OCR-AS-A2-Level-physics

OCR-AS-Unit-1-Mechanics (From page 1 to page 143)
OCR-AS-Unit-2-Electrons-Waves-and-Photons (From page 144 to page 309)
OCR-A2-Unit-4-The-Newtonian-World (From page 310 to page 467)
OCR-A2-Unit-5-Fields-and-Particles (From page 468 to page 640)

AS-Unit-1-Mechanics 1
Module 1: Motion 1
Chapter 1 Physical quantities and units 1
1-1 Units 1
1-1-1 Physical quantities and SI units 1
1-1-2 Prefixes 2
1-2 1 Worked examples 2
Chapter 2 Scalars and vectors 3
2-1 Addition of vectors 3
2-2 Resolving a vector into two perpendicular components 5
2-3 10 Worked examples 6
Chapter 3 Kinematics and Linear motion 14
3-1 Displacement and velocity 14
3-2 Acceleration 15
3-3 Equations for uniform acceleration 16
3-4 Displacement—time graphs 17
3-5 Velocity—time graphs 18
3-6 Free-fall motion 18
3-7 15 Worked examples 19
Module 2: Forces in action 37
Chapter 1 Force and Nonlinear motion 37
1-1 Force definition 37
1-2 Weight and g 37
1-3 Newton’s first law of motion 38
1-4 Newton’s second law 38
1-5 Newton’s third law of motion 40
1-6 drag force and terminal speed 40
1-7 9 Worked examples 43
Chapter 2 Equilibrium 49
2-1 balanced forces 49
2-2 moments 49
2-3 Couples and torque of a couple 50
2-4 the principle of moments 51
2-5 Centre of gravity and Determination of Centre of Gravity (c.g.) of irregular lamina using the plumb line method 52
2-6 Density and Pressure 53
2-7 23 Worked examples 54
Chapter 3 Car safety 78
3.1. Stopping distance 78
3.2. Car safety 78

3.3 2 Worked examples 81
Module 3: Work and energy 83
3.1 Work and conservation of energy 83
3.1.1 Work 83
3.1.2 Energy 84
3.1.3 Efficiency 85
3.2 Kinetic and potential energies 85
3.2.1 Kinetic energy Theorem of kinetic energy 85
3.2.2 Gravitational potential energy 87
3.2.3 The law of conservation of mechanical energy 87
3.3 Power 88
3.3.1 Power 88
3.3.2 Power and velocity 88
3.4 15 Worked examples 88
3.5 Behaviour of springs and materials 105
3-5-1 Deformation 105
3-5-2 tensile stress and tensile strain 105
3-5-3 Hooke's law 106
3-5-4 Young's modulus 106
3-5-5 Strain energy 107
3-5-6 Materials stretching 108
3-5-6-1 Stretching glass 108
3-5-6-2 stretching a metal 108
3.6 15 Worked examples 109
ocr_66622_pp_11_jun_gce_ug481 128

AS-Unit-1-Mechanics

Module 1: Motion

Chapter 1 Physical quantities and units

1-1 Units

1-1-1 Physical quantities and SI units

In general, a physical quantity is made up of two parts: numerical magnitude + unit.

For example, the distance from school to your home is 1000 m. then 1000 is the numerical magnitude and m (meter) is its unit.

(i) SI Units

There are seven SI Units shown in **table 1.1**:

Table 1.1 SI Units

Base quantity	SI Units	
	Name	Symbol
Mass	Kilogram	kg
Length	Meter	m
Time	Second	s
Thermodynamic temperature	Kelvin	K
Electric current	Ampere	A
Amount of substance	Mole	mol
Luminous intensity	candela	cd

Other units are derived from these: (**table 1.2**)

Table 1.2 Examples of SI derived Units

Physical quantity	Defining equation	Derived unit	Special symbol
Speed	Distance × time	$m \cdot s^{-1}$	--
Acceleration	Speed/time	$m \cdot s^{-2}$	--
Force	mass×acceleration	$kg \cdot m \cdot s^{-2}$	N(Newton)
Work	force×distance	$N \cdot m$	J(joule)
Density	Mass/volume	$kg \cdot m^{-3}$	--
Charge	current×time	$A \cdot s$	C(coulomb)
Pressure	Force/area	$N \cdot m^{-2}$	Pa(Pascal)
Resistance	Voltage/current	$V \cdot A^{-1}$	Ω (ohm)
voltage	Energy/charge	$J \cdot C^{-1}$	V(volt)

1-1-2 Prefixes

Prefixes can be added to SI and derived units to make larger or smaller units as shown in **table 1.3**:

Table 1.3 Prefixes

Value	prefix	symbol	Value	prefix	symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zeta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	**m**
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	**M**	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

For example:

1 kilometer = 1 km = 10^3 m

1 microgram = 1 μ g = 10^{-6} g

1 mega meter = 1 **M** m = 10^6 m

1 millimeter = 1 **m** m = 10^{-3} m

1-2 1 Worked examples

1. A volume is measured to be 25 mm^3, express the volume in m^3.

Solution:

$1mm = 10^{-3}\,m$, thus

$$(1mm)^3 = \left(10^{-3}\,m\right)^3 = 10^{-9}\,m^3$$

Chapter 2 Scalars and vectors

2-1 Addition of vectors

1.1 Definition of scalars and vectors

Scalar: quantity has direction only.

Examples of scalar: mass, temperatures, volume, work…

Vector: quantity both has magnitude and direction

Examples of vectors: force, acceleration, displacement, velocity, momentum…

Representation of vectors: any vectors can be represented by a straight line with an arrow whose length represents the magnitude of the vectors, and the direction of the arrow gives the direction of the vectors.

Vector Notation: use an arrow $\vec{A}$, $\vec{S}$, $\vec{B}$…

Or use the bold letter **A, B, S**…

When considering the magnitude of a vector only, we can use the italic letter *A, B, S*…

1.2 Addition of vectors:

When adding vectors, the units of the vectors must be the same, the direction must be taken into account.

Addition Principles:

i : if two vectors are in the same direction: the magnitude of the resultant vector is equal to the sum of their magnitudes, in the same direction.

ii : if two vectors are in the opposite direction: the magnitude of the resultant vector is equal to the difference of the magnitude of the two vectors and is in the direction of the greater vector.

iii: if two vectors are placed tail-to-tail at an angle θ, it can also be represented as a closed triangle (**Fig. 2.1**).

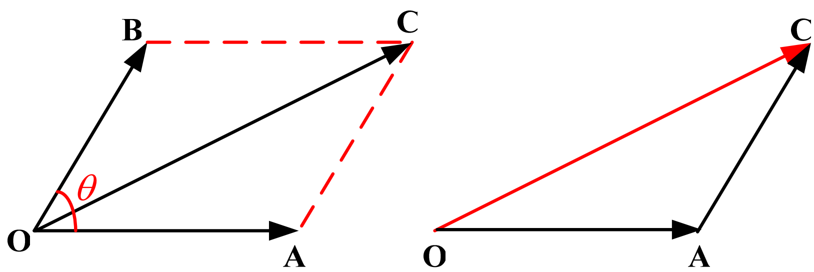


Fig. 2.1 addition principles of vectors

$\overrightarrow{OA}+\overrightarrow{AC}=\overrightarrow{OC}$ Because $\overrightarrow{OB}=\overrightarrow{AC}$

$\overrightarrow{OA}$ *and* $\overrightarrow{OB}$ are placed tail to tail to form two adjacent sides of a parallelogram and the diagonal $\overrightarrow{OC}$ gives the sum of the vectors $\overrightarrow{OA}$ *and* $\overrightarrow{OB}$. This is also called as 'parallelogram rule of vector addition.

