

# Contents

**WJEC Advanced Subsidiary GCE Mathematics  
and Related Subjects  
WJEC Advanced GCE in Mathematics  
and Related Subjects**

## **Examinations from 2009**



	<b>Page</b>
<b>Availability of Units</b>	<b>2</b>
<b>Summary of Assessment</b>	<b>3</b>
<b>Introduction</b>	<b>5</b>
<b>Aims</b>	<b>5</b>
<b>Specification Content</b>	<b>6</b>
<b>Key Skills</b>	<b>33</b>
<b>Assessment Objectives</b>	<b>33</b>
<b>Scheme of Assessment</b>	<b>35</b>
<b>Grade Descriptions</b>	<b>40</b>

Availability of Units	
January	June
C1, C2, C3, FP1, M1, S1	All units

**Qualification Accreditation Numbers**

**Advanced Subsidiary in Mathematics: 100/3423/7**  
**Advanced Subsidiary in Further Mathematics: 100/6024/8**  
**Advanced Subsidiary in Pure Mathematics: 100/6025/X**

**Advanced in Mathematics: 100/3424/9**  
**Advanced in Further Mathematics: 100/6026/1**  
**Advanced in Pure Mathematics: 100/6027/3**

## SUMMARY OF ASSESSMENT

This is a modular specification.

Candidates for an Advanced Subsidiary GCE qualification are required to sit three units.

Candidates for an Advanced GCE qualification are required to sit six units.

### Units Available

C1 Pure Mathematics 1

C2 Pure Mathematics 2

C3 Pure Mathematics 3

C4 Pure Mathematics 4

FP1 Further Pure Mathematics 1

FP2 Further Pure Mathematics 2

FP3 Further Pure Mathematics 3

M1 Mechanics 1

M2 Mechanics 2

M3 Mechanics 3

S1      Statistics 1

S2 Statistics 2

### S3 Statistics 3

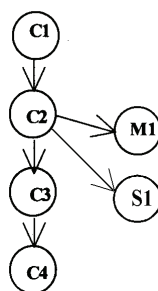
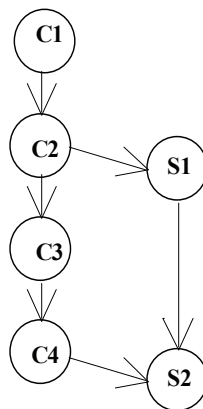
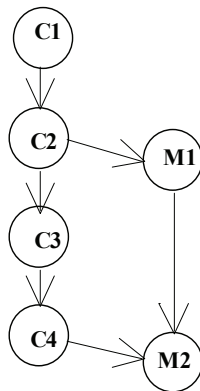
## Awards Available

WJEC Advanced Subsidiary GCE in: Mathematics  
Further Mathematics  
Pure Mathematics

WJEC Advanced GCE in:

	Mathematics
	Further Mathematics
	Pure Mathematics

The schematic diagrams below show the different routes of progress through the units which could lead to an Advanced GCE in Mathematics.



# MATHEMATICS and Related Subjects

## **1** INTRODUCTION

The Advanced Subsidiary (AS) and Advanced Level (AL) specifications in Mathematics have been designed to respond to changes in the post-16 curriculum following the Dearing Report on 16-19 qualifications. These specifications comply with the criteria for AS/AL Examinations published by ACCAC/QCA. Assessment for these qualifications will be carried out according to the code of practice published by the regulatory authorities. The assessment will be available in English and Welsh.

The specifications will:

- (a) provide a complete course in Mathematics for those who do not wish to proceed further in the subject;
- (b) provide a firm mathematical foundation for those who wish to proceed to further study or to employment;
- (c) complement other studies and provide support for those who are taking AS/AL courses in other subjects.

## **2** AIMS

The aims of the specification are to enable centres to provide courses in Mathematics which will encourage candidates to:

- (a) develop their understanding of Mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment;
- (b) develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs;
- (c) extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems;
- (d) develop an understanding of coherence and progression in Mathematics and of how different areas of Mathematics can be connected;
- (e) recognise how a situation may be represented mathematically and understand the relationship between 'real world' problems and standard and other mathematical models and how these can be refined and improved;

- (f) use Mathematics as an effective means of communication;
- (g) read and comprehend mathematical arguments and articles concerning applications of Mathematics;
- (h) acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations;
- (i) develop an awareness of the relevance of Mathematics to other fields of study, to the world of work and to society in general;
- (j) take increasing responsibility for their own learning and the evaluation of their own mathematical development.

This specification will enable centres to provide courses in Mathematics that will allow candidates to discriminate between truth and falsehood. The mathematical models of the real world will naturally raise moral and cultural issues, environmental and safety considerations and European developments for discussion during the course. Understanding of these issues will **not** be assessed on the examination.

### 3

## SPECIFICATION CONTENT

Mathematics is, inherently, a sequential subject. There is a progression of material through all levels at which the subject is studied. The specification content therefore builds on the knowledge, understanding and skills established in GCSE Mathematics.

### Knowledge, Understanding and Skills

The following requirements concerning **proof** will pervade the content material of those units which lead to an award entitled Mathematics.

These AS and AL specifications will require:

- (a) construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language;
- (b) correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient', and notation such as  $\therefore$ ,  $\Rightarrow$ ,  $\Leftarrow$  and  $\Leftrightarrow$ .

In addition, the AL specification will require:

- (c) methods of proof, including proof by contradiction and disproof by counter-example.

## Unit C1

### Pure Mathematics 1

Topics	Notes
1. Laws of indices for all rational exponents.	
Use and manipulation of surds.	To include simplification of fractions such as $\frac{2 + \sqrt{5}}{3 - \sqrt{5}}$ .
Quadratic functions and their graphs. The discriminant of a quadratic function. Completing the square. Solution of quadratic equations.	The nature of the roots of a quadratic equation. Condition for real roots and equal roots. To include finding the maximum or minimum value of a quadratic function.
Simultaneous equations: analytical solution by substitution, e.g. one linear and one quadratic.	To include finding the points of intersection or the point of contact of a line and a curve
Solution of linear and quadratic inequalities.	To include the solution of inequalities such as $1 - 2x < 4$ and $x^2 - 6x + 8 \geq 0$ .
Graphs of functions; sketching curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of curves to solve equations.	The equations will be restricted to the form $y = f(x)$ .
Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$ , $y = f(x) + a$ , $y = f(x + a)$ , $y = f(ax)$ .	
2. Equation of a straight line, including the forms $y = mx + c$ , $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$ . Conditions for two straight lines to be parallel or perpendicular to each other.	To include finding the gradient, equation, length and mid-point of a line joining two given points. To include finding the equations of lines which are parallel or perpendicular to a given line.
3. Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the Factor Theorem and the Remainder Theorem.	The use of the Factor Theorem and Remainder Theorem will be restricted to cubic polynomials and the solution of cubic equations.
Binomial expansion of $(1+x)^n$ for positive integer $n$ . The notations $n!$ and $\binom{n}{r}$ .	Binomial expansion of $(a+b)^n$ for positive integer $n$ is also required.

Topics	Notes
<p>4. The derivative of <math>f(x)</math> as the gradient of the tangent to the graph of <math>y = f(x)</math> at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.</p>	<p>To include finding from first principles the derivative of a polynomial of degree less than 3. The notations <math>\frac{dy}{dx}</math> or <math>f'(x)</math> may be used.</p>
<p>Differentiation of <math>x^n</math> and related sums and differences.</p>	<p>To include polynomials.</p>
<p>Application of differentiation to gradients, tangents and normals, maxima and minima, and stationary points, increasing and decreasing functions.</p>	<p>Equations of tangents and normals. The use of maxima and minima in simple optimisation problems. To include stationary points of inflection and simple curve sketching.</p>



## Unit C2

### Pure Mathematics 2

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Unit C1.

Topics	Notes
<p>1. Sequences, including those given by a formula for the <math>n</math>th term and those generated by a simple relation of the form <math>x_{n+1} = f(x_n)</math>.</p> <p>Arithmetic series. The sum of a finite arithmetic series. The sum of the first <math>n</math> natural numbers.</p> <p>Geometric series. The sum of a finite geometric series. The sum to infinity of a convergent geometric series.</p>	<p>Use and proof of <math>S_n = \frac{n}{2}[2a + (n-1)d]</math> and <math>S_n = \frac{n}{2}[a + l]</math>.</p> <p>Use and proof of <math>S_n = \frac{a(1-r^n)}{1-r}</math>.</p> <p>Use of <math>S_\infty = \frac{a}{1-r}</math> for <math> r  &lt; 1</math>.</p> <p>The <math>\Sigma</math> notation.</p>
<p>2. <math>y = a^x</math> and its graph.</p> <p>Laws of logarithms.</p> <p><math>\log_a x + \log_a y = \log_a (xy)</math></p> <p><math>\log_a x - \log_a y = \log_a (x/y)</math></p> <p><math>k \log_a x = \log_a (x^k)</math></p> <p>The solution of equations in the form <math>a^x = b</math>.</p>	<p>Use of the result that <math>y = a^x</math> implies <math>x = \log_a y</math>.</p> <p>Proof of the laws of logarithms.</p> <p>Use of the laws of logarithms.</p> <p>e.g. Simplify <math>\log_2 36 - 2\log_2 15 + \log_2 100</math>.</p> <p>Change of base will not be required.</p> <p>The use of a calculator to solve equations such as (i) <math>3^x = 2</math>, (ii) <math>25^x - 4 \times 5^x + 3 = 0</math>.</p>

Topics	Notes
<p>3. Coordinate geometry of the circle using the equation of a circle in the form <math>(x - a)^2 + (y - b)^2 = r^2</math> (and in the form <math>x^2 + y^2 + 2gx + 2fy + c = 0</math>), and including use of the following circle properties:</p> <ul style="list-style-type: none"> <li>(i) the angle in a semicircle is a right-angle;</li> <li>(ii) the perpendicular from the centre to a chord bisects the chord;</li> <li>(iii) the perpendicularity of radius and tangent.</li> </ul>	<p>Equations of tangents. Condition for two circles to touch internally or externally. To include finding the points of intersection or the point of contact of a line and a circle.</p>
<p>4. The sine and cosine rules, and the area of a triangle in the form <math>\frac{1}{2} ab \sin C</math>.</p> <p>Radian measure. Arc length, area of sector and area of segment.</p> <p>Sine, cosine and tangent functions. Their graphs, symmetries and periodicity.</p> <p>Knowledge and use of <math>\tan \theta = \frac{\sin \theta}{\cos \theta}</math></p> <p>and <math>\cos^2 \theta + \sin^2 \theta = 1</math>.</p> <p>Solution of simple trigonometric equations in a given interval.</p>	<p>To include the use of the sine rule in the ambiguous case. Use of the exact values of the sine, cosine and tangent of <math>30^\circ</math>, <math>45^\circ</math> and <math>60^\circ</math>.</p> <p>To include the solution of equations such as <math>3 \sin \theta = 1</math>, <math>\tan \frac{\theta}{2} = \sqrt{3}</math> and <math>2 \cos^2 \theta + \sin \theta - 1 = 0</math>.</p>
<p>5. Indefinite integration as the reverse of differentiation.</p> <p>Integration of <math>x^n</math> (<math>n \neq -1</math>).</p> <p>Approximation of area under a curve using the trapezium rule. Interpretation of the definite integral as the area under a curve. Evaluation of definite integrals.</p>	<p>Including sums, differences and polynomials.</p> <p>No consideration of error terms will be required in the examination.</p> <p>To include finding the area of a region between a straight line and a curve.</p>

## Unit C3

### Pure Mathematics 3

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1 and C2

Topics	Notes
<p>1. Knowledge of secant, cosecant and cotangent and of <math>\sin^{-1}</math>, <math>\cos^{-1}</math> and <math>\tan^{-1}</math>. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.</p> <p>Knowledge and use of <math>\sec^2 \theta \equiv 1 + \tan^2 \theta</math> and <math>\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta</math>.</p>	<p>Including their derivatives.</p> <p>Questions aimed <b>solely</b> at proving identities will not be set. The solution of trigonometric equations such as <math>\sec^2 \theta + 5 = 5 \tan \theta</math>.</p>
<p>2. Definition of a function. Domain and range of functions. Composition of functions.</p> <p>Inverse functions and their graphs.</p> <p>The modulus function.</p> <p>Combinations of the transformations on the graph of <math>y = f(x)</math> as represented by <math>y = af(x)</math>, <math>y = f(x) + a</math>, <math>y = f(x + a)</math> and <math>y = f(ax)</math>.</p>	<p>In the case of a function defined by a formula (with unspecified domain) the domain is taken to be the largest set such that the formula gives a unique image for each element of the set. The notation <math>fg</math> will be used for composition.</p> <p>To include solution of inequations such as <math> x - 3  \geq 5</math>.</p>
<p>3. The function <math>e^x</math> and its graph.</p> <p>The function <math>\ln x</math> and its graph; <math>\ln x</math> as the inverse function of <math>e^x</math>.</p>	
<p>4. Differentiation of <math>e^x</math>, <math>\ln x</math>, <math>\sin x</math>, <math>\cos x</math>, <math>\tan x</math>, and their sums and differences.</p> <p>Differentiation using the product rule, the quotient rule and the chain rule and by the use of <math>\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}</math>.</p> <p>Differentiation of simple functions defined implicitly.</p> <p>Differentiation of simple functions defined parametrically.</p>	<p>Derivatives of <math>\sin^{-1}x</math>, <math>\cos^{-1}x</math> and <math>\tan^{-1}x</math>.</p> <p>To include second derivatives.</p>

Topics	Notes
5. Integration of $e^x$ , $\frac{1}{x}$ , $\sin x$ , $\cos x$ .	Use of the result: if $\int f(x)dx = F(x) + k$ then $\int f(ax + b)dx = \frac{1}{a}F(ax + b) + c$ .
6. Location of roots of $f(x) = 0$ by considering changes in sign of $f(x)$ in an interval of $x$ in which $f(x)$ is continuous.  Approximate solutions of equations using simple iterative methods. Sequences generated by a simple recurrence relation of the form $x_{n+1} = f(x_n)$ .  Numerical integration of functions.	The iterative formula will be given. Consideration of the conditions for convergence will <b>not</b> be required.  Simpson's Rule. No consideration of error terms will be required in the examination.

