and latest event times. Earliest and latest start and finish times for activities.	
Total float. Gantt (cascade) charts. Scheduling.	

5. Linear programming

What students need to learn:

Formulation of problems as linear programs.

Graphical solution of two variable problems using ruler and vertex methods.

Consideration of problems where solutions must have integer values.

6. Matchings

What students need to learn:

Use of bipartite graphs for modelling matchings. Complete matchings and maximal matchings.	Students will be required to use the maximum matching algorithm to improve a matching by finding alternating paths. No consideration of assignment is required.
Algorithm for obtaining a maximum matching.	

Glossary for D1

1. Algorithms

In a list containing N items the 'middle' item has position $[\frac{1}{2}(N+1)]$ if N is odd $[\frac{1}{2}(N+2)]$ if N is even, so that if $N = 9$, the middle item is the 5th and if $N = 6$ it is the 4th.

2. Algorithms on graphs

A **graph** G consists of points (**vertices** or **nodes**) which are connected by lines (**edges** or **arcs**).

A **subgraph** of G is a graph, each of whose vertices belongs to G and each of whose edges belongs to G.

If a graph has a number associated with each edge (usually called its **weight**) then the graph is called a **weighted graph** or **network**.

The **degree** or **valency** of a vertex is the number of edges incident to it. A vertex is **odd** (**even**) if it has **odd** (**even**) degree.

A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, and in which no vertex appears more than once.

A **cycle** (**circuit**) is a closed path, i.e. the end vertex of the last edge is the start vertex of the first edge.

Two vertices are **connected** if there is a path between them. A graph is **connected** if all its vertices are connected.

If the edges of a graph have a direction associated with them they are known as **directed edges** and the graph is known as a **digraph**.

A **tree** is a connected graph with no cycles.

A **spanning tree** of a graph G is a subgraph which includes all the vertices of G and is also a tree.

A **minimum spanning tree** (MST) is a spanning tree such that the total length of its arcs is as small as possible. (MST is sometimes called a **minimum connector**.)

A graph in which each of the n vertices is connected to every other vertex is called a **complete graph**.

4. Critical path analysis

The **total float** $F(i, j)$ of activity (i, j) is defined to be $F(i, j) = l_j - e_i -$ duration (i, j), where e_i is the earliest time for event i and l_j is the latest time for event j.

6. Matchings

A **bipartite graph** consists of two sets of vertices X and Y. The edges only join vertices in X to vertices in Y, not vertices within a set. (If there are r vertices in X and s vertices in Y then this graph is $K_{r,s}$.)

A **matching** is the pairing of some or all of the elements of one set, X, with elements of a second set, Y. If every member of X is paired with a member of Y the matching is said to be a **complete matching**.

D Assessment and additional information

Assessment information

Assessment requirements

For a summary of assessment requirements and assessment objectives, see *Section B: Specification overview*.

Entering candidates for the examinations for this qualification

Details of how to enter candidates for the examinations for this qualification can be found in the International Information Manual, copies of which are sent to all examinations officers. The information can also be found at www.edexcel.com/international

Resitting of units

There is one resit opportunity allowed for each unit prior to claiming certification for the qualification. The best available result for each contributing unit will count towards the final grade.

After certification all unit results may be reused to count towards a new award. Students may re-enter for certification only if they have retaken at least one unit.

Results of units are held in the Pearson unit bank and have a shelf life limited only by the shelf life of this specification.

Awarding and reporting

The IAS qualification will be graded and certificated on a five-grade scale from A to E. The full International Advanced Level will be graded on a six-point scale A* to E. Individual unit results will be reported.

A pass in an International Advanced Subsidiary subject is indicated by one of the five grades A, B, C, D, E of which grade A is the highest and grade E the lowest. A pass in an International Advanced Subsidiary subject is indicated by one of the six grades A*, A, B, C, D, E of which Grade A* is the highest and Grade E the lowest.

For International Advanced Level in Mathematics, A* will be awarded to candidates who have achieved grade A overall (at least 480 of the 600 maximum uniform mark) and at least 180 of the 200 maximum uniform mark for the C34 unit.