Addition Methods:

(i): Graphical Methods----using scale drawings

For example:

$\mathbf{F_1}$ and $\mathbf{F_2}$ are at right angle, and $\mathbf{F_1}$ = 3 N, $\mathbf{F_2}$ = 4 N, determine the resultant force **F** (**Fig. 2.2**).

Let 1cm=1N

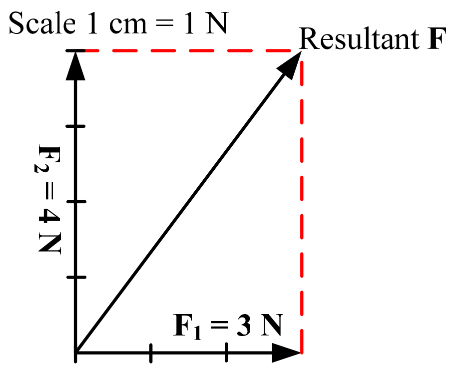


Fig. 2.2 addition methods—scale drawings

Measure the length of the resultant vector, we get length = 5cm, then resultant force, F = 5 N.

(ii) Algebraic Methods

For example:

$\mathbf{F_1}$ and $\mathbf{F_2}$ are at right angle, and $\mathbf{F_1}$ = 3 N, $\mathbf{F_2}$ = 4 N, determine the resultant force **F (Fig. 2.3)**.

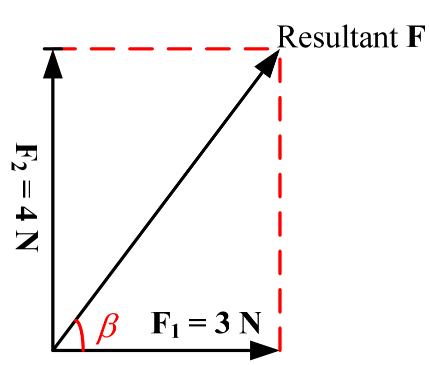


Fig. 2.3 Algebraic methods

Using the Pythagorean Theorem:

Magnitude of the resultant force, $F = \sqrt{F_1^{\,2} + F_2^{\,2}} = \sqrt{3^2 + 4^2} = 5N$

The angle β between F and F_1 is given by:

$$\tan\beta = \frac{F_2}{F_1} = \frac{4}{3}$$

Or

$$\sin\beta = \frac{F_2}{F} = \frac{4}{5}$$

Or

$$\cos\beta = \frac{F_1}{F} = \frac{3}{5}$$

2-2 Resolving a vector into two perpendicular components

For example, for a vector $\overrightarrow{OC}$, θ is known, resolving it horizontally and vertically (**Fig. 2.4**).

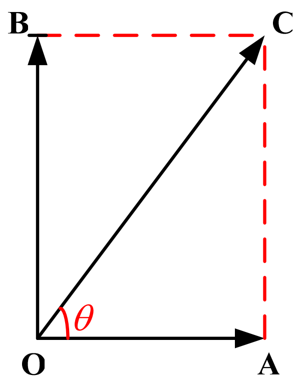


Fig. 2.4 Resolving a vector into two perpendicular components

Magnitude of Horizontally component $OA = OC\cos\theta$

Magnitude of vertically component $OB = OC\sin\theta$

Thus, a force can be resolved into two perpendicular components (**Fig. 2.5**): F and θ are known.

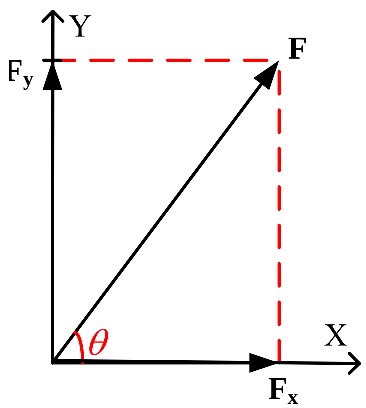


Fig. 2.5 Resolving a force into two perpendicular components

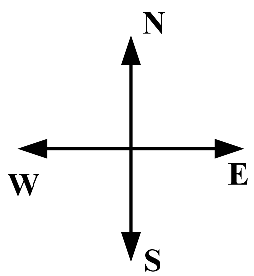
$F_x = F\cos\theta$ $F_y = F\sin\theta$

2-3 10 Worked examples

1. Representation of vectors:

(i) A displacement of 500 m due east

Represent the displacement:



Let scale: $1cm = 100m$

Then

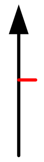
Note: of course you can also let scale: $1cm = 250m$

Then:

(ii) A force of $\vec{F} = 100N$ (or **F**=100N) due north.

Let scale: $1cm = 50N$

Then



2. Addition of the vectors

(i) are in the same direction.

Magnitude of the resultant $F = F_1 + F_2 = 11N$

Direction: the same direction of $\vec{F}_1$ *and* $\vec{F}_2$

(ii) $\vec{F}_1$ *and* $\vec{F}_2$ are in the opposite direction.

Magnitude of the resultant $F = F_2 - F_1 = 4N$

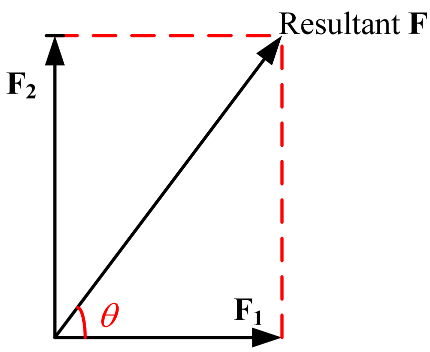
Direction: the same direction of $\vec{F}_2$

(iii) $\vec{F}_1$ *and* $\vec{F}_2$ are at right angles to each other.

Using the algebraic methods:

Magnitude of the resultant:

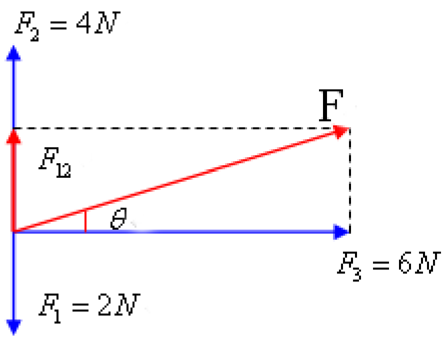
$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{12.25 + 68.5} = 9N$$



Direction:

$$\tan\theta = \frac{F_2}{F_1} = \frac{7.5}{3.5} = 2.14 \quad \textit{Then} \quad \theta = \arctan 2.14$$

3. Calculate the resultant force of F_1, F_2, F_3



Strategy: ① calculate the resultant of F_1 *and* F_2

$F_{12} = F_2 - F_1 = 2N$

② calculate the resultant force of F_{12} *and* F_3, that is the resultant force of F_1, F_2 *and* F_3

Magnitude of resultant force:

$$F = \sqrt{{F_{12}}^2 + {F_3}^2} = \sqrt{2^2 + 6^2} = 6.32N$$

Direction:

$$\tan\theta = \frac{F_{12}}{F_3} = \frac{2}{6} = \frac{1}{3}$$

$$\theta = \arctan\frac{1}{3}$$

4. A crane is used to raise one end of a steel girder off the ground, as shown in **Fig. 4.1**. When the cable attached to the end of the girder is at 20°to the vertical, the force of the cable on the girder is 6.5kN. Calculate the horizontal and vertical components of this force.