## Unit C4

### Pure Mathematics 4

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1, C2 and C3.

Topics	Notes
<p>1. Binomial series for any rational <math>n</math>.</p> <p>Simplification of rational expressions including factorising and cancelling, and algebraic division.</p> <p>Rational functions. Partial fractions (denominators not more complicated than repeated linear terms).</p>	<p>To include the expansion, in ascending powers of <math>x</math>, of expressions such as</p> $(2-x)^{\frac{1}{2}} \quad \text{and} \quad \frac{(4-x)^{\frac{3}{2}}}{(1+2x)},$ <p>including the condition for convergence.</p> <p>With denominators of the form <math>(ax+b)(cx+d)</math>, and <math>(ax+b)(cx+d)^2</math>. The degree of the numerator will be less than the degree of the denominator. Candidates will <b>not</b> be expected to sketch the graphs of rational functions.</p>
<p>2. Knowledge and use of formulae for <math>\sin(A \pm B)</math>, <math>\cos(A \pm B)</math> and <math>\tan(A \pm B)</math>; of double angle formulae; and of expressions for <math>a\cos\theta + b\sin\theta</math> in the equivalent forms <math>r\cos(\theta \pm \alpha)</math> or <math>r\sin(\theta \pm \alpha)</math>.</p>	<p>Use of these formulae to solve equations in a given range, such as (i) <math>\sin 2\theta = \sin \theta</math>, (ii) <math>3\cos\theta + \sin\theta = 2</math>. Applications to integration e.g. <math>\int \cos^2 x dx</math>.</p> <p>Application to finding greatest and least values, e.g. the least value of <math>\frac{1}{3\cos\theta + 4\sin\theta + 10}</math>.</p>
<p>3. Cartesian and parametric equations of curves and conversion between the two forms.</p>	<p>To include finding the equations of tangents and normals to curves defined parametrically or implicitly. Knowledge of the properties of curves other than the circle will <b>not</b> be expected.</p>
<p>4. Formation of simple differential equations.</p> <p>Exponential growth and decay.</p>	<p>Questions may be set on the modelling of practical problems e.g. population growth, radioactive decay. Questions on the kinematics of a particle will <b>not</b> be set.</p>

Topics	Notes
<p>5. Evaluation of volume of revolution.</p> <p>Simple cases of integration by substitution and integration by parts. These methods as the reverse processes of the chain rule and the product rule respectively.</p> <p>Simple cases of integration using partial fractions.</p> <p>Analytical solution of simple first order differential equations with separable variables.</p>	<p>Rotation about the <math>x</math>-axis only.</p> <p>Substitutions will be given.</p>
<p>6. Vectors in two and three dimensions.</p> <p>Magnitude of a vector.</p> <p>Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.</p> <p>Position vectors. The distance between two points. Vector equations of lines.</p> <p>The scalar product. Its use for calculating the angle between two lines.</p>	<p>Unit vectors. Use of unit vectors <math>\mathbf{i}</math>, <math>\mathbf{j}</math>, <math>\mathbf{k}</math>.</p> <p>Condition for two vectors to be parallel.</p> <p>Use of <math>\mathbf{AB} = \mathbf{b} - \mathbf{a}</math>. Use and derivation of the position vector of a point dividing a line in a given ratio. Intersection of two lines.</p> <p>Condition for two vectors to be perpendicular.</p>

## Unit FP1

### Further Pure Mathematics 1

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1, C2, C3 and C4.

Topics	Notes
<p>1. Mathematical induction.</p> <p>Summation of a finite series. Use of formulae for  <math display="block">\sum_{r=1}^n r, \sum_{r=1}^n r^2 \text{ and } \sum_{r=1}^n r^3.</math></p>	<p>Including application to the proof of the binomial theorem for a positive integral power and, for example, the proof of the divisibility of <math>5^{2n} - 1</math> by 24.</p> <p>Including mathematical induction and difference methods. Summation of series such as  <math display="block">\sum_{r=1}^n \frac{1}{r(r+1)}, \sum_{r=1}^n rx^r \text{ and } \sum_{r=1}^n (2r+1)^3.</math></p>
<p>2. The algebraic and trigonometric forms of a complex number, its modulus, argument and conjugate.</p> <p>Representation of complex numbers by points in an Argand diagram. Algebraic and geometric treatment of the addition, subtraction, multiplication and division of complex numbers. Equating real and imaginary parts.</p> <p>Loci in an Argand diagram. Simple cases of transformations of lines and curves defined by <math>w = f(z)</math>.</p>	<p><math>z = x + iy</math> and <math>z = r(\cos\theta + i\sin\theta)</math> where <math>\theta = \arg(z)</math> may be taken to be in either <math>[0, 2\pi)</math> or <math>(-\pi, \pi]</math>. The complex conjugate of <math>z</math> will be denoted by <math>\bar{z}</math>.</p> <p>Including the solution of equations such as  <math display="block">z + 2\bar{z} = \frac{1 + 2i}{1 - i}.</math></p> <p>For example, <math> z - 1  = 2 z + i </math>. For example, the image of the line <math>x + y = 1</math> under the transformation defined by <math>w = z^2</math>.</p>
<p>3. The number and nature of the roots of polynomial equations, real and complex roots. Repeated roots.</p> <p>The occurrence of the non-real roots of a polynomial equation with real coefficients in conjugate pairs.</p> <p>Relationships between the roots and coefficients of a polynomial equation.</p>	<p>Candidates will be expected to know that an <math>n</math>th degree polynomial equation has <math>n</math> roots. Including the result that, if <math>\alpha</math> is a repeated root of <math>f(x) = 0</math>, then <math>\alpha</math> is also a root of <math>f'(x) = 0</math>.</p> <p>The degree of the polynomial will not exceed 3.</p>

4. Matrices, equality, multiplication by a scalar, addition and multiplication.  
Identity matrices, the determinant of a square matrix, singular matrices.  
Transpose of a matrix, adjugate matrix, inverse of a non-singular matrix.

Determinantal condition for the solution of simultaneous equations which have a unique solution.

Solution of simultaneous equations by reduction to echelon form and by the use of matrices.

The order of matrices will be at most  $3 \times 3$ .

To include equations which  
(a) have a unique solution,  
(b) have non-unique solutions,  
(c) are not consistent.

5. Transformations in the plane, translation, rotation and reflection, and combinations of these, in matrix form and otherwise.  
Identification of fixed points.

6. The general definition of a derivative.  
Finding, from first principles, the derivative of simple algebraic functions.  
Logarithmic differentiation.

For example,  $x^3 - x$  and  $\frac{1}{2x+1}$ .

For example, finding the derivatives of

$$(\sin x)^x \text{ and } \frac{(x^2 + 3)^{\frac{1}{2}}}{(5 \sin x - 1)^{\frac{2}{3}}}.$$



## Unit FP2

### Further Pure Mathematics 2

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1, C2, C3, C4 and FP1.

Topics	Notes
1. Further partial fractions.	To include denominators $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx^2 + d)$ .
2. Knowledge and use of de Moivre's Theorem.	Proof by induction of de Moivre's Theorem for positive integer values of $n$ .
Applications to trigonometry.	For example, showing that $\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1$ and $\cos^4 \theta = \frac{1}{8}(\cos 4\theta + 4 \cos 2\theta + 3)$ .
Calculation of the $n$ th roots of complex numbers.	
3. Trigonometric equations.	Questions aimed solely at proving identities will not be set.
Use of formulae for $\sin A \pm \sin B$ , $\cos A \pm \cos B$ and for $\sin x$ , $\cos x$ and $\tan x$ in terms of $t$ , where $t = \tan \frac{1}{2}x$ .	For example, $\cos \theta + \cos 2\theta + \cos 3\theta = 0$ and $2 \sin x - \tan \frac{1}{2}x = 0$ .
General solution of trigonometric equations.	
4. Real functions. The language of set theory, image and inverse image of a set under a function.	Notation to be used for image and inverse image: $f(A) = \{f(x) : x \in A \text{ and } f(x) \text{ is defined}\}$ $f^{-1}(B) = \{x : f(x) \text{ is defined and } f(x) \in B\}$
Odd functions, even functions, strictly increasing functions, strictly decreasing functions, bounded functions.	
Functions defined piecewise on their domain.	Informal treatment only.

Topics	Notes
The idea of continuity.	
Sketching graphs of rational functions, including those in which the degree of the numerator exceeds that of the denominator.	For example: $y = \frac{9(x-3)}{(x+1)(x-2)}; \quad y = \frac{(x+1)^2}{(2x-3)}$
Asymptotes.	Including asymptotes which are not parallel to a coordinate axis.
<p>5. Loci in Cartesian and parametric form. Finding intersections, chords, tangents and normals using algebra and calculus.</p> <p>The derivation of the standard forms of the equations of conics including the focus-directrix properties. Forms derived from these by translation.</p>	<p>Candidates will be expected to translate geometrical ideas into algebraic form and vice versa, and should know basic methods of general application.</p> <p>Apart from the focus-directrix properties of conics, no knowledge of particular curves other than the circle will be expected.</p>
<p>6. Further integration.</p> <p>Basic properties of the definite integral.</p> <p>Integration of</p> $\frac{1}{\sqrt{a^2 - x^2}} \text{ and } \frac{1}{a^2 + x^2}.$	<p>To include the use of partial fractions.</p> <p>To include differentiation of an integral with respect to a variable limit.</p>

## Unit FP3

### Further Pure Mathematics 3

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1, C2, C3, C4, FP1 and FP2.

Topics	Notes
<p>1. Definition and basic properties of the six hyperbolic functions. The inverse functions <math>\sinh^{-1}</math>, <math>\cosh^{-1}</math> and <math>\tanh^{-1}</math>, including their logarithmic forms.</p> <p>Differentiation and integration of <math>\sinh</math>, <math>\cosh</math> and <math>\tanh</math>.</p> <p>Knowledge and use of the identity <math>\cosh^2 x - \sinh^2 x \equiv 1</math> and its equivalents.</p> <p>Knowledge and use of the formulae for <math>\sinh(A \pm B)</math>, <math>\cosh(A \pm B)</math>, <math>\tanh(A \pm B)</math>, <math>\sinh 2A</math>, <math>\cosh 2A</math> and <math>\tanh 2A</math>.</p>	<p>Questions aimed solely at proving identities will not be set.</p> <p>Application to integration, for example <math>\int \sinh^2 x dx</math>.</p>
<p>2. Further integration.</p> <p>The arc length of a curve and the curved surface area of a solid of revolution (for rotations about the <math>x</math>-axis only).</p> <p>Reduction formulae.</p>	<p>Integration of functions such as <math>\frac{1}{ax^2 + bx + c}</math>, <math>\frac{1}{\sqrt{ax^2 + bx + c}}</math>, <math>\sqrt{ax^2 + bx + c}</math>, <math>e^{ax} \cos bx</math> and <math>\frac{1}{a + b \cos x}</math>.</p> <p>Substitutions will not always be given.</p> <p>The equation of the curve will be given in either Cartesian or parametric form.</p>
<p>3. The use of Maclaurin and Taylor series. Applications to approximations.</p>	<p>No consideration of error terms will be required in the examination.</p>

Topics	Notes
<p>4. The approximate solution of equations using graphs and the Intermediate Value Theorem.</p> <p>Iterative methods for an equation in the form <math>x = f(x)</math>.</p> <p>The Newton-Raphson method.</p>	<p>Condition for convergence of the iterative sequence for solving <math>x = f(x)</math>, i.e. <math> f'(x)  &lt; 1</math> in an appropriate interval.</p> <p>Knowledge and use of the Newton-Raphson formula. Graphical treatment of convergence.</p>
<p>5. Polar coordinates <math>(r, \theta)</math>.</p> <p>The relationship between Cartesian and polar coordinates.</p> <p>The intersection of curves defined by their polar equations.</p> <p>The location of points at which tangents are parallel to, or perpendicular to, the initial line.</p> <p>Calculation of area using <math>\frac{1}{2} \int r^2 d\theta</math>.</p>	<p>Where <math>r \geq 0</math> and the value of <math>\theta</math> may be taken to be in either <math>[0, 2\pi)</math> or <math>(-\pi, \pi]</math>.</p> <p>Candidates will be expected to sketch simple curves such as <math>r = a(b + c \cos \theta)</math> and <math>r = a \cos n\theta</math>.</p>

## Unit M1

### Mechanics 1

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1 and C2.