For International Advanced Level in Further Mathematics, A* will be awarded to candidates who have achieved a grade A overall (at least 480 of the 600 maximum uniform mark) and at least 270 of the 300 combined maximum uniform mark for their **best** three IA2 units (whether pure or application units).

For International Advanced Level in Pure Mathematics, A* will be awarded to candidates who have achieved a grade A overall (at least 480 of the 600 maximum uniform mark) and at least 270 of the 300 combined maximum uniform mark for their IA2 units.

Students whose level of achievement is below the minimum judged by Pearson to be of sufficient standard to be recorded on a certificate will receive an unclassified U result.

Grade descriptions

Grade descriptions indicate the level of attainment characteristic of grades A, C and E at International Advanced Level. See *Appendix 1* for the grade descriptions for this subject.

Unit results

The minimum uniform marks required for each grade for each unit:

Unit C12, C34

Unit grade	**A**	**B**	**C**	**D**	**E**
Maximum uniform mark = 200	**160**	**140**	**120**	**100**	**80**

Students who do not achieve the standard required for a grade E will receive a uniform mark in the range 0–79.

Unit F1, F2, F3, M1, M2, M3, S1, S2, S3, D1

Unit grade	**A**	**B**	**C**	**D**	**E**
Maximum uniform mark = 100	**80**	**70**	**60**	**50**	**40**

Students who do not achieve the standard required for a grade E will receive a uniform mark in the range 0–39.

Qualification results

The minimum uniform marks required for each grade:

International Advanced Subsidiary cash-in code XMA01, XFM01, XPM01

Qualification grade	A	B	C	D	E
Maximum uniform mark = 300	**240**	**210**	**180**	**150**	**120**

Students who do not achieve the standard required for a grade E will receive a uniform mark in the range 0–119.

International Advanced Level cash-in code YMA01, YFM01, YPM01

Qualification grade	A	B	C	D	E
Maximum uniform mark = 600	**480**	**420**	**360**	**300**	**240**

Students who do not achieve the standard required for a grade E will receive a uniform mark in the range 0–239.

For information on A*, see *Awarding and Reporting* section.

Language of assessment

Assessment of this specification will be available in English only. Assessment materials will be published in English only and all work submitted for examination and moderation must be produced in English.

Additional information

Malpractice	For up-to-date information on malpractice, please refer to the latest Joint Council for Qualifications (JCQ) Suspected Malpractice in Examinations and Assessments: Policies and Procedures document, available on the JCQ website: www.jcq.org.uk
Access arrangements and special requirements	Pearson's policy on access arrangements and special considerations for GCE, GCSE, IAL and Entry Level is designed to ensure equal access to qualifications for all students (in compliance with the Equality Act 2010) without compromising the assessment of skills, knowledge, understanding or competence. Please see the JCQ website (www.jcq.org.uk) for their policy on access arrangements, reasonable adjustments and special considerations. Please see our website (www.edexcel.com) for: ■ the forms to submit for requests for access arrangements and special considerations ■ dates for submissions of the forms.
Equality Act 2010	Please see our website (www.edexcel.com) for information on the Equality Act 2010.
Prior learning and progression	**Prior learning** Students who would benefit most from studying an International Advanced Level in Mathematics, Further Mathematics and Pure Mathematics are likely to have a Level 2 qualification such as a International GCSE in Mathematics at grades A* – C. Students embarking on IAS and IAL in Mathematics are expected to have covered all the material in the International GCSE Mathematics Higher Tier. This material is regarded as assumed background knowledge. **Progression** This qualification supports progression into further education, training or employment.
Combinations of entry	Only units achieved from this qualification may contribute to the certification of the International Advanced Subsidiary in Mathematics, Further Mathematics, Pure Mathematics, or the International Advanced Level in Mathematics, Further Mathematics, Pure Mathematics.

Conditions of dependency

The units in the areas of Core Mathematics, Mechanics and Statistics are consecutively numbered in order of study. The study of a unit is dependent on the study of all preceding units within that area of mathematics; for example, the study of C34 is dependent on the study of C12.

The exception is F2 and F3, which are independent units, their study is dependent on the study of C12, C34 and F1.