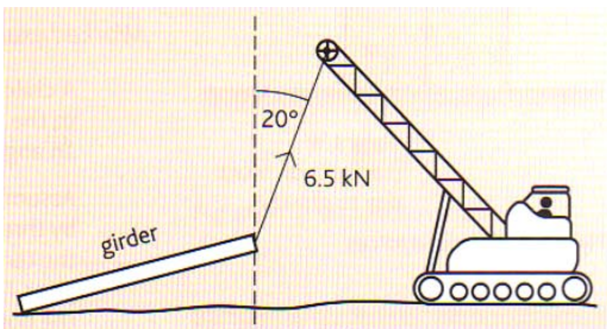
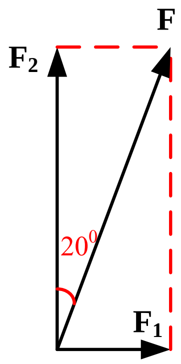


Fig. 4.1

Strategy:

Resolving the force F = 6.5 kN



$F_1 = F\sin 20^o = 6.5\sin 20^o = 2.2kN$ (Horizontal components of the force)

$F_2 = F\cos 20^o = 6.5\cos 20^o = 6.1kN$ (Vertical components of the force)

5. (a) (i) State what is meant by a scalar quantity.

Scalar quantity: quantity has direction only.

(ii) State two examples of scalar quantities.

Example 1: mass

Example 2: temperatures

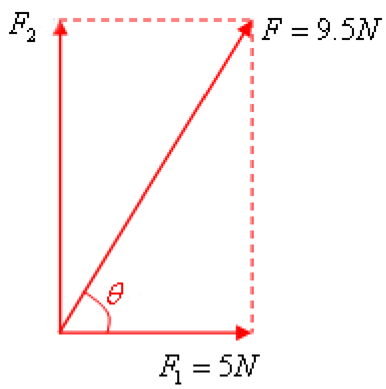
(b) An object is acted upon by two forces at right angles to each other. One of the forces has a magnitude of 5.0 N and the resultant force produced on the object is 9.5 N.

Determine

(i) The magnitude of the other force,

Strategy: adding of vectors, using the Algebraic Methods

Draw the forces below:



And $F_1^2 + F_2^2 = F^2$

$5^2 + F_2^2 = 9.5^2$

So, $F_2 = 8.1N$

(ii) The angle between the resultant force and the 5.0 N force.

$\cos\theta = \frac{F_1}{F} = \frac{5}{9.5} = 0.53$

So $\theta = \arccos 0.53 = 58^0$

6. (a) State the difference between vector and scalar quantities.

Answers: Vector quantities have direction and scalar quantities do not.

(b) State one example of a vector quantity (other than force) and one example of a scalar quantity.

Vector quantity: velocity, acceleration.

Scalar quantity: mass, temperature.

(c) A 12.0 N force and a 8.0 N force act on a body of mass 6.5 kg at the same time. For this body, calculate

(i) The maximum resultant acceleration that it could experience,

Strategy: by the Newton's second law, $F = ma$, the maximum resultant acceleration when the body has the maximum resultant force. And when the two forces are at the same direction, the body has the maximum resultant force.

So, resultant force, $F = F_1 + F_2 = 12 + 8 = 20N$

So the maximum resultant acceleration, $a = \frac{F}{m} = \frac{20}{6.5} = 3.1ms^{-2}$

(ii) The minimum resultant acceleration that it could experience.

Strategy: by the Newton's second law, $F = ma$, the minimum resultant acceleration when the body has the minimum resultant force. And when the two forces are at the opposite direction, the body has the minimum resultant force.

That is, resultant force, $F = F_1 - F_2 = 12 - 8 = 4N$

So the minimum resultant acceleration, $a = \frac{F}{m} = \frac{4}{6.5} = 0.62ms^{-2}$

7. **Figure 7.1** shows a uniform steel girder being held horizontally by a crane. Two cables are attached to the ends of the girder and the tension in each of these cables is T.

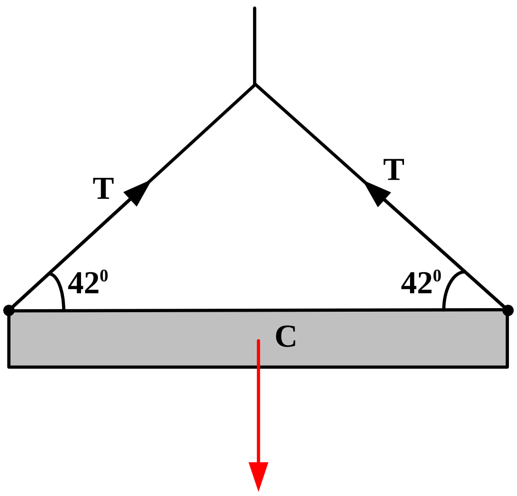


Fig. 7.1

(a) If the tension, T, in each cable is 850 N, calculate

(i) The horizontal component of the tension in each cable,

Answers: $T_h = T\cos 42 = 850 \times \cos 42 = 632\ N$

(ii) The vertical component of the tension in each cable,

$T_v = T\sin 42 + T\sin 42 = 1138N$

(iii) The weight of the girder.

Strategy: the girder is at a uniform state, so the weight of the girder is equal to the vertical component of the tension.

So weight, $W = T_v = 1138N$

(b) On **Figure 7.1** draw an arrow to show the line of action of the weight of the girder.

8. Which of the following contains three scalar quantities?

A	Mass	Charge	Speed
B	Density	Weight	Mass
C	Speed	Weight	Charge
D	Charge	Weight	Density

Solution:

Scalar: quantity has direction only.

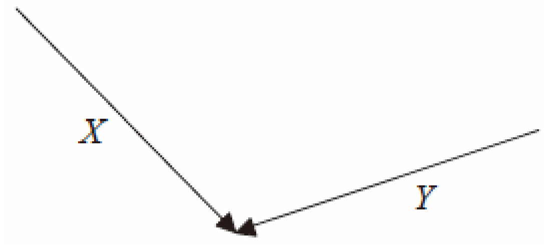
Examples of scalar: mass, temperatures, volume, work…

Vector: quantity both has magnitude and direction

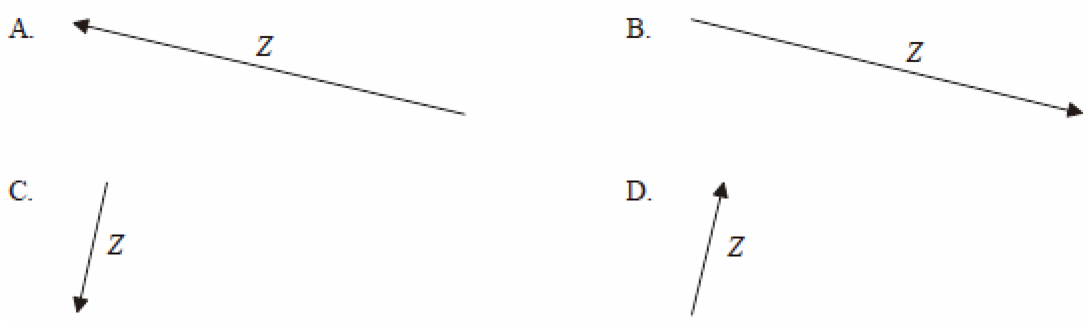
Examples of vectors: force, acceleration, displacement, velocity, momentum…

And $weight = m \cdot \vec{g}$ is a vector. Thus choose (A)

9. The diagram below shows two vectors **X** and **Y**.

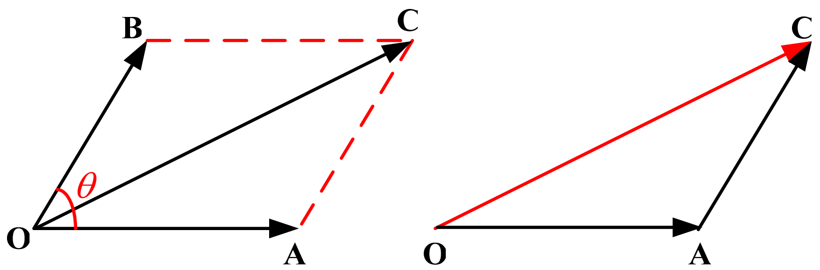


Which of the following best represents the vector **Z** = **X** – **Y**.