Candidates will also be expected to use appropriate mathematical modelling techniques and be aware of underlying modelling assumptions.

Topics	Notes
<b>1.</b> Rectilinear motion.	
Motion under uniform acceleration.	Candidates may quote the equations of uniformly accelerated motion. To include the sketching and interpretation of velocity-time graphs.
Vertical motion under gravity.	The value $9.8 \text{ ms}^{-2}$ will be used for the acceleration due to gravity.
<b>2.</b> Dynamics of a particle.	
Newton's Laws of Motion.	Forces will be constant and will include weight, friction, normal reaction, tension and thrust. To include problems on lifts. To include motion on an inclined plane. The motion of particles connected by strings passing over fixed pulleys or pegs; one particle will be freely hanging and the other particle may be <ul style="list-style-type: none"> <li>(i) freely hanging,</li> <li>(ii) on a horizontal plane,</li> <li>(iii) on an inclined plane.</li> </ul>
<b>3.</b> Friction.	
Laws of friction. Coefficient of friction. Limiting friction.	Use of $F \leq \mu R$ . No distinction will be made between the coefficients of dynamic and static friction.

Topics	Notes
<p><b>4.</b> Momentum and impulse. Conservation of momentum.</p> <p>Newton's Experimental Law for (i) the direct impact of two bodies moving in the same straight line, (ii) the impact of a body moving at right-angles to a plane.</p>	<p>Problems will be restricted to the one-dimensional case.</p>
<p><b>5.</b> Statics</p> <p>Composition and resolution of forces.</p> <p>Equilibrium of a particle under the action of coplanar forces which may include friction.</p> <p>The moment of a force about a point.</p> <p>Equilibrium of a rigid body under the action of parallel coplanar forces.</p> <p>Centre of mass of a coplanar system of particles.</p> <p>Centre of mass of uniform laminae: triangles, rectangles, circles and composite shapes.</p> <p>Simple cases of equilibrium of a plane lamina or a coplanar system of particles connected by <b>light</b> rods.</p>	<p>Candidates will be expected to know that, when particle is in equilibrium, the sum of the resolved parts of the forces acting on the particle in any direction is zero.</p> <p>Candidates will be expected to be familiar with the term 'centre of gravity'.</p> <p>The use of integration is <b>not</b> required.</p> <p>The lamina or system of particles may be suspended from a fixed point.</p>

## Unit M2

### Mechanics 2

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1, C2, C3, C4 and M1.

Candidates will also be expected to use appropriate mathematical modelling techniques and be aware of underlying modelling assumptions.

Topics	Notes
<p>1. Rectilinear motion.</p> <p>Formation and solution of simple equations of motion in which velocity or acceleration is given as a function of time.</p>	<p>To include the use of <math>\frac{d^2x}{dt^2} = \frac{dv}{dt}</math>.</p>
<p>2. Dynamics of a particle.</p> <p>Light strings and springs obeying Hooke's Law.</p> <p>Motion under forces dependent on time.</p> <p>Work, energy and power.</p> <p>Gravitational potential energy, kinetic energy, elastic energy.</p> <p>Conservation of energy.</p> <p>Work-energy Principle.</p>	<p>Calculation of work done by using change of energy.</p>
<p>3. Motion under gravity in two dimensions.</p>	<p>To include finding the speed and direction of motion of the projectile at any point on its path.</p> <p>The maximum horizontal range of a projectile for a given speed of projection.</p> <p>In examination questions, candidates may be expected to derive the general form of the formulae for the range, the time of flight, the greatest height or the equation of path.</p> <p>In questions where derivation of formulae has not been requested, the quoting of these formulae will <b>not</b> gain full credit.</p> <p>Questions will <b>not</b> involve resistive forces.</p>

Topics	Notes
<p>4. Vectors in two- and three-dimensions.</p> <p>Magnitude of a vector, unit vectors, addition, scalar multiplication. Use of unit vectors <b>i</b>, <b>j</b>, <b>k</b>. The scalar product.</p> <p>Differentiation and integration of a vector in component form with respect to a scalar variable. Vector quantities including displacement, velocity, acceleration, force and momentum.</p>	<p>Including position vectors.</p> <p>Condition for two vectors to be parallel.</p> <p>Condition for two vectors to be perpendicular. Applications including calculation of work, energy, power and resolute.</p> <p>Resultants of vector quantities. Simple applications including the relative motion of two objects and the determination of the shortest distance between them.</p>
<p>5. Circular Motion.</p> <p>Motion of a particle in a horizontal circle with uniform angular speed.</p> <p>Motion in a vertical circle.</p>	<p>Angular speed <math>\omega</math> and the use of <math>v = r\omega</math>.</p> <p>Radial acceleration in circular motion in the form <math>r\omega^2</math> and <math>\frac{v^2}{r}</math>.</p> <p>Problems on banked tracks including the condition for no side slip. The conical pendulum. The motion of a particle in a horizontal circle where the particle is</p> <ol style="list-style-type: none"> <li>constrained by two strings,</li> <li>threaded on one string,</li> <li>constrained by one string and a smooth horizontal surface.</li> </ol> <p>To include the determination of points where the circular motion breaks down (e.g. loss of contact with a surface or a string becoming slack). The condition for a particle to move in complete vertical circles when</p> <ol style="list-style-type: none"> <li>it is attached to a light string,</li> <li>it is attached to a light rigid rod,</li> <li>it moves on the inside surface of a sphere.</li> </ol> <p>The tangential component of the acceleration is <b>not</b> required.</p>



## Unit M3

### Mechanics 3

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1, C2, C3, C4, M1 and M2.

Candidates will also be expected to use appropriate mathematical modelling techniques and be aware of underlying modelling assumptions.

Topics	Notes
<p>1. Formation and solution of first order differential equation with separable variables. Formation and solution of second order differential equation of the form</p> $a \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + cx = \alpha t + \beta$ <p>where <math>a, b, c, \alpha, \beta</math> are constants.</p>	<p>The ability to express stated laws as differential equations and interpret their solutions.</p> <p>The auxiliary equation, complementary function, the particular integral.</p> <p>The roots of the auxiliary equation may be real or complex, distinct or equal.</p> <p>Candidates will be expected to quote and use the appropriate form of the complementary function in each case.</p>
<p>2. Rectilinear motion.</p> <p>Formation and solution of simple equations of motion in which</p> <p>(i) acceleration is given as a function of time, displacement or velocity,</p> <p>(ii) velocity is given as a function of time or displacement.</p>	<p>To include use of <math>\frac{d^2 x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx}</math>.</p>
<p>3. Simple Harmonic Motion.</p>	<p>Candidates will be expected to set up the differential equation of motion, identify the period, amplitude and appropriate forms of solution.</p> <p>Candidates may quote formulae in problems unless the question specifically requires otherwise.</p> <p>Questions may involve light elastic strings or springs.</p> <p>Questions may require the refinement of the mathematical model to include damping.</p> <p>Angular S.H.M. is <b>not</b> included.</p>

Topics	Notes
<b>4.</b> Impulsive tensions in strings	
Motion of connected particles involving impulse.	Two-dimensional problems may be set.
<b>5.</b> Statics.	
Equilibrium of a single rigid body under the action of coplanar forces where the forces are not all parallel	Problems may include rods resting against rough or smooth walls and on rough ground. Considerations of jointed rods is not required. Questions involving toppling will <b>not</b> be set.

## Unit S1

### Statistics 1

Candidates will be expected to be familiar with

- (a) the knowledge, skills and understanding implicit in Units C1 and C2,
- (b) the exponential function and its series representation.

Candidates will also be expected to use appropriate mathematical modelling techniques and be aware of underlying modelling assumptions.

Topics	Notes
<b>1.</b> Random experiments, sample space as the set of all possible outcomes. Events described verbally and as subsets of the sample space. Complementary events.  The addition law for mutually exclusive events.   The generalised addition law. Conditional probability. Multiplication law for independent events. Multiplication law for dependent events. Exhaustive events, the Law of Total Probability and Bayes' Theorem. Probabilities for samples drawn with replacement and without replacement.	Venn diagrams may be used.  The complement of $A$ will be denoted by $A'$ . $P(A \cup B) = P(A) + P(B)$ and results following from this, e.g. $P(A') = 1 - P(A)$ and $P(A) = P(A \cap B) + P(A \cap B')$ .  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ . $P(A \cap B) = P(A)P(B)$ . $P(A \cap B) = P(A)P(B A) = P(B)P(A B)$ .  Tree diagrams may be used. With at most 3 exhaustive events. Including simple problems involving permutations and combinations.
<b>2.</b> Discrete probability distributions. Mean, variance and standard deviation of a discrete random variable.	Use of the result that $P[(X_1 = x_1) \cap (X_2 = x_2)]$ $= P(X_1 = x_1) \times P(X_2 = x_2),$ where $X_1, X_2$ are independent observations of a discrete random variable.
Use of the results: $E(aX + b) = aE(X) + b$ , $\text{Var}(aX + b) = a^2\text{Var}(X)$ .	
Expected value of a function of a discrete random variable.	$E[g(X)] = \sum g(x)P(X = x)$ .

Topics	Notes
<p>3. Bernoulli trials and the binomial distribution.  The Poisson distribution.  Mean and variance of the binomial and Poisson distributions.</p> <p>Poisson approximation to a binomial.</p>	<p>Use of the binomial formula and tables.</p> <p>Use of the Poisson formula and tables.</p> <p>Derivations will <b>not</b> be assessed.</p>
<p>4. Continuous probability distributions.  Probability density and cumulative distribution functions and their relationships.  Median, quartiles and percentiles.  Mean, variance and standard deviation.  Use of the results:  <math>E(aX + b) = aE(X) + b</math>,  <math>\text{Var}(aX + b) = a^2\text{Var}(X)</math>.  Expected value of a function of a continuous random variable.</p>	<p><math>f(x) = F'(x)</math></p> <p><math>F(x) = \int_{-\infty}^x f(t)dt</math></p> <p><math>E[g(X)] = \int g(x)f(x) dx</math></p> <p>Simple functions only, e.g.  <math>\frac{1}{X^2}</math> and <math>\sqrt{X}</math>.</p>

## Unit S2

### Statistics 2

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1, C2, C3, C4 and S1.

Candidates will also be expected to use appropriate mathematical modelling techniques and be aware of underlying modelling assumptions.

Topics	Notes
1. The uniform (rectangular) distribution.	Including mean and variance.
2. Description and use of the normal distribution.	Use of tables of the standard normal cumulative distribution function and its inverse. Linear interpolation in tables will <b>not</b> be required.
The normal distribution as an approximation to the binomial and Poisson distributions.	To include the use of a continuity correction.
3. Use of the result $E(aX + bY) = aE(X) + bE(Y)$ . For independent $X$ and $Y$ , use of the results $E(XY) = E(X)E(Y)$ , $\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$ . Generalisation to $n$ random variables. Application of the result that the sum of independent Poisson random variables has a Poisson distribution. Application of the result that a linear combination of independent normally distributed random variables has a normal distribution.	Derivation of these results will not be assessed.
4. Definition of a random sample of observations of a random variable. Distribution of the mean of a random sample from a normal distribution with known mean and variance. Use of the result that the mean of a large random sample from any distribution with known mean and variance is approximately normally distributed (the Central Limit Theorem).	The mean and variance will be given.  The mean and variance will be given.

Topics	Notes
<p>5. Hypothesis testing, null hypothesis and alternative hypothesis. Test statistic, significance level and critical region. <math>p</math>-value.</p>	<p>Specifying a value for a parameter. One-sided and two-sided.</p> <p>The <math>p</math>-value is the probability that the observed result or a more extreme one will occur under the null hypothesis <math>H_0</math>. For uniformity, interpretations of a <math>p</math>-value should be along the following lines:  <math>p &lt; 0.01</math>; there is very strong evidence for rejecting <math>H_0</math>.  <math>0.01 \leq p \leq 0.05</math>; there is strong evidence for rejecting <math>H_0</math>.  <math>p &gt; 0.05</math>; there is insufficient evidence for rejecting <math>H_0</math>.</p>
<p>Test for</p> <ul style="list-style-type: none"> <li>(a) a population proportion or binomial probability parameter,</li> <li>(b) the mean of a Poisson distribution,</li> <li>(c) the mean of a normal distribution of known variance.</li> <li>(d) a specified difference between the means of two normal distributions whose variances are known.</li> </ul>	<p>Using a binomial distribution or a normal approximation as appropriate. Using a Poisson distribution or a normal approximation as appropriate.</p> <p>The specified difference may be different from zero.</p>
<p>6. Confidence limits for</p> <ul style="list-style-type: none"> <li>(a) the mean of a normal distribution with known variance,</li> <li>(b) the difference between the means of two normal distributions whose variances are known.</li> </ul>	<p>Candidates will be expected to be familiar with the term 'confidence interval', including its interpretation.</p>

## Unit S3

### Statistics 3

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Units C1, C2, C3, C4, S1 and S2.