Students who wish to take IAS or IAL in Further Mathematics will be expected to have obtained (or to be obtaining concurrently) an IAS or IAL in Mathematics. Units that contribute to an award in Mathematics may not also be used for an award in Further Mathematics. Students who have obtained or who are in the process of obtaining IAL in Mathematics with an awarding body other than Pearson should contact the awarding body to check requirements for IAL Further Mathematics.

Student recruitment

Pearson's access policy concerning recruitment to our qualifications is that:

- they must be available to anyone who is capable of reaching the required standard
- they must be free from barriers that restrict access and progression
- equal opportunities exist for all students.

E Support, training and resources

Support

Pearson aim to provide the most comprehensive support for our qualifications. Here are just a few of the support services we offer:

- Subject Advisor – subject experts are on-hand to offer their expertise to answer any questions you may have on delivering the qualification and assessment.
- Subject Page – written by our Subject Advisors, the subject pages keep you up to date with the latest information on your subject.
- Subject Communities – exchange views and share information about your subject with other teachers.
- Training – see 'Training' below for full details.

For full details of all the teacher and student support provided by Pearson to help you deliver our qualifications, please visit www.edexcel.com/ial/maths/support

Training

Our programme of professional development and training courses, covering various aspects of the specification and examinations, are arranged each year on a regional basis. Pearson training is designed to fit you, with an option of face-to-face, online or customised training so you can choose where, when and how you want to be trained.

Face-to-face training
Our programmes of face-to-face training have been designed to help anyone who is interested in, or currently teaching, a Pearson Edexcel qualification. We run a schedule of events throughout the academic year to support you and help you to deliver our qualifications.

Online training
Online training is available for international centres who are interested in, or currently delivering, our qualifications. This delivery method helps us run training courses more frequently to a wider audience.

To find out more information or to book a place please visit:
www.edexcel.com/training

Alternatively, email internationaltfp@pearson.com or
telephone +44 (0) 44 844 576 0025

Resources

Pearson is committed to ensuring that teachers and students have a choice of resources to support their teaching and study.

Teachers and students can continue to use their existing GCE A level resources for International Advanced Levels.

To search for Pearson GCE resources, please visit www.pearsonschools.co.uk

To search for endorsed resources from other publishers, please visit www.edexcel.com/resources

Specifications, Sample Assessment Materials and Teacher Support Materials

Specifications, Sample Assessment Materials (SAMs) and Teacher Support Materials (TSMs) can be downloaded from the International Advanced Level subject pages.

To find a complete list of supporting documents, including the specification, SAMs and TSMs, please visit www.edexcel.com/ial/maths

F Appendices

Appendix 1 Grade descriptions

The following grade descriptions indicate the level of attainment characteristic of grades A, C and E at International Advanced Level. They give a general indication of the required learning outcomes at the specified grades. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

Grade A

Students recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Students manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Students recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model.

Students comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical information from extended pieces of prose having mathematical content. They can comment meaningfully on the mathematical information.

Students make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Grade C

Students recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Students manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Students recall or recognise most of the standard models that are needed and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation; they sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model.

Students comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical content. They give some useful comments on this mathematical information.

Students usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Grade E

Students recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Students manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Students recall or recognise some of the standard models that are needed and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation.

Students sometimes comprehend or understand the meaning of translations in mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical content. They attempt to comment on this mathematical information.

Students often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

Appendix 1 Grade descriptions

Appendix 2 Codes

Type of code	Use of code	Code number
Unit codes	Each unit is assigned a unit code. This unit code is used as an entry code to indicate that a student wishes to take the assessment for that unit. Centres will need to use the entry codes only when entering students for their examination.	Please see *Summary of assessment requirements*
Cash-in codes	The cash-in code is used as an entry code to aggregate the student's unit scores to obtain the overall grade for the qualification. Centres will need to use the entry codes only when entering students for their qualification.	IAS in Mathematics (XMA01) IAS in Further Mathematics (XFM01) IAS in Pure Mathematics (XPM01) IAL in Mathematics (YMA01) IAL in Further Mathematics (YFM01) IAL in Pure Mathematics (YPM01)
Entry codes	The entry codes are used to: 1 enter a student for the assessment of a unit 2 aggregate the student's unit scores to obtain the overall grade for the qualification.	Please refer to the *Pearson Information Manual*, available on the Edexcel website.