Strategy:

If two vectors are placed tail-to-tail at an angle θ, it can also be represented as a closed triangle.

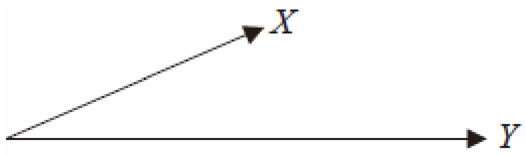


$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$ Because $\overrightarrow{OB} = \overrightarrow{AC}$

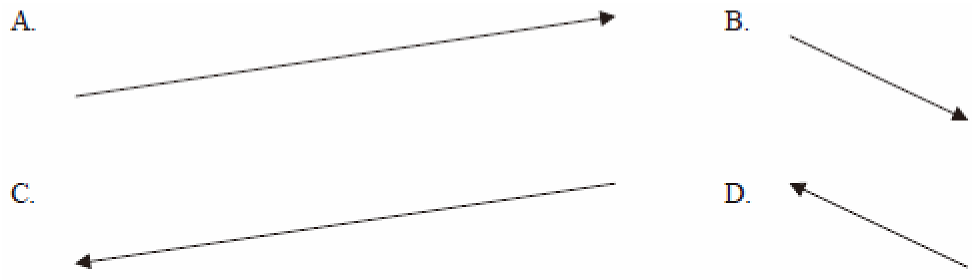
Solution:

And **X** = **Z** + **Y, thus choose (B)**

10. The magnitude and direction of two vectors **X** and **Y** are represented by the vector diagram below.

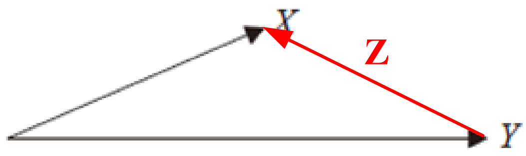


Which of the following best represents the vector (**X**–**Y**)?



Solution:

Let **X** minus **Y** to be **Z**: **X**–**Y** = **Z,** thus **X** = **Z** + **Y**



Choose (**D**):

Chapter 3 Kinematics and Linear motion

3-1 Displacement and velocity

Distance: is the magnitude of the path covered, is a scalar.

SI unit: metre (m)

Displacement: the change in position between the starting point and the end point.

SI unit: metre (m)

Displacement is a vector; its direction is from the starting point to end point.

For example:

(i) An ant crawl along the arc that start from O to A (**Fig. 3.1**),

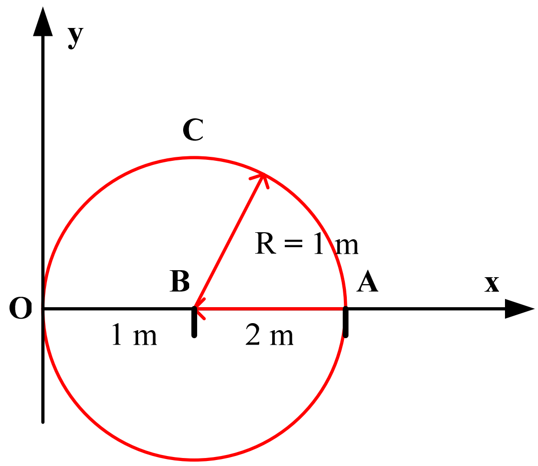


Fig. 3.1

Then:

$\text{Distance} = \pi R = \pi = 3.14m$

$\text{Displacement} = \overrightarrow{OA} = 2m$

(ii) The ant now goes on crawling from A to B,

$\text{Distance} = \overset{\frown}{OCA} + AB = \pi R + 1 = 4.14m$

$\text{Displacement} = \overrightarrow{OB} = 1m$

(iii) The ant now goes back from B to O,

Note: the ant start from O then go back to O. that is starting point is O, the end point is O.

$\text{Distance} = \overset{\frown}{OCA} + AO = \pi R + 2 = 5.14m$

$\text{Displacement} = \overrightarrow{OO} = 0m$

Speed: the distance traveled by a moving object over a period of time.

Constant speed: the moving object doesn't change its speed.

$$(average)speed = \frac{distance}{time\ taken}$$

$$v = \frac{s}{t}$$

Unit: m/s or ms^{-1}

Velocity: the speed in a given direction.

Average velocity: the change in position (displacement) over a period of time.

$$\vec{v}_{average} = \frac{change\ in\ position}{time\ taken} = \frac{displacement}{time\ taken} = \frac{\Delta\vec{x}}{t} = \frac{\vec{s}}{t}$$

Where $\vec{s}$ is displacement

Unit: m/s or ms^{-1}

Velocity is a vector; the direction is the same as the direction of the displacement.

Instantaneous velocity: the velocity that the moving object has at any one instance

3-2 Acceleration

Changing velocity (non-uniform) means an acceleration is present.

We can define acceleration as the change of velocity per unit time.

Uniform acceleration: the acceleration is constant, means the velocity of the moving object changes the same rate.

Average acceleration: change in velocity over a period of time.

$$\text{Average acceleration} = \frac{change\ in\ velocity}{time\ taken}$$

In symbol:

$$\vec{a}_{average} = \frac{\Delta\vec{v}}{t} = \frac{v-u}{t}$$

Where, v is the final velocity, u is the initial velocity.

SI unit: Meters per second squared (m/s^2)

Acceleration is a vector; the direction is the same as the direction of the change of velocity.

3-3 Equations for uniform acceleration

Consider a body is moving along a straight line with uniform acceleration, and its velocity increases from *u* (initial velocity) to *v* (final velocity) in time *t*.

First equation:

$$\text{acceleration} = \frac{\textit{change in velocity}}{\textit{time taken}}$$

$$a = \frac{v-u}{t}$$

So

$$at = v - u \quad \text{or} \quad v = u + at \ \ldots\ldots \text{①}$$

Second equation:

$$\textit{average velocity} = \frac{\textit{change in position}}{\textit{time taken}} = \frac{\textit{displacement}}{\textit{time taken}}$$

$$\bar{v} = \frac{\Delta\vec{x}}{t} = \frac{\vec{s}}{t}$$

Because the body is moving along a straight line in one direction, the magnitude of the displacement is equal to the distance.