Candidates will also be expected to use appropriate mathematical modelling techniques and be aware of underlying modelling assumptions.

Topics	Notes
<p>1. Samples and populations. General discussion of statistics and their sampling distributions.</p> <p>A statistic as an estimator of a population parameter. Unbiased estimators. The variance criterion for choosing between unbiased estimators. Unbiased estimators of a probability and of a population mean and their standard errors. Unbiased estimator of a population variance.</p>	<p>To include the sampling distributions of statistics obtained by selecting random samples, with or without replacement, from a small population.</p> <p>The use of the term ‘estimate’ as the calculated value of an ‘estimator’.</p> <p>Divisor <math>n - 1</math>.</p>
<p>2. Further hypothesis testing. Tests for</p> <p>(a) a specified mean of any distribution whose variance is estimated from a large sample,</p> <p>(b) a specified difference between the means of two populations whose variances are estimated from large samples,</p> <p>(c) a specified mean of a normal distribution with unknown variance.</p>	<p>Using the Central Limit Theorem.</p> <p>Using the Central Limit Theorem. The specified difference may be different from zero.</p> <p>Estimating the variance from the data and using the Student’s <math>t</math>-distribution. The significance level will be given and questions involving the Student’s <math>t</math>-distribution will not require the calculation of <math>p</math>-values.</p>
<p>3. Confidence limits for the mean of a normal distribution with unknown variance. Approximate confidence limits, given large samples, for</p> <p>(a) a probability or a proportion,</p> <p>(b) the mean of any distribution whose variance is unknown,</p> <p>(c) the difference between the means of two populations whose variances are unknown.</p>	<p>Estimating the variance from the data and using the Student’s <math>t</math>-distribution.</p> <p>Using a normal approximation. Using the Central Limit Theorem.</p> <p>Using the Central Limit Theorem.</p>

Topics	Notes
<p>4. The principle of least squares, with particular reference to its use for estimating a linear relationship <math>y = \alpha + \beta x</math> given a set of observations <math>(x,y)</math> where the observed <math>x</math> values are accurate and the observed <math>y</math> values are subject to independent random errors that are normally distributed with zero mean and specified variance. Confidence limits and hypothesis tests for <math>\alpha</math>, <math>\beta</math> and the true value of <math>y</math> for a given value of <math>x</math>.</p>	<p>Familiarity with the formulae and sampling distributions given in the Formula Booklet is expected but their derivations will not be assessed.</p> <p>To include confidence limits and hypothesis tests for the increase in the true value of <math>y</math> for a given increase in <math>x</math>.</p>



## 4

**KEY SKILLS**

Key skills are integral to the study of Mathematics and a number of them may be assessed in the context of the subject as indicated from page 53 in this in the specification. In particular, candidates may be assessed on their ability to organise and present information, ideas, descriptions and arguments clearly and logically, to plan and interpret information from different types of sources including a large data set, carry out multi-stage calculations, interpret results of calculations and present findings. In addition, candidates will have the opportunity for developing and, where appropriate, being assessed on the wider key skills of Working with Others, Improving Own Learning and Performance, and Problem Solving.

## 5

**ASSESSMENT OBJECTIVES**

All candidates for AS and AL Mathematics will be required to meet the following assessment objectives.

<b>Assessment Objectives</b>	
Candidates should be able to:	
AO1	recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts;
AO2	construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form;
AO3	recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models;
AO4	comprehend translations of common realistic contexts into Mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications;
AO5	use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations; give answers to appropriate accuracy.

These assessment objectives apply to the whole of both the AS and AL specifications. In each assessment scheme for Mathematics, these assessment objectives will be weighted, in terms of marks, according to the following ranges.

AO1	30% - 40%
AO2	30% - 40%
AO3	10% - 20%
AO4	5% - 15%
AO5	5% - 15%

The relationship between the marks available in the assessment units and the assessment objectives is given in the following table, but clearly the marks available for any particular assessment scheme will satisfy the above overall percentage requirements. For each permitted combination of units, the set of papers will satisfy the overall proportions of marks required.

	AO1	AO2	AO3	AO4	AO5
<b>C1</b>	30-35	30-35	0-3	0-8	0-2
<b>C2</b>	25-30	25-30	0-3	0-8	6-10
<b>C3</b>	25-30	25-30	0-3	0-8	6-10
<b>C4</b>	25-30	25-30	0-3	0-8	6-10
<b>FP1</b>	40-50	20-30	0-5	0-5	0-10
<b>FP2</b>	40-50	20-30	0-5	0-5	0-10
<b>FP3</b>	40-50	20-30	0-5	0-5	0-10
<b>M1</b>	15-19	15-19	23-26	12-15	6-10
<b>M2</b>	15-19	15-19	23-26	12-15	6-10
<b>M3</b>	15-19	15-19	23-26	12-15	6-10
<b>S1</b>	15-19	15-19	23-26	12-15	6-10
<b>S2</b>	15-19	15-19	23-26	12-15	6-10
<b>S3</b>	15-19	15-19	23-26	12-15	6-10

Assessment Objective AO2 requires candidates to make appropriate use of precise statements and logical deduction. It is therefore important that candidates for each paper use clear, precise and appropriate mathematical language.

## 6

**SCHEME OF ASSESSMENT**

Papers will be set on each of the following assessment units.

Pure Mathematics (core) group	{	C1	Pure Mathematics 1
		C2	Pure Mathematics 2
		C3	Pure Mathematics 3
		C4	Pure Mathematics 4
Further Pure Mathematics group	{	FP1	Further Pure Mathematics 1
		FP2	Further Pure Mathematics 2
		FP3	Further Pure Mathematics
Mechanics group	{	M1	Mechanics 1
		M2	Mechanics 2
		M3	Mechanics 3
Statistics group	{	S1	Statistics 1
		S2	Statistics 2
		S3	Statistics 3

**AS/A2 Units**

The Advanced GCE examination consists of two parts:

Part 1 (Advanced Subsidiary - AS) and Part 2 (A2). Papers C1, C2, M1 and S1 are deemed to be AS units. The remaining papers are deemed to be A2 units.

**Subject Core**

The subject core for AS Mathematics is included in each combination of papers leading to an AS award entitled 'Mathematics'.

The subject core for Advanced Level Mathematics is included in each combination of papers leading to an AL award entitled 'Mathematics'.

**Papers**

Each paper will be of 1½ hours' duration and will consist of questions with varying mark allocations which will be indicated on the paper.

The total number of marks allocated to each paper will be 75. Candidates will be expected to answer all questions.

### Synoptic Assessment

Synoptic assessment in Mathematics addresses candidates' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the A Level course through using and applying methods developed at earlier stages of study in solving problems. Making and understanding connections in this way is intrinsic to learning Mathematics.

At least 20% of the overall marks will be allocated to synoptic assessment.

Questions set in any paper in a group may require knowledge of any of the previous units in the group. For example, problems on the topic 'co-ordinate geometry of the circle' may well require earlier methods such as the solution of simultaneous equations (one linear, one quadratic) and gradients of parallel or perpendicular lines.

Questions set in any 'applications' paper may require knowledge of the content of particular Pure Mathematics group units. The particular Pure Mathematics group units required will be stated in the specification of each unit.

### Calculators

In Paper C1, calculators are not allowed. In all other units, candidates are permitted to use calculators, including graphical calculators. The use of graphical calculators should be encouraged in the study of the subject wherever appropriate. The use of computers and calculators with computer algebra functions is not permitted in any paper. However, their use should be encouraged in the study of the subject whenever appropriate.

### Advanced Subsidiary (AS) Options

An Advanced Subsidiary examination comprises three units. The following combinations of papers and subject titles will be offered.

<i>Option</i>	<i>Title of Award</i>
C1, C2, M1	Mathematics
C1, C2, S1	Mathematics
C1, C2, C3	Pure Mathematics

In each option, the papers will be of equal weighting.

In addition to the above options, candidates may sit an AS examination in Further Mathematics. The allowed options are listed in the section dealing with Further Mathematics.

### Advanced Level Mathematics Options

An Advanced Level examination comprises six units. The following combinations of papers are allowed.

C1	C2	C3	C4	M1	M2
C1	C2	C3	C4	S1	S2
C1	C2	C3	C4	M1	S1

In each option the papers will be of equal weighting.

### Advanced Subsidiary (AS) Further Mathematics Options

An Advanced Subsidiary Further Mathematics examination comprises three units. Candidates may be expected to have obtained (or to be obtaining concurrently) an Advanced GCE in Mathematics.

The following combinations are allowed.

A Level Mathematics Option Taken	AS Further Mathematics Option Allowed
C1, C2, C3, C4, M1, M2	FP1, FP2, FP3 or FP1, FP2, M3 or FP1, FP2, S1 or FP1, M3, S1 or FP1, S1, S2
C1, C2, C3, C4, S1, S2	FP1, FP2, FP3 or FP1, FP2, S3 or FP1, FP2, M1 or FP1, S3, M1 or FP1, M1, M2
C1, C2, C3, C4, M1, S1	FP1, FP2, FP3 or FP1, FP2, M2 or FP1, FP2, S2 or FP1, M2, M3 or FP1, M2, S2 or FP1, S2, S3

### Advanced Level Further Mathematics Options

An Advanced Level Further Mathematics option comprises six units. Candidates may be expected to have obtained (or to be obtaining concurrently) an Advanced GCE in Mathematics.

The following combinations are allowed.

A Level Mathematics Option Taken	A Level Further Mathematics Option Allowed
C1, C2, C3, C4, M1, M2	FP1, FP2, FP3, M3, S1, S2 or FP1, FP2, FP3, S1, S2, S3 or FP1, FP2, M3, S1, S2, S3
C1, C2, C3, C4, S1, S2	FP1, FP2, FP3, S3, M1, M2 or FP1, FP2, FP3, M1, M2, M3 or FP1, FP2, M1, M2, M3, S3
C1, C2, C3, C4, M1, S1	FP1, FP2, FP3, M2, M3, S2 or FP1, FP2, FP3, M2, S2, S3 or FP1, FP2, M2, M3, S2, S3

In each option, the papers will be of equal weighting.

## Advanced Level Pure Mathematics Option

An Advanced Level Pure Mathematics option comprises the units C1, C2, C3, C4, FP1 and FP2.

Each paper will be of equal weighting.

### 6.1 Availability of Units

There are two assessment dates - January and June. In January the following papers will be available.

C1, C2, C3, FP1, M1 and S1.

In June, all papers will be available.

### 6.2 Awarding and Reporting

Candidates' results for both AS and AL will be graded on the six point scale A-E and U (unclassified). The result for each unit will be reported to the candidate.

Individual assessment unit results, prior to certification for a qualification, have shelf-life limited only by the shelf-life of the specification.

### 6.3 Forbidden Combinations/Overlap with other Qualifications

- (a) Units that contribute to an award in A Level Mathematics may not also be used for an award in Further Mathematics. Candidates who are awarded certificates in both A Level Mathematics and A Level Further Mathematics must use unit results from 12 different teaching modules. Candidates who are awarded certificates in both A Level Mathematics and AS Further Mathematics must use unit results from 9 different teaching modules.
- (b) The AL subject 'Pure Mathematics' may not be taken at the same sitting with any other Mathematics subject at AS or AL.
- (c) The classification codes for this specification are as follows:

Mathematics	2210
Pure Mathematics	2230
Further Mathematics	2330

Centres should be aware that candidates who enter for more than one qualification with the same classification code will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.

## 6.4 Mathematical Notation

The notation used in this specification is consistent with the inter-board agreed list of notation. This list is reproduced in Appendix 1 to this specification.

## 6.5 Formula Booklet

A formula booklet will be required in the examination. Copies of this booklet may be obtained from the WJEC. It has been agreed that certain formulae in Pure Mathematics will not be included in the formulae booklets of any board. These formulae are listed in Appendix 2 to this specification.

## 6.6 Statistical Tables

Candidates for Papers S1, S2 and S3 will require a book of statistical tables.

The following books of statistical tables are allowed in the examination.

- (i) Statistical Tables (Murdoch and Barnes, Macmillan)
- (ii) Elementary Statistical Tables (RND/WJEC Publications)

## 6.7 Calculating Aids

- (i) The calculator must be of a size suitable for use on the desk at which the candidate will attempt the examination.
- (ii) The power supply for the calculator is the responsibility of the candidate and must be integral.
- (iii) The working condition of the calculator is the responsibility of the candidate.
- (iv) A fault in a calculator will not normally be considered as justifying the giving of special consideration to the user.
- (v) Calculator cases, instruction leaflets and similar materials must not be in the possession of candidates during the examination.
- (vi) Calculators must not be borrowed from other candidates in the course of an examination for any reason, although the invigilator may provide a candidate with a replacement calculator.
- (vii) Programmable calculators may be used but no prepared programs may be taken into the examination room.

*(Information and/or programs stored in the calculator's memory must be cleared before the examination. Retrieval of information and/or programs during the examination is an infringement of the regulations.)*

- (viii) Candidates are responsible for clearing any information and/or programs stored in the calculator before the examination.