Appendix 3 Notation

The following notation will be used in all mathematics examinations:

1 Set notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that …
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	the universal set
A'	the complement of the set A
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \notin \mathbb{Z}^+\right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0{}^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geqslant 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0{}^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geqslant 0\}$
$\mathbb{C}$	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B, i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval, $\{x \in \mathbb{R} : a \leqslant x \leqslant b\}$
$[a, b), [a, b[$	the interval $\{x \in \mathbb{R} : a \leqslant x < b\}$
$(a, b],]a, b]$	the interval $\{x \in \mathbb{R} : a < x \leqslant b\}$

(a, b), $]a, b[$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y \, R \, x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2 Miscellaneous symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leqslant$, $\ngtr$	is less than or equal to, is not greater than
$>$	is greater than
$\geqslant$, $<$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

3 Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$

$\sqrt{a}$	the positive square root of a
$\lvert a \rvert$	the modulus of a
$n!$	n factorial
$\binom{n}{r}$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$ for $n \in \mathbb{Z}^+$ $\dfrac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

4 Functions

$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f} : A \rightarrow B$	f is a function under which each element of set A has an image in set B
$\mathrm{f} : x \rightarrow y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx, δx	an increment of x
$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	the first, second, ... , nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\dfrac{\partial V}{\partial x}$	the partial derivative of V with respect to x
$\dot{x}, \ddot{x}, \ldots$	the first, second, ... derivatives of x with respect to t

5 Exponential and logarithmic functions

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$, $\log_\mathrm{e} x$	natural logarithm of x
$\lg x$, $\log_{10} x$	logarithm of x to base 10

6	**Circular and hyperbolic functions**	
	sin, cos, tan, cosec, sec, cot	the circular functions
	arcsin, arccos, arctan, arccosec, arcsec, arccot	the inverse circular functions
	sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions
	arsinh, arcosh, artanh, arcosech, arsech, arcoth	the inverse hyperbolic functions
7	**Complex numbers**	
	i, j	square root of -1
	z	a complex number, $z = x + \mathrm{i}y$
	$\mathrm{Re}\, z$	the real part of z, $\mathrm{Re}\, z = x$
	$\mathrm{Im}\, z$	the imaginary part of z, $\mathrm{Im}\, z = y$
	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{(x^2 + y^2)}$
	$\arg z$	the argument of z, $\arg z = \theta$, $-\pi < \theta \leqslant \pi$
	z^*	the complex conjugate of z, $x - \mathrm{i}y$
8	**Matrices**	
	$\mathbf{M}$	a matrix $\mathbf{M}$
	$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
	$\det \mathbf{M}$ or $\lvert\mathbf{M}\rvert$	the determinant of the square matrix $\mathbf{M}$
9	**Vectors**	
	$\mathbf{a}$	the vector $\mathbf{a}$
	$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
	$\lvert\mathbf{a}\rvert$, a	the magnitude of $\mathbf{a}$
	$\lvert\overrightarrow{AB}\rvert$, AB	the magnitude of $\overrightarrow{AB}$
	$\mathbf{a} \cdot \mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
	$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10 Probability and statistics

A, B, C, etc	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A
$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
X, Y, R, etc	random variables
x, y, r, etc	values of the random variables X, Y, R, etc
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\mathrm{p}(x)$	probability function $\mathrm{P}(X = x)$ of the discrete random variable X
p_1, p_2	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
$\mathrm{f}(x), \mathrm{g}(x)$	the value of the probability density function of a continuous random variable X
$\mathrm{F}(x), \mathrm{G}(x)$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of a continuous random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}\,(X)$	variance of the random variable X
$\mathrm{G}(t)$	probability generating function for a random variable which takes the values $0, 1, 2, \ldots$
$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}, m$	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \dfrac{1}{n-1}\sum(x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$

Φ	corresponding cumulative distribution function
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
$\mathrm{Cov}(X, Y)$	covariance of X and Y

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