And for the acceleration is uniform,

$$\textit{the average velocity}, \bar{v} = \frac{v+u}{2}$$

So

$$\bar{v} = \frac{v+u}{2} = \frac{s}{t} \quad \text{or} \quad s = \frac{(v+u)}{2}t \ \ldots\ldots \text{②}$$

Third equation:

From equation ①, $v = u + at$ and equation ②, $s = \frac{(v+u)}{2}t$

$$s = \frac{(u+at+u)}{2}t = ut + \frac{1}{2}at^2 \ \ldots\ldots \text{③}$$

Fourth equation:

From equation, $v = u + at$

We get:

$v^2 = (u+at)^2$

$v^2 = u^2 + 2uat + a^2t^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$

But $s = ut + \frac{1}{2}at^2$

So

$$v^2 = u^2 + 2as \quad \ldots\ldots \text{④}$$

3-4 Displacement—time graphs

Note: for a body moving along a straight line, we can only draw the displacement—time graphs (**Fig. 3.2**)

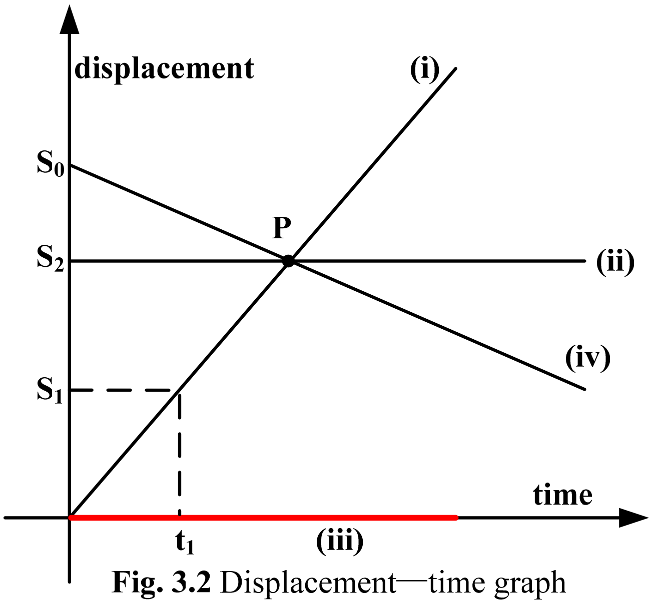


Fig. 3.2 Displacement—time graph

(i) Represents the body moving along a straight line with constant velocity; And the slope or gradient of the displacement—time graph represents the velocity of the body.

(ii) The body keeps rest with displacement S_2.

(iii) The body keeps rest with zero displacement.

(iv) The body moving along the opposite direction with constant velocity and initial displacement S_0.

(v) The point P means the displacement when the objects meeting with each other.

(vi) Displacement of the body is S_1 at time t_1.

3-5 Velocity—time graphs

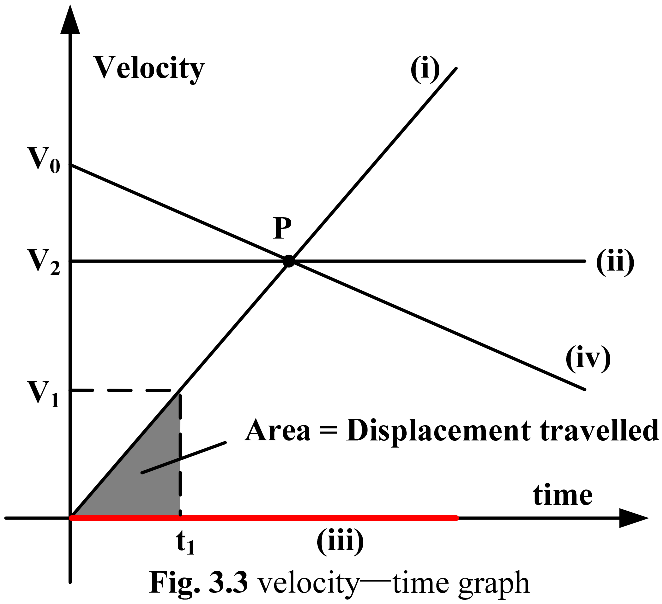


Fig. 3.3 velocity—time graph

(i) represents the body moving along a straight line with constant acceleration; And the slope or gradient of the velocity—time graph represents the acceleration of the body.

(ii) The body moving with constant velocity V_2.

(iii) The body keeps rest with zero velocity.

(iv)The body moving along a straight line with constant deceleration with initial velocity V_0; and the slope or gradient of the velocity—time graph represents the deceleration of the body

(v)The point P means the same velocity when the objects meeting with each other.

(vi)Velocity of the body is V_1 at time t_1 and the area under a velocity—time graph measures the displacement traveled.

3-6 Free-fall motion

6.1 Free-fall motion

The motion of a body that is only acted on by gravity and falls down from rest is called **free-fall motion.** This motion can occur only in a space without air. If air resistance is quite small and neglectable, the falling of a body in the air can also be referred to as a free-fall motion.

Galileo pointed out: free-fall motion is a uniformly accelerated rectilinear motion with zero initial velocity.

6.2 Acceleration of free-fall body

All bodies in free-fall motion have the same acceleration. This acceleration is called **free-fall acceleration** or **gravitational acceleration**. It is usually denoted by g.

The magnitude of gravitational acceleration g / ($m \cdot s^{-2}$)

Standard value: $g = 9.80665 m/s^2$

The direction of gravitational acceleration g is always vertically downward. Its magnitude can be measured through experiments.

Precise experiments show that the magnitude of g varies in different places on the earth. For example, at the equator $g = 9.780 m/s^2$. We take $9.81 m/s^2$ for g in general calculations. In rough calculations, $10 m/s^2$ is used.

As free-fall motion is uniformly accelerated rectilinear motion with zero initial velocity, the fundamental equations and the deductions for uniformly accelerated rectilinear motion are applicable for free-fall motion. What is only needed is to take zero for the initial velocity (*u*) and replace acceleration *a* with *g*.

3-7 15 Worked examples

1. An aero plane taking off accelerates uniformly on a runway from a velocity of $3ms^{-1}$ to a velocity of $90ms^{-1}$ in 45s.

Calculate:

(i) Its acceleration.

(ii) The distance on the runway.

Solution: data: $u = 3ms^{-1}$ $v = 93ms^{-1}$ $t = 45s$

Strategy: $v = u + at \Rightarrow a = \frac{v-u}{t}$, $s = ut + \frac{1}{2}at^2$

Answers:

$$a = \frac{v-u}{t} = \frac{93-3}{45} = 2ms^{-1}$$

$$s = ut + \frac{1}{2}at^2 = 3 \times 45 + \frac{1}{2} \times 2 \times 45^2 = 2160m = 2.16km$$

2. A car accelerates uniformly from a velocity of $15ms^{-1}$ to a velocity of $25ms^{-1}$ with a distance of 125m.

Calculate:

(i) Its acceleration

(ii) The time taken

Solution:

Data: $u = 15ms^{-1}$ $v = 25ms^{-1}$ $s = 125m$

Strategy: $v^2 = u^2 + 2as \Rightarrow a = \frac{(v^2 - u^2)}{2s}$

$$v = u + at \Rightarrow t = \frac{v-u}{a}$$

Answers: $a = \frac{(v^2 - u^2)}{2s} = \frac{25^2 - 15^2}{2 \times 125} = 1.6ms^{-2}$

$$v = u + at \Rightarrow t = \frac{v-u}{a} = \frac{25-15}{1.6} = 6.25s$$

3. A racing car starts from rest and accelerates uniformly at $2ms^{-2}$ in 30seconds, it then travels at a constant speed for 2min and finally decelerates at $3ms^{-2}$ until it stops, determine the maximum speed in km/h and the total distance in km it covered.

Strategy:

First stage: $u = 0ms^{-1}$ $a = 2ms^{-2}$ $t = 30s$, $v = u + at = 60ms^{-1}$

Second stage: moving with a constant speed $60ms^{-1}$ for 2min.

Third stage: $u = 60ms^{-1}$ $v = 0ms^{-1}$ $a = -3ms^{-2}$ (*deceleration*)

Answers:

First stage: $v = u + at = 60ms^{-1}$

$$s_1 = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 30^2 = 900m$$

Second stage: the final speed of the first stage is the constant speed of the second stage.