Calculators which have non-numerical functions or give non-numerical information are not permitted. Such prohibited facilities include data banks, dictionaries, language translators, text retrieval and calculators with facilities which are capable of carrying out symbolic algebra. The use of any calculator which is capable of communicating with other machines for sending/receiving messages is strictly prohibited and the use of such calculators by candidates will be regarded as malpractice.

## 6.7 Candidates with Particular Requirements

Details of the special arrangements and special consideration for candidates with particular requirements are contained in the Joint Council for General Qualifications document *Candidates with Special Assessment Needs: Regulations and Guidance*. Copies of this document are available from the WJEC.

# 7

## GRADE DESCRIPTIONS

The following grade descriptions indicate the level of attainment characteristic of the given grade at Advanced Level. They give a general indication of the required learning outcomes at each specified grade. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

### Grade A

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts and usually make sensible comments or predictions. They can distil the essential mathematical information from extended pieces of prose having mathematical content. They can comment meaningfully on the mathematical information.

Candidates make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.



**Grade C**

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Candidates recall or recognise most of the standard models that are needed and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation. They sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical content. They give some useful comments on this mathematical information.

Candidates usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

**Grade E**

Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Candidates recall or recognise some of the standard models that are needed and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation.

Candidates sometimes comprehend or understand the meaning of translations in Mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical information.

Candidates often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

## APPENDIX 1: AGREED MATHEMATICAL NOTATION

### 1. Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
$\{x : \dots\}$	the set of all $x$ such that ...
$n(A)$	the number of elements in set $A$
$\emptyset$	the empty set
$\mathbb{E}$	the universal set
$A'$	the complement of the set $A$
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \dots\}$
$\mathbb{Z}_n$	the set of integers modulo $n$ , $\{0, 1, 2, \dots, n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
$(x, y)$	the ordered pair $x, y$
$A \times B$	the Cartesian product of sets $A$ and $B$ i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval, $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b), [a, b[$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b], ]a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
$(a, b), ]a, b[$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y R x$	$y$ is related to $x$ by the relation $R$
$y \sim x$	$y$ is equivalent to $x$ , in the context of some equivalence relation

**2. Miscellaneous Symbols**

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leq, \nless$	is less than or equal to, is not greater than
$>$	is greater than
$\geq, \ngtr$	is greater than or equal to, is not less than
$\infty$	infinity
$p \wedge q$	$p$ and $q$
$p \vee q$	$p$ or $q$ (or both)
$\sim p$	not $p$
$p \Rightarrow q$	$p$ implies $q$ (if $p$ then $q$ )
$p \Leftarrow q$	$p$ is implied by $q$ (if $q$ then $p$ )
$p \Leftrightarrow q$	$p$ implies and is implied by $q$ ( $p$ is equivalent to $q$ )
$\exists$	there exists
$\forall$	for all

**3. Operations**

$a + b$	$a$ plus $b$
$a - b$	$a$ minus $b$
$a \times b, ab, a.b$	$a$ multiplied by $b$
$a \div b, \frac{a}{b}, a/b$	$a$ divided by $b$
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
$\prod_{i=1}^n a_i$	$a_1 \times a_2 \times \dots \times a_n$
$\sqrt{a}$	the positive square root of $a$
$ a $	the modulus of $a$
$n!$	$n$ factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbb{Z}^+$ $\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

**4. Functions**

$f(x)$	the value of the function $f$ at $x$
$f : A \rightarrow B$	$f$ is a function under which each element of set $A$ has an image in set $B$
$f : x \rightarrow y$	the function $f$ maps the element $x$ to the element $y$
$f^{-1}$	the inverse function of the function $f$
$g \circ f, gf$	the composite function of $f$ and $g$ which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as $x$ tends to $a$
$\Delta x, \delta x$	an increment of $x$
$\frac{dy}{dx}$	the derivative of $y$ with respect to $x$
$\frac{d^n y}{dx^n}$	the $n$ th derivative of $y$ with respect to $x$
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ..., $n$ th derivatives of $f(x)$ with respect to $x$
$\int y \, dx$	the indefinite integral of $y$ with respect to $x$
$\int_a^b y \, dx$	the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of $V$ with respect to $x$
$\dot{x}, \ddot{x}, \dots$	the first, second, ... derivatives of $x$ with respect to $t$

**5. Exponential and Logarithmic Functions**

$e$	base of natural logarithms
$e^x, \exp x$	exponential function of $x$
$\log_a x$	logarithm to the base $a$ of $x$
$\ln x, \log_e x$	natural logarithm of $x$
$\lg x, \log_{10} x$	logarithm of $x$ to base 10

**6. Circular and Hyperbolic Functions**

$\left. \begin{array}{l} \sin, \cos, \tan, \\ \operatorname{cosec}, \sec, \cot \end{array} \right\}$	the circular functions
$\left. \begin{array}{l} \sin^{-1}, \cos^{-1}, \tan^{-1}, \\ \operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1} \\ \text{OR} \\ \arcsin, \arccos, \arctan, \\ \operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot} \end{array} \right\}$	the inverse circular functions
$\left. \begin{array}{l} \sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$	the hyperbolic functions

$\sinh^{-1}, \cosh^{-1}, \tanh^{-1},$ $\operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \operatorname{coth}^{-1}$ OR $\operatorname{ar}(c)\sinh, \operatorname{ar}(c)\cosh, \operatorname{ar}(c)\tanh,$ $\operatorname{ar}(c)\operatorname{cosech}, \operatorname{ar}(c)\operatorname{sech}, \operatorname{ar}(c)\operatorname{coth}$	} the inverse hyperbolic functions
--	------------------------------------

## 7. Complex Numbers

$i, j$	square root of $-1$
$z$	a complex number, $z = x + iy$ $= r(\cos \theta + i \sin \theta)$
$\operatorname{Re} z$	the real part of $z$ , $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of $z$ , $\operatorname{Im} z = y$
$ z $	the modulus of $z$ , $ z  = \sqrt{(x^2 + y^2)}$
$\arg z$	the argument of $z$ , $\arg z = \theta$ , $-\pi < \theta \leq \pi$
$z^*$	the complex conjugate of $z$ , $x - iy$

## 8. Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
$\mathbf{M}^T$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$ or $ \mathbf{M} $	the determinant of the square matrix $\mathbf{M}$

## 9. Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\vec{AB}$	the vector represented in magnitude and direction by the directed line segment $AB$
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the Cartesian co-ordinate axes
$ \mathbf{a} , a$	the magnitude of $\mathbf{a}$
$\left  \vec{AB} \right , AB$	the magnitude of $\vec{AB}$
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

## 10. Probability and Statistics

$A, B, C$ , etc.	events
$A \cup B$	union of the events $A$ and $B$
$A \cap B$	intersection of the events $A$ and $B$
$P(A)$	probability of the event $A$
$A'$	complement of the event $A$
$P(A \mid B)$	probability of the event $A$ conditional on the event $B$

$X, Y, R$ , etc.	random variables
$x, y, r$ , etc.	values of the random variables $X, Y, R$ , etc.
$x_1, x_2, \dots$	observations
$f_1, f_2, \dots$	frequencies with which the observations $x_1, x_2, \dots$ occur
$p(x)$	probability function $P(X = x)$ of the discrete random variable $X$
$p_1, p_2, \dots$	probabilities of the values $x_1, x_2, \dots$ of the discrete random variable $X$
$f(x), g(x), \dots$	the value of the probability density function of a continuous random variable $X$
$F(x), G(x), \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of a continuous random variable $X$
$E(X)$	expectation of the random variable $X$
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable $X$
$G(t)$	probability generating function for a random variable which takes the values 0, 1, 2, ...
$B(n, p)$	binomial distribution with parameters $n$ and $p$
$N(\mu, \sigma^2)$	normal distribution with mean $\mu$ and variance $\sigma^2$
$\mu$	population mean
$\sigma^2$	population variance
$\sigma$	population standard deviation
$\bar{x}, m$	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$
$\phi$	probability density function of the standardised normal variable with distribution $N(0,1)$
$\Phi$	corresponding cumulative distribution function
$\rho$	product moment correlation coefficient for a population
$r$	product moment correlation coefficient for a sample
$\text{Cov}(X, Y)$	covariance of $X$ and $Y$

## APPENDIX 2: FORMULAE FOR AS AND A LEVEL MATHEMATICS

Candidates will be expected to remember any formula required in the specification which is **not** included in the formula booklet

The list below gives some of the more complicated formulae which candidates will be expected to remember, together with an indication of the unit in which it will **first** be examined.

### Core Material

### Unit first examined

#### Quadratic equations

$$ax^2 + bx + c = 0 \text{ has roots } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{C1}$$

#### Laws of logarithms

$$\log_a x + \log_a y \equiv \log_a (xy) \quad \text{C2}$$

$$\log_a x - \log_a y \equiv \log_a \left( \frac{x}{y} \right) \quad \text{C2}$$

$$k \log_a x \equiv \log_a (x^k) \quad \text{C2}$$

#### Trigonometry

In the triangle  $ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{C2}$$

$$\text{Area} = \frac{1}{2} ab \sin C \quad \text{C2}$$

$$\cos^2 A + \sin^2 A \equiv 1 \quad \text{C2}$$

$$\sec^2 A \equiv 1 + \tan^2 A \quad \text{C3}$$

$$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A \quad \text{C3}$$

$$\sin 2A \equiv 2 \sin A \cos A \quad \text{C4}$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \quad \text{C4}$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A} \quad \text{C4}$$

**Differentiation**

function	derivative	
$x^n$	$nx^{n-1}$	C1
$\sin kx$	$k \cos kx$	C3
$\cos kx$	$-k \sin kx$	C3
$e^{kx}$	$k e^{kx}$	C3
$\ln x$	$\frac{1}{x}$	
$f(x) + g(x)$	$f'(x) + g'(x)$	C1
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$	C3
$f(g(x))$	$f'(g(x))g'(x)$	C3

**Integration**

function	integral	
$x^n$	$\frac{1}{n+1} x^{n+1} + c, n \neq -1$	C2
$\cos kx$	$\frac{1}{k} \sin kx + c$	C3
$\sin kx$	$-\frac{1}{k} \cos kx + c$	C3
$e^{kx}$	$\frac{1}{k} e^{kx} + c$	C3
$\frac{1}{x}$	$\ln  x  + c, x \neq 0$	C3
$f'(x) + g'(x)$	$f(x) + g(x) + c$	C2
$f'(g(x))g'(x)$	$f(g(x)) + c$	C3

**Area**

area under a curve $= \int_a^b y \, dx \quad (y \geq 0)$	C2
--	----

**Vectors**

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$	C4
--	----



**Further Pure Mathematics****Summation**

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1) \quad \text{FP1}$$

**Trigonometry**

If  $t = \tan \frac{x}{2}$  : FP2

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\tan x = \frac{2t}{1-t^2}$$

$$\frac{dx}{dt} = \frac{2}{1+t^2} dt$$

<i>Equation</i>	<i>General Solution</i> (where $\alpha$ is a particular solution)	FP2
-----------------	--	-----

$\sin \theta = k$	$\theta = 180^\circ n + (-1)^n \alpha$	
-------------------	--	--

$\cos \theta = k$	$\theta = 360^\circ n = \alpha$	
-------------------	---------------------------------	--

$\tan \theta = k$	$\theta = 180^\circ n + \alpha$	
-------------------	---------------------------------	--

**Mechanics****Uniform Acceleration**

$v = u + at$	M1
--------------	----

$s = ut + \frac{1}{2}at^2$	M1
----------------------------	----

$s = \frac{1}{2}(u + v)t$	M1
---------------------------	----

$v^2 = u^2 + 2as$	M1
-------------------	----

**Momentum***Conservation of Momentum*

$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$	M1
-------------------------------------	----

*Newton's Experimental Law*

$v_1 - v_2 = -e(u_1 - u_2)$	M1
-----------------------------	----

**Elastic Strings/Springs**

$$\text{Hooke's Law: } T = \frac{\lambda x}{l} \quad \text{M2}$$

$$\text{Elastic energy} = \frac{\lambda x^2}{2l} \quad \text{M2}$$

**Vectors**

$$(a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}) \cdot (b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}) = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad \text{M2}$$

$$\mathbf{a} \cdot \mathbf{i} = |a| |b| \cos \theta \quad \text{M2}$$

**Simple Harmonic Motion**

$$\ddot{x} = -\omega^2 x \quad \text{M3}$$

$$v^2 = \omega^2 (a^2 - x^2) \quad \text{M3}$$

$$x = a \sin \omega t, v = \omega a \cos \omega t \text{ (} t \text{ measured from centre)} \quad \text{M3}$$

$$x = a \cos \omega t, v = -\omega a \sin \omega t \text{ (} t \text{ measured from end-point)} \quad \text{M3}$$

$$T = \frac{2\pi}{\omega} \quad \text{M3}$$

**Statistics****Probability**

$$P(A') = 1 - P(A) \quad \text{S1}$$

$$P(A) = P(A \cap B) + P(A \cap B') \quad \text{S1}$$

If  $E_1, E_2, E_3$  form a set of mutually exclusive and exhaustive events, then

$$P(A) = P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3) \quad \text{S1}$$

**Probability distributions**

$$E[aX + b] = aE[X] + b \quad \text{S1}$$

$$\text{Var}[aX + b] = a^2 \text{Var}(X) \quad \text{S1}$$

$$E[aX + bY] = aE[X] + bE[Y] \quad \text{S2}$$

## THE EXEMPLIFICATION OF KEY SKILLS

The following tables give some examples of Mathematics contexts in which naturally occurring key skills evidence could be accumulated.