$s_2 = vt = 60 \times 2\min = 60 \times 2 \times 60 = 7200m$

$(1\min = 60s)$

Third stage: $v^2 = u^2 + 2as \Rightarrow s_3 = \frac{v^2 - u^2}{2a} = \frac{0 - 60^2}{2 \times (-3)} = 600m$

So

Maximum speed = $60ms^{-1} = \frac{60}{1000} \times 60 \times 60 = 216km/h$

Total distance = $s_1 + s_2 + s_3 = 900 + 7200 + 600 = 8700m = 8.7km$

4. Figure 4.1 shows the shuttle spacecraft as it is launched into space.

Fig. 4.1 shuttle spacecraft launching into space

During the first 5 minutes of the launch the average acceleration of the shuttle is $14.5ms^{-2}$.

a. Calculate the speed of the shuttle after the first 5 minutes.

b. Calculate how far the shuttle travels in the first 5 minutes.

Data: $u = 0ms^{-1}$, $\overline{a} = 14.5ms^{-2}$, $t = 5\min = 300\sec$

Strategy: $v = u + at$, $s = ut + \frac{1}{2}at^2$

Answers: a. $v = u + at = 0 + 14.5 \times 300 = 4350m = 4.35km$

b. $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 14.5 \times 300^2 = 652500m = 652.5km$

5. Figure 5.1 shows an incomplete velocity—time graph for a boy running a distance of 100m.

a. What is his acceleration during the first 4 seconds?

b. How far does the boy travel during (i) the first 4 seconds, (ii) the next 9 seconds?

c. Copy and complete the graph showing clearly at what time he has covered the distance of 100m. Assume his speed remains constant at the value shown by the horizontal portion of the graph.

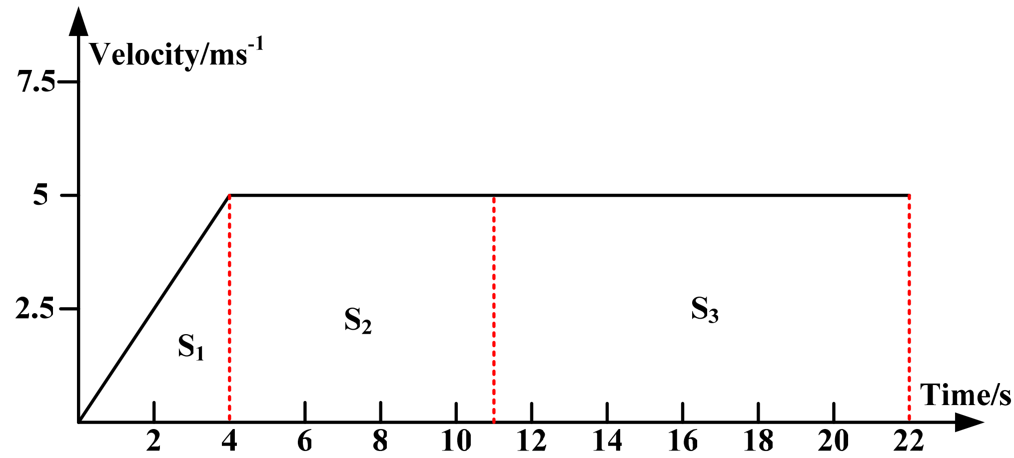


Fig. 5.1 velocity—time graph

Solution:

a. the gradient of the velocity—time graph represents the acceleration of the body.

During the first 4 seconds, $\text{gradient} = \frac{5}{4} = 1.25$

$\text{acceleration} = 1.25ms^{-2}$

b. (i) the area under a velocity—time graph measures the displacement traveled.

$area\ S_1 = \frac{1}{2} \times 4 \times 5 = 10$

$\text{Displacement} = 10m$

(ii) The next 9 seconds, $\text{area}\ S_2 = 9 \times 5 = 45$

$\text{Displacement} = 45m$

c. during the first 13 seconds, the distance covered is 10 + 45 = 55m,

The area needed $S_3 = 100 - 55 = 45$

So from 13s to 22 s, he covers $S_3 = 45$ m.

6. A constant resultant horizontal force of 1.8×10^3 N acts on a car of mass 900 kg, initially at rest on a level road.

(a) Calculate

(i) The acceleration of the car,

Strategy: by the Newton's second law, $F = ma$, $a = \frac{F}{m}$

So $a = \frac{F}{m} = \frac{1.8 \times 10^3}{900Kg} = 2ms^{-2}$

(ii) The speed of the car after 8.0 s,

Strategy: initial velocity, $u=0$, $t=8.0s$, $a=2ms^{-2}$. And from the equation: $v=u+at$, gives

$v=0+2\times 8=16ms^{-1}$

(iii) The momentum of the car after 8.0 s,

Strategy: The product of an object's mass m and velocity v is called its momentum:

$momentum=mv=900\times 16=1.44\times 10^{4}kgms^{-1}$

(iv) The distance traveled by the car in the first 8.0 s of its motion,

Strategy: $s=ut+\frac{1}{2}at^{2}$

$S=0+\frac{1}{2}\times 2\times 8^{2}=64m$

(v) The work done by the resultant horizontal force during the first 8.0 s.

Strategy: Work done=force × distance moved in direction of force.

$W=FS=1.8\times 10^{3}\times 64=115.2kJ$

(b) On the axes below (**Fig. 6.1**) sketch the graphs for speed, *v*, and distance traveled, *s*, against time, *t*, for the first 8.0 s of the car's motion.

Strategy: for the first 8.0 s, the car is moving with constant acceleration, $a=2ms^{-2}$, so the gradient of the v—t graph is equal to $2ms^{-2}$

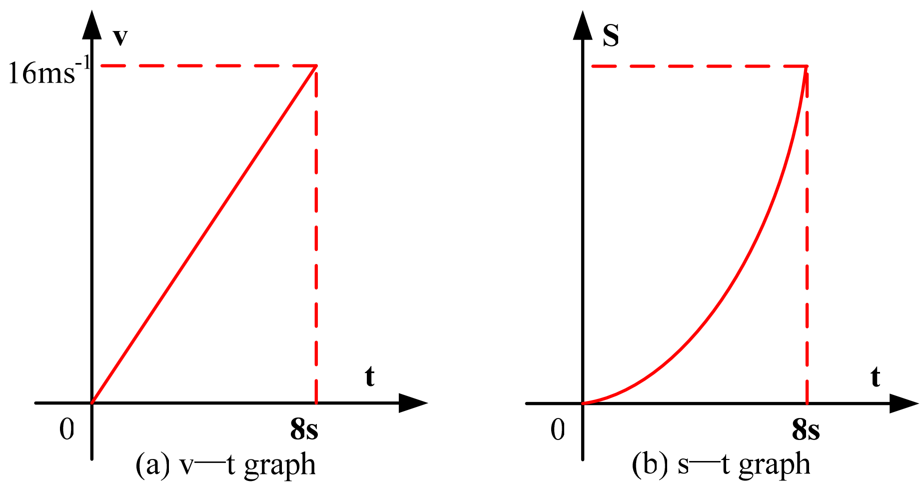


Fig. 6.1 (a) v—t graph (b) s—t graph

(c) In practice the resultant force on the car changes with time. Air resistance is one factor that affects the resultant force acting on the vehicle.

You may be awarded marks for the quality of written communication in your answer.

(i) Suggest, with a reason, how the resultant force on the car changes as its speed increases.

Answers: the resultant force decreases as its speed increases, because the air resistance increases as its speed increases, and the engine force of the car is constant, so the constant force decreases.