**Note: If producing certain types of evidence creates difficulties due to disability or other factors, the candidate may be able to use other ways to show achievement. The candidate should ask the tutor or supervisor for further information.**

### COMMUNICATION: LEVEL 1

C1.1 TAKE PART IN A DISCUSSION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
take part in a <b>one-to-one</b> discussion and a <b>group</b> discussion about different straightforward subjects.	<ul style="list-style-type: none"> <li>provide information that is relevant to the subject and purpose of the discussion</li> <li>speak clearly in a way that suits the situation</li> <li>listen and respond appropriately to what others say.</li> </ul>	Records from an assessor who observed each discussion and noted how the student met the requirements of the Unit, or an audio/video tape of the discussions.	Classroom discussion, possibly in question and answer form, relating to a straightforward topic e.g. experiments in probability (S1).

C1.2 INFORMATION GATHERING			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
read and obtain information from <b>two</b> different types of documents about straightforward subjects, including at least <b>one</b> image.	<ul style="list-style-type: none"> <li>read relevant material</li> <li>identify accurately the main points and ideas in material</li> <li>use the information to suit the purpose.</li> </ul>	A record of what the student reads and why, including a note or copy of the image. Notes, highlighted text or answers to questions about the material read. Records of how the student used the information. <i>Eg</i> in discussions for C1.1 or writing for C1.3.	Collection of data from a survey to determine to what extent the distribution of the data can be modelled by a normal distribution (S2).

C1.3 WRITING			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
write <b>two</b> different types of documents about straightforward subjects. Include at least <b>one</b> image in one of the documents.	<ul style="list-style-type: none"> <li>present relevant information in a form that suits the purpose</li> <li>ensure text is legible</li> <li>make sure that spelling, punctuation and grammar are accurate so the meaning is clear.</li> </ul>	The two different documents might include a letter, a short report or essay, with an image such as a chart or sketch.	Presentation of statistical information by compiling a short written report on the testing of hypotheses (S2).

## COMMUNICATION: LEVEL 2

C2.1a CONTRIBUTE TO A DISCUSSION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
contribute to a discussion about a straightforward subject.	<ul style="list-style-type: none"> <li>make clear and relevant contributions in a way that suits the purpose and situation</li> <li>listen and respond appropriately to what others say</li> <li>help to move the discussion forward.</li> </ul>	A record from an assessor who observed the discussion and noted how the student met the requirements of the Unit, or an audio/video tape of the discussion.	e.g. Discussion on how distance/time and velocity/time graphs can be used to demonstrate rectilinear motion (M1).

C2.1b GIVE A SHORT TALK			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
give a short talk about a straightforward subject using an image.	<ul style="list-style-type: none"> <li>speak clearly in a way that suits the subject, purpose and situation</li> <li>keep to the subject and structure the talk to help listeners follow what the student says</li> <li>use an image to illustrate clearly the main points.</li> </ul>	A record from an assessor who observed the talk, or an audio/video tape of the talk. Notes from preparing and giving the talk. A copy of the image used.	e.g. A short talk on quadratic functions and their graphs (C1).

C2.2 INFORMATION GATHERING			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
read and summarise information from <b>two</b> extended documents about a straightforward subject. One of the documents should include <b>at least one</b> image.	<ul style="list-style-type: none"> <li>select and read relevant material</li> <li>identify accurately the lines of reasoning and main points from text and images</li> <li>summarise the information to suit the purpose.</li> </ul>	A record of what is read and why, including a note or copy of the image. Notes, highlighted text or answers to questions about the material read. Evidence of summarising information could include the student's notes for the talk, or one of the documents written.	e.g. Use a substantial collection of data from 2 sources in order to demonstrate various distributions (S2).

C2.3 WRITING			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
write <b>two</b> different types of documents about straightforward subjects. One piece of writing should be an extended document and include <b>at least one</b> image.	<ul style="list-style-type: none"> <li>present relevant information in an appropriate form</li> <li>use a structure and style of writing to suit the purpose</li> <li>ensure the text is legible and that spelling, punctuation and grammar are accurate, so the meaning is clear.</li> </ul>	The two different documents might include a report or an essay, with an image such as a chart, graph or diagram, a business letter or notes.	e.g. Presentation of the results of opinion polls (S2).

### COMMUNICATION: LEVEL 3

C3.1a TAKE PART IN A DISCUSSION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
contribute to a group discussion about a complex subject.	<ul style="list-style-type: none"> <li>make clear and relevant contributions</li> <li>listen and respond appropriately</li> <li>create opportunities for others to take part.</li> </ul>	A record from someone who has observed discussion or has made video/ audio tape of discussion.	Classroom discussion, possibly in Q and A form relating to a complex topic e.g. the nature of mathematical proof. (C1-C4, FP1-FP3)

C3.1b MAKE A PRESENTATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
make a presentation about a complex subject, using at least <b>one</b> image to show complex points.	<ul style="list-style-type: none"> <li>speak clearly and use suitable style</li> <li>structure ideas and information</li> <li>use a range of techniques.</li> </ul>	A record from someone who has observed discussion or has made video/ audio tape of discussion or preparatory notes with images.	Classroom presentation relating to a complex topic e.g. S.H.M. (M3).

C3.2 INFORMATION GATHERING			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
select and synthesise information from <b>two</b> extended documents that deal with a complex subject  <b>One</b> of these documents should include at least one image.	<ul style="list-style-type: none"> <li>select and read material that contains information needed</li> <li>identify accurately, and compare, the lines of reasoning and main points from texts and images</li> <li>synthesise the key information in a suitable form.</li> </ul>	A record of what was read and why, including a note of the image. Notes, highlighted text or answers to questions about material read. Evidence of synthesising information from notes of a presentation or a written document.	Use of library and internet resources to explore a complex topic e.g. damping in S.H.M. (M3).

C3.3 WRITING			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
write <b>two</b> different types of documents about complex subjects. <b>One</b> piece of writing should be an extended document and include at least <b>one</b> image.	<ul style="list-style-type: none"> <li>select and use appropriate style of writing</li> <li>organise relevant information clearly and coherently, using specialist vocabulary</li> <li>ensure text is legible, spelling, punctuation and grammar are accurate, and that meaning is clear.</li> </ul>	The two different documents might include an extended essay or report, with an image such as a chart, graph or diagram and a letter or memo.	Written summaries of complex topics developed in classroom discussion e.g. circular motion. (Unit M2).

## APPLICATION OF NUMBER: LEVEL 1

N1.1 INTERPRET STRAIGHTFORWARD INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Interpret straightforward information from <b>two</b> different sources. At least <b>one</b> source should be a table, chart, diagram or line graph.	<ul style="list-style-type: none"> <li>Obtain the information needed to meet the purpose of the task; and</li> <li>Identify suitable calculations to get the results needed.</li> </ul>	<p>Description of the tasks and purposes. Copies of source material.</p> <p>A statement from an assessor who checked the accuracy of the student's measurements or observations (if this was done).</p> <p>Records of the information obtained and the types of calculations identified to get the results needed.</p>	Interpretation of: e.g. Distance/time graphs (M1).

N1.2 CARRY OUT STRAIGHTFORWARD CALCULATIONS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Carry out straightforward calculations to do with: <ol style="list-style-type: none"> <li>amounts and sizes;</li> <li>scales and proportion;</li> <li>handling statistics.</li> </ol>	<ul style="list-style-type: none"> <li>Carry out calculations to the levels of accuracy the student has been given; and</li> <li>Check the results make sense.</li> </ul>	Records of the calculations (for a, b and c) and how the student checked them.	Calculations involving e.g. <ol style="list-style-type: none"> <li>Areas and volumes (C1)</li> <li>Ratio and Proportion (C1).</li> </ol>

N1.3 INTERPRET THE RESULTS OF CALCULATIONS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Interpret the results of the calculations and present her/his findings. The student must use <b>one</b> chart and <b>one</b> diagram.	<ul style="list-style-type: none"> <li>Choose suitable ways to present findings;</li> <li>Present findings clearly; and</li> <li>Describe how the results of the calculations meet the purpose of the task.</li> </ul>	<p>Descriptions of the findings and how the results of the calculations met the purpose of the tasks.</p> <p>At least <b>one</b> chart and <b>one</b> diagram presenting the findings.</p>	e.g. Using charts <b>and</b> diagrams to present findings of a probability experiment (S1).

## APPLICATION OF NUMBER: LEVEL 2

Candidates must carry through at least **one** substantial activity that includes a number of straightforward **related tasks** for N2.1, N2.2 and N2.3.

N2.1 INTERPRET INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Interpret information from <b>two</b> different sources, including material containing a graph.	<ul style="list-style-type: none"> <li>Choose how to obtain the information needed to meet the purpose of the activity;</li> <li>Obtain the relevant information; and</li> <li>Select appropriate methods to get the results needed.</li> </ul>	<p>A description of the substantial activity.</p> <p>Copies of source material, including the graph, and/or a statement from someone who has checked the accuracy of the student's measurements and observations.</p> <p>Records of the information obtained and the methods selected for getting the results needed.</p>	Interpret given information from e.g. Graphs of polynomial functions (C1).

N2.2 CARRY OUT CALCULATIONS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Carry out calculations to do with: <ol style="list-style-type: none"> <li>amounts and sizes;</li> <li>scales and proportion;</li> <li>handling statistics;</li> <li>using formulae.</li> </ol>	<ul style="list-style-type: none"> <li>Carry out calculations, clearly showing methods and levels of accuracy; and</li> <li>Check methods to identify and correct any errors, and making sure the results make sense.</li> </ul>	Records of calculations (for a, b, c and d), showing methods used and levels of accuracy. Notes on how the student checked methods and results.	e.g. <ol style="list-style-type: none"> <li>Volumes of cone and sphere (C1)</li> <li>Similar figures (C1)</li> <li>Changing the subject of a formula (C1).</li> </ol>

N2.3 INTERPRETING THE RESULTS OF CALCULATIONS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Interpret the results of calculations and present findings. The student must use at least <b>one</b> graph, <b>one</b> chart and <b>one</b> diagram.	<ul style="list-style-type: none"> <li>Select effective ways of presenting findings;</li> <li>Present findings clearly, describing methods; and</li> <li>Explain how the results of the calculations meet the purpose of the study.</li> </ul>	Descriptions of findings and methods. Notes on how the results from the calculations met the purpose of the activity. At least <b>one</b> graph, <b>one</b> chart and <b>one</b> diagram presenting the findings.	e.g. Using a graph, chart <b>and</b> diagram to present findings of a probability experiment (S1).

## APPLICATION OF NUMBER: LEVEL 3

Candidates must plan and carry through at least **one** substantial and complex activity that includes a number of **related** tasks for N3.1, N3.2 and N3.3.

N3.1 INTERPRET INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Plan and interpret information from <b>two</b> different sources, including a large data set.	<ul style="list-style-type: none"> <li>Plan how to obtain the information required to meet the purpose of the activity;</li> <li>Obtain the relevant information; and</li> <li>Choose appropriate methods for obtaining the results needed and justify the choice.</li> </ul>	A description of the activity and tasks. Copies of source material, including a note of the large data set. A statement from someone who has checked the accuracy of any measurements or observations. Records and a justification of methods selected.	Experiment which involves the collection of a large data set, e.g. determining a probability. (S2).

N3.2 CARRY OUT MULTI-STAGE CALCULATIONS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Carry out multi-stage calculations to do with: <ol style="list-style-type: none"> <li>amounts and sizes;</li> <li>scales and proportion;</li> <li>handling statistics;</li> <li>rearranging and using formulae.</li> </ol>	<ul style="list-style-type: none"> <li>Carry out calculations to appropriate levels of accuracy, clearly showing methods; and</li> <li>Check methods and results to help ensure errors are found and corrected.</li> </ul>	Records of calculations (for a, b, c and d). Showing methods used and levels of accuracy. Notes on the large data set and how the methods and results were checked.	Multi-stage calculations pervade the specification in Mathematics. (Appropriate to all units).

N3.3 INTERPRETING THE RESULTS OF CALCULATIONS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Interpret the results of calculations, present findings and justify methods. The student must use at least <b>one</b> graph, <b>one</b> chart and <b>one</b> diagram.	<ul style="list-style-type: none"> <li>Select appropriate methods of presentation and justify choice;</li> <li>Present findings effectively; and</li> <li>Explain how the results of the calculations relate to the purpose of the activity.</li> </ul>	Report justifying methods and explanation of how results relate to the activity. At least <b>one</b> graph, <b>one</b> chart and <b>one</b> diagram.	Findings presented using graphs, charts and diagrams as appropriate and within acceptable degrees of accuracy e.g. graphical treatment of functions. (C3).



## INFORMATION TECHNOLOGY: LEVEL 1

IT 1.1 FIND, STORE AND DEVELOP INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
find, explore and develop information for <b>two</b> different purposes.	<ul style="list-style-type: none"> <li>find and select relevant information</li> <li>enter and bring in information, using formats that help development</li> <li>explore and develop information to meet the student's purpose.</li> </ul>	Print-outs and copies of the information the student selects to use. A record from an assessor who observed the student using IT when exploring and developing information or working drafts with notes of how the student met the requirements of the Unit.	e.g. Use of mathematics software package in order to study the graphs or straight lines (C1).