(ii) Explain, using Newton's laws of motion, why the vehicle has a maximum speed.

As the velocity increases, the air resistance increases, so the resultant force decreases, which means the acceleration of the car decreases, but the velocity is still increasing till the resultant force is zero (acceleration of the car is zero), according to the Newton's first law, then the vehicle has a maximum speed.

7. Fig. 7.1 represents the motion of two cars, A and B, as they move along a straight, horizontal road.

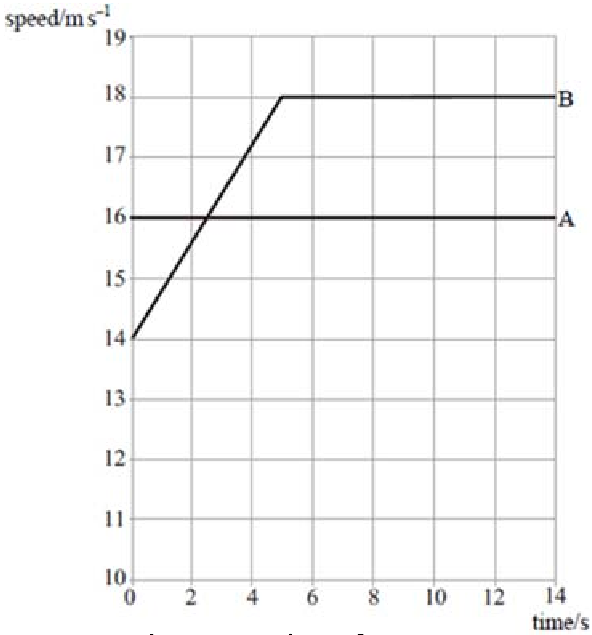


Fig. 7.1 motion of two cars

(a) Describe the motion of each car as shown on the graph.

(i) Car A: is moving with constant speed $16ms^{-1}$

(ii) Car B: accelerates in the first 5 seconds, and then moving with constant speed $18ms^{-1}$.

(b) Calculate the distance traveled by each car during the first 5.0 s.

(i) Car A:

Strategy: car A moving with constant speed, so distance of car A, So $S_A = ut = 16 \times 5 = 80m$

(ii) Car B:

Strategy: in the first 5 seconds, car B accelerates, and from the graph, the gradient of the v—t graph for B is $\frac{18-14}{5} = 0.8$, that is the acceleration of B is $a = 0.8ms^{-2}$

So $S_B = ut + \frac{1}{2}at^2 = 14 \times 5 + \frac{1}{2} \times 0.8 \times 5^2 = 80m$

(c) At time t = 0, the two cars are level. Explain why car A is at its maximum distance ahead of B at t = 2.5 s

Because car A is faster than car B at the first 2.5s, so for the first 2.5s, the distance between them increases till they have the same speed at 2.5s. After 2.5s, car B is faster than car A, so the distance then decreases. So at the time 2.5s, car A is at its maximum distance ahead of B.

8. A car accelerates from rest to a speed of $26ms^{-1}$. **Table 8.1** shows how the speed of the car varies over the first 30 seconds of motion.

Table 8.1

Time/s	0	5.0	10.0	15.0	20.0	25.0	30.0
Speed/ms^{-1}	0	16.5	22.5	24.5	25.5	26.0	26.0

(a) Draw a graph of speed against time on the grid provided (**Fig. 8.1**).

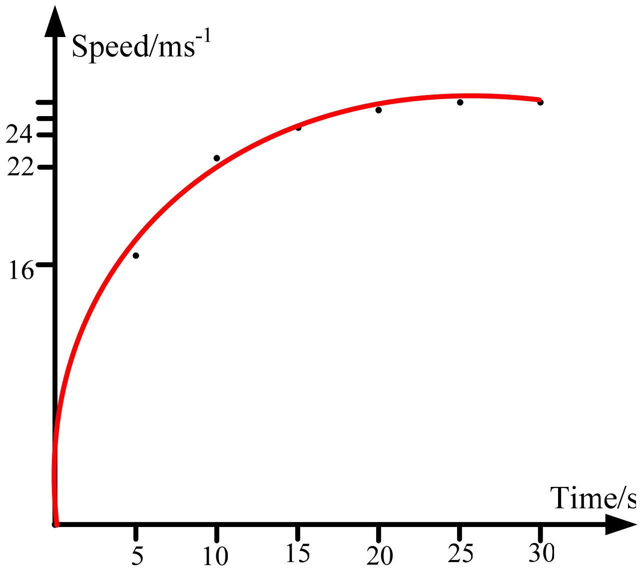


Fig. 8.1 speed—time graph

Note: you must draw the right scales and the six points are correctly plotted, and it is a trend line not a straight line.

(b) Calculate the average acceleration of the car over the first 25 s.

$$\text{Strategy: } \bar{a} = \frac{\Delta v}{\Delta t} = \frac{26}{25} = 1.04ms^{-2}$$

(c) Use your graph to estimate the distance traveled by the car in the first 25 s.

Strategy: area under the v—t graph represents the distance traveled.
So from the graph, its distance is 510m

(d) Using the axes below, sketch a graph to show how the resultant force acting on the car varies over the first 30 s of motion.

Solution:

From table 8.1, the rate of change of speed decreases to zero, thus the resultant force decreases to zero. As shown in **Fig. 8.2**.

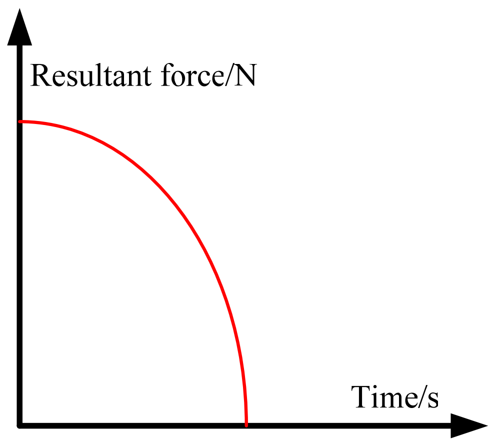


Fig. 8.2 resultant force—time graph

(e) Explain the shape of the graph you have sketched in part (d), with reference to the graph you plotted in part (a).

Because the first graph shows that the gradient of the car decreases, which means that the acceleration of the car decreases, and by the Newton's second law, $F = ma$, the force, F, decreases, and as the acceleration is changing in the first 25s, so the force is also changing, so the graph of the force is not a straight line.

9. A supertanker of mass $4.0\times10^{8}kg$, cruising at an initial speed of $4.5m/s$, takes one hour to come to rest.

(a) Assuming that the force slowing the tanker down is constant, calculate

(i) The deceleration of the tanker,

Solution:

The force slowing the tanker down is constant, so the tanker decelerates uniformly. Therefore, deceleration of the tanker is given by

$$a = \frac{0-4.5}{t} = \frac{-4.5}{1\times 60\times 60} = 1.25\times 10^{-3} m/s^2$$

(ii) The distance travelled by the tanker while slowing to a stop.

Solution:

The average speed is given by

$$\bar{v} = \frac{0+4.5}{2} = 2.25 m/s$$

So the distance traveled: $s = \bar{v}t = 2.25\times 1\times 60\times 60 = 8100m$

(b) Sketch, using the axes below, a distance-time graph representing the motion of the tanker until it stops.