IT 1.2 PRESENT INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
present information for <b>two</b> different purposes. The student's work must include <b>at least one</b> example of text, <b>one</b> example of images, and <b>one</b> example of numbers.	<ul style="list-style-type: none"> <li>use appropriate layouts for presenting information in a consistent way</li> <li>develop the presentation so it is accurate, clear and meets the purpose</li> <li>save information so it can be found easily.</li> </ul>	Working drafts showing how the student developed the presentation or records from an assessor who saw the student's screen displays. Print-outs or prints of a static or dynamic screen display of the students final work, including examples of text, images and numbers. Records of how the student saved information.	e.g. Presentation and manipulation of data using a spreadsheet package on simulated experiments using a PC (S1).

## INFORMATION TECHNOLOGY: LEVEL 2

IT 2.1 SEARCH FOR AND SELECT INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
search for and select information for <b>two</b> different purposes.	<ul style="list-style-type: none"> <li>identify the information needed and suitable sources</li> <li>carry out effective searches</li> <li>select information that is relevant to the student's purpose.</li> </ul>	Print-outs of the relevant information with notes of sources and how the student made searches, or a record from an assessor who observed the student using IT when searching for information.	e.g. Use of mathematical software package in order to study the graphs of polynomial functions (C1).

IT 2.2 EXPLORE AND DEVELOP INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
explore and develop information, and derive new information, for <b>two</b> different purposes.	<ul style="list-style-type: none"> <li>enter and bring together information using formats that help developments</li> <li>explore information as needed for the purpose</li> <li>develop information and derive new information as appropriate.</li> </ul>	Print-outs, or a record from an assessor who observed the student using IT, with notes to show how the student explored and developed information and derived new information.	e.g. Enter information gathered from a variety of sources into a database. Search the database to collate, analyse and present information (S1).

IT 2.3 PRESENT COMBINED INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
present combined information for <b>two</b> different purposes. The student's work must include at least <b>one</b> example of text, <b>one</b> example of images and <b>one</b> example of numbers.	<ul style="list-style-type: none"> <li>select and use appropriate layouts for presenting combined information in a consistent way</li> <li>develop the presentation to suit the purpose and the types of information</li> <li>ensure the work is accurate, clear and saved appropriately.</li> </ul>	Working drafts, or a record from an assessor who observed the screen displays, with notes to show how the student developed content and presentation. Print-outs, or prints of static or dynamic screen displays, of the final work, including examples of text, images and numbers. Records of how the information was saved.	e.g. Presentation and manipulation of data using a statistical software package to find the mean and variance (S1).

### INFORMATION TECHNOLOGY: LEVEL 3

Candidates must plan and carry through at least **one** substantial activity that includes a number of related tasks for IT3.1, IT3.2 and IT3.3.

IT 3.1 SEARCH AND SELECT INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
compare and use different sources to search for, and select, information required for <b>two</b> different purposes.	<ul style="list-style-type: none"> <li>plan how to obtain and use information</li> <li>choose appropriate techniques for searches</li> <li>make selections based on judgements.</li> </ul>	<p>Print-outs with notes of sources and how searches made and selected information</p> <p>A record from someone who observed use of IT to search for and explore information.</p>	Use a Mathematics software package in order to study functions e.g. use of the "Derive" package. (C1-C4, FP1-FP3).

IT 3.2 DEVELOP INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
explore, develop and exchange information and derive new information to meet <b>two</b> different purposes.	<ul style="list-style-type: none"> <li>bring together information in consistent form</li> <li>create and use appropriate structures</li> <li>use methods for exchanging information.</li> </ul>	<p>Print-outs or record of someone who observed use of IT showing how information has been exchanged, explored and developed.</p> <p>Notes of automated routines</p>	Use up-to-date data obtained from the internet with appropriate software packages e.g. databases used to develop statistical techniques. (S1-S3, MS1, MS2).

IT 3.3 PRESENT INFORMATION			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
present information from different sources for <b>two</b> different purposes and audiences. <b>One</b> example of text, <b>one</b> of images and <b>one</b> of numbers.	<ul style="list-style-type: none"> <li>develop structures and content</li> <li>present information effectively</li> <li>ensure work is accurate.</li> </ul>	<p>Working drafts or a record from an assessor who observed screen displays, showing how developed for presentation.</p> <p>Print-outs or a static or dynamic screen display of final work, including text, images and numbers.</p>	Presentation and manipulation of data using a variety of software packages e.g. use of spread-sheets in iterative processes. (C3, FP3).

## WIDER KEY SKILLS

## PROBLEM SOLVING: LEVEL 1

Candidates must provide at least **two** examples of meeting the standard for PS1.1, PS1.2 and PS1.3.

PS 1.1 CONFIRM PROBLEMS AND IDENTIFY OPTIONS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
confirm understanding of the given problem and identify at least <b>two</b> options for solving it, with help from an appropriate person.	<ul style="list-style-type: none"> <li>check that the problem is understood, and how to succeed in solving it</li> <li>identify different ways of tackling the problem</li> <li>decide, with help, which options are most likely to be successful.</li> </ul>	<p>Descriptions of the two problems and how they have been solved.</p> <p>Descriptions of ways for solving the two problems and the options most likely to be successful.</p> <p>Records of help given.</p>	<p>Identify a task for investigation e.g. probability of assessment</p> <p>(a) by experiment,</p> <p>(b) using theoretical methods (S1).</p>

PS 1.2 PLAN AND TRY OUT OPTIONS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
plan and try out at least <b>one</b> option for solving the problem, using given evidence and support.	<ul style="list-style-type: none"> <li>confirm with an appropriate person the option to be tried for solving the problem</li> <li>plan how to carry out this option</li> <li>follow through the plan, making use of advice and support given by others.</li> </ul>	<p>Statements on how the student confirmed the options to be tried out.</p> <p>A plan for trying out each option.</p> <p>Records of what was done in following the plan, with notes on the advice and support given.</p>	<p>Discuss an appropriate plan of action with the class teacher for carrying out an investigation.</p> <p>(Appropriate to all units).</p>

PS 1.3 CHECK AND DESCRIBE RESULTS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
check if the problem has been solved and describe the results, including ways to improve the approach.	<ul style="list-style-type: none"> <li>check whether the problem has been solved successfully</li> <li>describe clearly the results of tackling the problem</li> <li>identify ways of improving the approach to problem solving.</li> </ul>	<p>Records of the methods given and they were used.</p> <p>Descriptions of the results of tackling the problems and ways to improve the approach to problem solving.</p>	<p>Produce a suitable report on an investigation identifying the methods used and giving suggestions for improving the approach to the problem.</p> <p>(Appropriate to all units).</p>

**PROBLEM SOLVING: LEVEL 2**

Candidates must provide at least **two** examples of meeting the standard for PS2.1, PS2.2 and PS2.3.

<b>PS 2.1 IDENTIFY PROBLEMS AND OPTIONS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
identify the problem and come up with <b>two</b> options for solving it.	<ul style="list-style-type: none"> <li>identify the problem, accurately describing its main features and how to show success in solving it</li> <li>come up with different ways of tackling the problem</li> <li>decide which options have a realistic chance of success, using help from others when appropriate.</li> </ul>	<p>Descriptions of the two problems and how the student is going to show they have been solved successfully.</p> <p>Descriptions of ways for solving the two problems.</p> <p>Records of how the student decided which options were most realistic, including the help obtained.</p>	<p>Identify a topic for investigation e.g. probability of an event</p> <p>(a) by experiment,</p> <p>(b) by computer simulation (S1).</p>
<b>PS 2.2 PLAN AND TRY OUT OPTIONS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
plan and try out at least <b>one</b> option for solving the problem, obtaining support and making changes to the plan when necessary.	<ul style="list-style-type: none"> <li>confirm with an appropriate person the option to be tried for solving the problem, and plan how to carry it out</li> <li>follow the plan, organising the relevant tasks and making changes to the plan when necessary</li> <li>obtain and effectively use support needed.</li> </ul>	<p>Statements on how the options were confirmed and tried out.</p> <p>A plan for trying out each option.</p> <p>Records of what was done, including any changes made to the plan.</p> <p>Notes of the support obtained and how this was used effectively.</p>	<p>Discuss an appropriate plan of action with the class teacher for carrying out the investigation.</p> <p>(Appropriate to all units).</p>
<b>PS 2.3 CHECK AND DESCRIBE RESULTS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
check if the problem has been solved by applying given methods, describe the results and explain the approach to problem solving.	<ul style="list-style-type: none"> <li>apply accurately the methods given to check whether the problem has been solved successfully</li> <li>describe clearly the results, and explain the decisions taken at each stage of tackling the problem</li> <li>identify the strengths and weaknesses of the approach to problem solving and describe what would be done differently if a similar problem were met.</li> </ul>	<p>Records of the methods used, the results of the checks carried out and explanations of the decisions taken.</p> <p>Descriptions of the strengths and weaknesses of the approach to the problem solving activities, and what would be done differently in future.</p>	<p>Identify a topic for investigation and evaluate the outcomes.</p> <p>(Appropriate to all units).</p>

**PROBLEM SOLVING: LEVEL 3**

Candidates must provide at least **one** substantial example of meeting the standard of PS3.1, PS3.2 and PS3.3.

<b>PS 3.1 EXPLORE PROBLEMS AND OPTIONS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
explore a complex problem, come up with <b>three</b> options for solving it and justify the options selected for taking it forward.	<ul style="list-style-type: none"> <li>explore the problem, accurately analysing its features, and agree with others on how to show success in solving it</li> <li>select and use a variety of methods to come up with different ways of tackling the problem</li> <li>compare the main features of each possible option, including risk factors, and justify the option selected to take it forward.</li> </ul>	<p>Description of the problem, the analysis of its features and methods used for exploring it</p> <p>Statements endorsed by appropriate people of how problem was going to be solved</p> <p>Descriptions of the <b>three</b> options for solving the problem, with notes on the methods used for coming up with these and comparisons of their main features</p> <p>A note to justify the chosen option.</p>	<p>Class discussion on solving a mathematical problem. Consideration by class and teacher of at least 3 solutions, suggested by pupils. e.g. investigating the probability of an event via</p> <p>(i) class experiment, (ii) computer simulation, (iii) theoretical techniques. (S1).</p>

<b>PS 3.2 PLAN AND IMPLEMENT OPTIONS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
plan and implement at least <b>one</b> option for solving a problem, review progress and revise approach as necessary.	<ul style="list-style-type: none"> <li>plan how to carry out the chosen option and obtain agreement to go ahead from an appropriate person</li> <li>implement plan, effectively using support and feedback from others</li> <li>review progress towards solving the problem and revise approach as necessary.</li> </ul>	<p>A plan, with notes of changes made, and endorsed statement of how agreement to go ahead with chosen option was obtained</p> <p>Records of how plan is implemented, including how support and feedback was used and how progress was reviewed.</p>	<p>Pupils should develop a technique for solving a mathematical problem and present a report which should include conclusions from their results e.g. introduction to S.H.M., using an oscillating spring. (M3).</p>

<b>PS 3.3 CHECK AND REVIEW APPROACH</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
apply agreed methods to check if the problem has been solved, describe the results and review approach to problem solving.	<ul style="list-style-type: none"> <li>agree, with an appropriate person, methods to check if the problem has been solved</li> <li>apply these methods accurately, draw conclusions and fully describe the results</li> <li>review approach problem solving, including whether alternative methods and options might have proved more effective.</li> </ul>	<p>Description of the methods used, the results and conclusions</p> <p>Records of review, including notes of any alternative methods and options which might be predicted to have been more effective.</p>	<p>Discussion by class and teacher of a solution to a mathematical problem to ensure that the problem has been addressed appropriately. (Appropriate to all units).</p>

**WORKING WITH OTHERS: LEVEL 1**

Students must provide at least **one** example of meeting the standard for WO1.1, WO1.2 and WO1.3

- **one** example must show work in one-to-one situations
- **one** example must show work in group situations

<b>WO 1.1 CONFIRM WHAT TO DO</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
confirm what needs to be done to achieve given objectives, including responsibilities and working arrangements.	<ul style="list-style-type: none"> <li>• check understanding of the objectives the student has been given for working together</li> <li>• identify what needs to be done to achieve them and suggest ways the student could help</li> <li>• make sure that the student is clear about her/ his responsibilities and working arrangements.</li> </ul>	Records from someone who observed the student's discussions with others or audio/video tapes. Notes of the objectives, responsibilities and working arrangements.	Plan an investigation into a mathematical problem with others in the group or with selected individuals e.g. friction experiments on inclined planes (M1).

<b>WO 1.2 WORK TOWARDS OBJECTIVES</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
work with others towards achieving the given objectives, carrying out tasks to meet responsibilities.	<ul style="list-style-type: none"> <li>• carry out tasks to meet responsibilities</li> <li>• work safely, and accurately follow the working methods the student has been given</li> <li>• ask for help and offer support to others, when appropriate.</li> </ul>	Records of how the student carried out tasks to meet responsibilities. Notes of the help given and the support the student offered others. These records could include a log, statements written by others with whom the student worked, audio/video tape recordings, photographs with notes.	Establish links with other individuals, with the group with a view to collecting relevant data to investigate a mathematical problem e.g. experiments relating to the laws of mechanics e.g. Fletcher's Trolley (M1).

<b>WO 1.3 IDENTIFY PROGRESS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
identify progress and ways of improving work with others to help achieve given objectives.	<ul style="list-style-type: none"> <li>• identify what has gone well and less well in working with others</li> <li>• report any difficulties in meeting responsibilities and what was done about them</li> <li>• suggest ways of improving work with others to help achieve objectives.</li> </ul>	Statements (written or recorded). Records of answers to questions about any difficulties and what the student did about them. Notes of ways to improve work with others.	Develop a time-plan. Arrange meetings to monitor progress. Reflect on ways in which collaborative working could be improved. (Appropriate to all units).

**WORKING WITH OTHERS: LEVEL 2**

Students must provide at least **two** examples of meeting the standard WO2.1, WO2.2 and WO3.3

- **one** example must show working in one-to-one situations
- **one** example must show working in group situations

<b>WO 2.1 PLAN WORK</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
plan straightforward work identifying objectives and clarifying responsibilities and confirm working arrangements.	<ul style="list-style-type: none"> <li>• identify the objectives of working together and what needs to be done to achieve these objectives</li> <li>• exchange relevant information to clarify responsibilities</li> <li>• confirm working arrangements with those involved.</li> </ul>	Records from someone who observed the student's discussions with others or audio/video tapes. Note of the information provided, with details of the identified objectives, responsibilities and working arrangements of those involved.	Plan an investigation into a mathematical problem with others in the group or with selected individuals e.g. survey of particular attributes of objects to determine probabilities (S1).

<b>WO 2.2 WORK TOWARDS OBJECTIVES</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
work co-operatively with others towards achieving the identified objectives, organising tasks to meet responsibilities.	<ul style="list-style-type: none"> <li>• organise own tasks so the student can be effective in meeting responsibilities;</li> <li>• carry out tasks accurately and safely, using appropriate working methods</li> <li>• support co-operative ways of working, seeking advice from an appropriate person when needed.</li> </ul>	Records of how the student organised and carried out tasks, supported co-operative work and sought advice. These records could include a log, statements written by others with whom the student worked, audio/video tape recordings, photographs with notes.	Establish lines with other individuals within the group with a view to collecting relevant data to investigate a mathematical problem e.g. survey of particular attributes of objects to determine probabilities (S1).

<b>WO 2.3 EXCHANGE INFORMATION ON PROGRESS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
exchange information on progress and agree ways of improving work with others to help achieve objectives.	<ul style="list-style-type: none"> <li>• provide information on what has gone well and less well in working with others, including the quality of work</li> <li>• listen and respond appropriately to progress reports from others</li> <li>• agree ways of improving work with others to help achieve objectives.</li> </ul>	Statements on progress (written or recorded) including details about the quality of work and how the student responded to other reports on progress. Notes of what the student agreed to do to improve work with others and help achieve objectives.	Develop a time-plan. Arrange meetings to monitor progress. Reflect on ways in which collaborative working could be improved. (Appropriate to all units).



**WORKING WITH OTHERS: LEVEL 3**

Students must provide at least **one** substantial example of meeting the standard for WO3.1, WO3.2 and WO3.3 in both one-to-one and group situations.

<b>WO 3.1 PLAN WORK</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
plan complex work with others, agreeing objectives, responsibilities and working arrangements.	<ul style="list-style-type: none"> <li>agree realistic objectives for working together and what needs to be done to achieve them</li> <li>exchange information, based on appropriate evidence to help agree responsibilities</li> <li>agree suitable working arrangements with those involved.</li> </ul>	Reports which describe how the student planned work with others, including objectives, responsibilities, and working arrangements. Records from someone who observed the discussions with others or audio/video tape.	Plan an investigation into a mathematical problem with others in the class or with selected individuals e.g. investigating the laws of friction. (M1).

<b>WO 3.2 WORK TOWARDS OBJECTIVES</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
seek to establish & maintain cooperative working relationships over an extended period of time, agreeing changes to achieve agreed objectives.	<ul style="list-style-type: none"> <li>organise and carry out tasks to show effectiveness and efficiency in meeting responsibilities and produce the quality of work required</li> <li>seek to establish and maintain cooperative working relationships, agreeing ways to overcome any difficulties</li> <li>exchange accurate information on progress of work, agreeing changes where necessary to achieve objectives.</li> </ul>	Records of how the student organized and carried out tasks and maintained cooperative working relationships, including a progress report. These records could include a log, statements written by others with whom the student worked, audio/video tape recordings, photographs, or products made, with notes.	Establish links with other individuals within the class or outside (e.g. local people, businesses, record offices etc.) with a view to collecting relevant data to solve a mathematical problem or to confirm a theoretical result e.g. to determine to what extent the distribution of data can be modelled by the normal distribution (S2).

<b>WO 3.3 REVIEW WORK</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
review work with others and agree ways of improving collaborative work in the future.	<ul style="list-style-type: none"> <li>agree the extent to which work with others has been successful and the objectives have been met</li> <li>identify factors that have influenced the outcomes</li> <li>agree ways of improving work with others in the future.</li> </ul>	Statements (written or recorded) from both the student and others on the extent to which the agreed objectives were achieved. Reports produced with others on ways to improve future collaborate work.	Monitor progress made in collecting mathematical evidence, reflecting on ways in which collaborative working could be improved. (Appropriate to all units).

**IMPROVING OWN LEARNING AND PERFORMANCE: LEVEL 1**

The candidate must provide at least **two** examples of meeting the standards for LP1.1, LP1.2 and LP1.3.

<b>LP 1.1 CONFIRM TARGETS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
confirm understanding of targets and how these will be met, with the person setting them.	<ul style="list-style-type: none"> <li>make sure targets clearly show what is wanted to be achieved</li> <li>identify action points and deadlines for each target</li> <li>make sure the dates for reviewing progress and how to get support needed are known.</li> </ul>	<p>Records of discussions which show the student checked her/his understanding of targets and knew how to get the support.</p> <p>Two action plans with action points, deadlines and dates for reviewing progress.</p>	Establish, with teachers and others, through one-to-one discussion, targets for enhancing performance. (Appropriate to all units).

<b>LP 1.2 FOLLOW A PLAN</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
<p>follow plans, using support given by others to help meet targets. Improved performance by</p> <ul style="list-style-type: none"> <li>studying a straightforward subject</li> <li>learning through a straightforward practical activity.</li> </ul>	<ul style="list-style-type: none"> <li>work through the action points to complete tasks on time</li> <li>use support given by others to help in the meeting of targets</li> <li>use different ways of learning suggested by supervisor and make changes suggested by the person supervising the student, when needed.</li> </ul>	<p>A log of study-based and activity-based learning, with notes of the support given.</p> <p>Records from those who have seen the work and which shows the tasks were completed on time and how any suggested changes were made.</p>	Monitor progress by producing a log, seeking teacher support as or when necessary. (Appropriate to all units).

<b>LP 1.3 REVIEW PROGRESS AND ACHIEVEMENTS</b>			
<b>Candidates must:</b>	<b>Evidence must show candidates can:</b>	<b>Examples of evidence:</b>	<b>Suggested context:</b>
review achievements and progress in meeting targets, with an appropriate person.	<ul style="list-style-type: none"> <li>say what it is thought has gone well and less well, what was learned and ways learning took place</li> <li>identify targets met and evidence of achievements</li> <li>check that the student understood how to improve.</li> </ul>	<p>Records of</p> <ul style="list-style-type: none"> <li>what was said about the student's progress</li> <li>her/his achievements</li> <li>what to do to improve</li> </ul> <p>Examples of work which show the student learned from studying two subjects and two practical learning activities to show targets met.</p>	Students to keep a record of all work completed and marked during the course and how they have learnt and improved from the comments made. (Appropriate to all units).

## IMPROVING OWN LEARNING AND PERFORMANCE: LEVEL 2

The candidate must provide at least **two** examples of meeting the standard for LP2.1, LP2.2 and LP2.3.

LP 2.1 SET TARGETS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
help set short-term targets with an appropriate person and plan how these will be met.	<ul style="list-style-type: none"> <li>provide accurate information to help set realistic targets for achieving what is to be done</li> <li>identify clear action points for each target</li> <li>plan how time will be used effectively to meet targets, including use of support and a date for reviewing progress.</li> </ul>	Records information provided to help set targets. Two action plans with action points, timetable and notes of support needed.	Establish, with teachers, through one-to-one discussion, targets for enhancing performance. (Appropriate to all units).

LP 2.2 USE A PLAN			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
Take responsibility for some decisions about learning, using a plan and support from others to help meet targets. Improve performance by <ul style="list-style-type: none"> <li>studying a straightforward subject</li> <li>learning through a straightforward practical activity.</li> </ul>	<ul style="list-style-type: none"> <li>use action points to help manage time well and complete tasks</li> <li>identify when support is needed and use this effectively to help the meeting of targets</li> <li>select and use different ways of learning to improve performance.</li> </ul>	A log of learning, with notes of: <ul style="list-style-type: none"> <li>when the student asked for support and it was used</li> <li>when and how the student worked without close supervision</li> <li>any changes made to the plan.</li> </ul> Records from those who saw the work which show the student managed her/his time well and completed tasks.	Establish, with teachers and others, through one-to-one discussion, targets for enhancing performance. (Appropriate to all units).

LP 2.3 REVIEW PROGRESS AND ACHIEVEMENTS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
review progress with an appropriate person and provide examples of evidence of achievements, including how learning was used from one task to meet the demands of a new one.	<ul style="list-style-type: none"> <li>identify what and how was learnt, including what has gone well and what has gone less well</li> <li>identify targets met, and examples of evidence of achievements</li> <li>identify ways of improving own performance.</li> </ul>	Records of information provided on progress and ways of improving performance. Examples of work which show what was learned from two study-based and two activity-based learning activities. Notes on personal action plans to show targets met.	Establish, with teachers and others, through one-to-one discussion, targets for enhancing performance. (Appropriate to all units).

### IMPROVING OWN LEARNING AND PERFORMANCE: LEVEL 3

Candidates must provide at least **one** substantial example of meeting the standard for LP3.1, LP3.2 and LP3.3.

LP 3.1 AGREE TARGETS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
agree targets and plan how these will be met over an extended period of time, using support from appropriate people.	<ul style="list-style-type: none"> <li>seek information on the ways to achieve what they want and identify factors that might affect plans</li> <li>use this information to agree realistic targets with appropriate people</li> <li>plan how time will be effectively managed and use support to meet targets, including alternative action for overcoming possible difficulties.</li> </ul>	<p>Records to show how the student obtained and used information to agree targets</p> <p>An action plan for an extended period of time (eg. about three months) including alternative courses of action and a note of supported needed.</p>	Establish with teachers and others, through one-to-one discussion, targets for enhancing performance (e.g. organising work programmes, researching a problem). (Appropriate to all units).

LP 3.2 USE A PLAN			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
<ul style="list-style-type: none"> <li>take responsibility for learning by using plan, seeking feedback and support from relevant sources, to help meet targets</li> <li>improve performance by:               <ul style="list-style-type: none"> <li>studying a complex subject</li> <li>learning through a complex practical activity</li> <li>further study or practical activity involving independent learning.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>manage time effectively to complete tasks, revising plan if necessary</li> <li>seek and actively use feedback and support from relevant sources to meet targets</li> <li>select and use different ways of learning to improve performance, adapting approaches to meet new demands.</li> </ul>	<p>A log of learning, with notes of:</p> <ul style="list-style-type: none"> <li>how the student learned in different ways and adapted his/her approach</li> <li>when the student sought feedback and support and how he/she used it</li> <li>any revisions made to the plan</li> </ul> <p>Records from those who have seen the work managed effectively and tasks were completed.</p>	Monitor progress by producing a log. This should include details of the approach to a mathematical problem together with the sources of the evidence collected. (Appropriate to all units).

LP 3.3 REVIEW PROGRESS AND ACHIEVEMENTS			
Candidates must:	Evidence must show candidates can:	Examples of evidence:	Suggested context:
review progress on <b>two</b> occasions and establish evidence of achievement, including how learning from other tasks has been used to meet new demands.	<ul style="list-style-type: none"> <li>provide information on the quality of learning and performance, including factors that have affected the outcome</li> <li>identify targets met, seek relevant sources to establish evidence of achievements</li> <li>exchange views with appropriate people to agree ways to further improve performance.</li> </ul>	<p>Records of information provided by the student on his/her learning and performance, including how he/she used learning from other tasks to meet new demands</p> <p>Examples of work which show what the student learned from studying complex subjects, through practical activity and independent learning</p> <p>Records of discussions which show how the student sought evidence of his/her achievements and exchanged views on ways to improve performance</p> <p>Note on action plan to show targets that have been met.</p>	Keep a portfolio of all tasks which have been assessed during a course of study and how, possibly through a log, you have learnt and improved performance from comments, both verbal and written, made by the teacher and others. (Appropriate to all units).