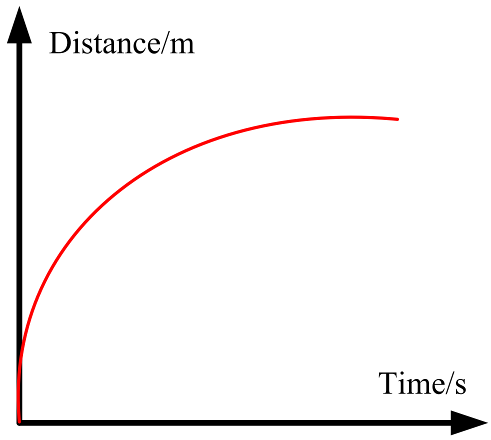


Fig. 9.1 Distance—time graph

(c) Explain the shape of the graph you have sketched in part (b).

Solution:

Because the speed is decreasing, the gradient of the curve decreases in the distance-time graph.

10. (a) A cheetah accelerating uniformly from rest reaches a speed of $29m/s$ in 2.0 s and then maintains this speed for 15 s. Calculate

(i) Its acceleration,

Solution:

Using $a = \frac{v-u}{t} = \frac{29-0}{2} = 14.5m/s^2$

(ii) The distance it travels while accelerating,

Solution:

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 14.5 \times 2^2 = 29m$

(iii) The distance it travels while it is moving at constant speed.

Solution:

$s = vt = 29 \times 15 = 435m$

(b) The cheetah and an antelope are both at rest and 100 m apart. The cheetah starts to chase the antelope. The antelope takes 0.50 s to react. It then accelerates uniformly for 2.0 s to a speed of $25m/s$ and then maintains this speed. **Fig. 10.1** shows the speed-time graph for the cheetah.

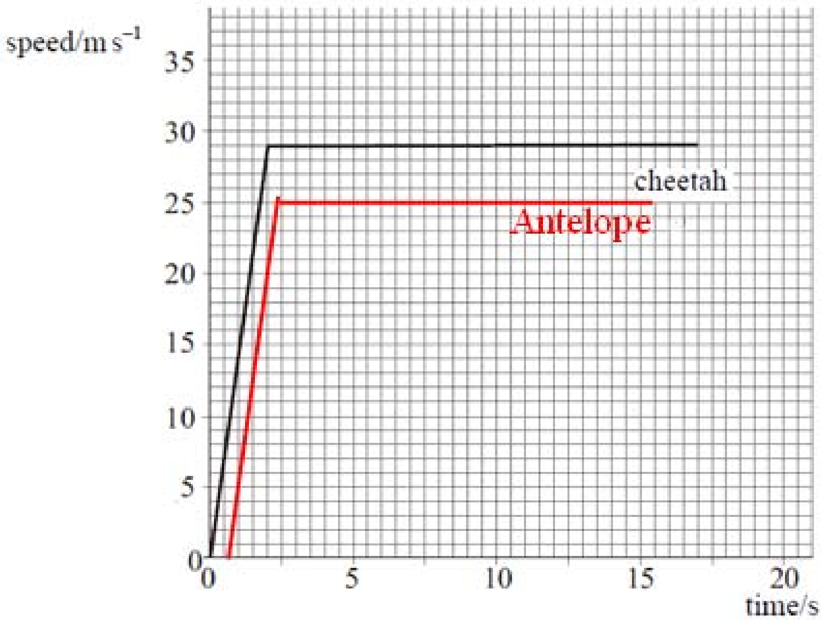


Fig. 10.1 speed—time graph for cheetah and antelope

(i) Using the same axes plot the speed-time graph for the antelope during the chase.

Solution:

The antelope takes 0.50 s to react and accelerates uniformly for 2.0 s to a speed of 25 m/s. thus we can get the speed-time graph beginning with 0.50 s.

(ii) Calculate the distance covered by the antelope in the 17 s after the cheetah started to run.

Solution:

The antelope accelerates from rest, and reaches to a speed of 25 m/s in 2 s. then maintains this speed. Thus the distance is given by

$s = \frac{v+u}{2} \times 2 + 25 \times (17 - 2 - 0.5) = 12.5 \times 2 + 25 \times 14.5 = 387.5m$

(iii) How far apart are the cheetah and the antelope after 17 s?

Solution:

From (a), the distance of cheetah is $s_1 = 435 + 29 = 464m$

And at the beginning, they are 100 m apart. Thus

$$\Delta s = s + 100 - s_1 = 387.5 + 100 - 464 = 23.5m$$

11. Figure 11.1 shows a distance-time graphs for two runners, A and B, in a 100 m race.

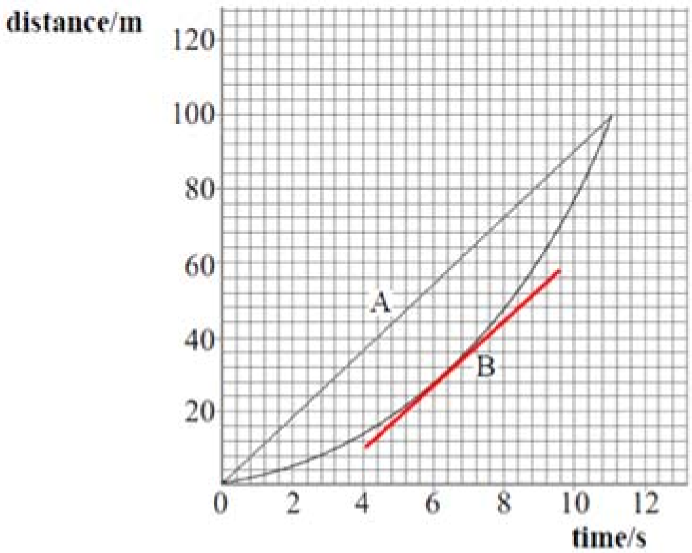


Fig. 11.1 distance—time graph for two runners

(a) Explain how the graph shows that athlete B accelerates throughout the race.

Solution:

The gradient is changing (increasing)

(b) Estimate the maximum distance between the athletes.

Solution:

When B's speed is the same as A's, it has the maximum distance between the athletes. From the graph is the gradient of B curve is the same that of A.

From the graph, the maximum distance is 25 m.

(c) Calculate the speed of athlete A during the race.

Solution:

For A, it has a distance in time 11 s, thus

$$speed = \frac{distance}{time} = \frac{100m}{11s} = 9.1m/s$$

(d) The acceleration of athlete B is uniform for the duration of the race.

(i) State what is meant by uniform acceleration.

(ii) Calculate the acceleration of athlete B.

Solution:

(i) The acceleration keeps the same or the velocity increases uniformly with time.

(ii) For B, its initial velocity is u = 0 m/s, distance s = 100 m, time taken t = 11 s.

Thus, using $s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$, gives

$$a = \frac{2s}{t^2} = \frac{2\times100}{11^2} = 1.7m/s^2$$

12.An aircraft accelerates horizontally from rest and takes off when its speed is 82 m s^{-1}. The mass of the aircraft is $5.6\times10^4 kg$ and its engines provide a constant thrust of $1.9\times10^5 N$.

(a) Calculate

(i) The initial acceleration of the aircraft,

Solution:

(i) Initially, the resultant force $F = 1.9\times10^5 N$, from Newton's second law: $F = ma$, we can get that

$$a = \frac{F}{m} = \frac{1.9\times10^5 N}{5.6\times10^4 kg} = 3.4m/s^{-2}$$

(ii) The minimum length of runway required, assuming the acceleration is constant.

Solution: let the minimum length of the runway required L. thus

$v^2 - u^2 = 2aL$

Therefore

$$L = \frac{v^2 - u^2}{2a} = \frac{82^2 - 0}{2\times3.4} = 989m$$

(b) In practice, the acceleration is unlikely to be constant. State a reason for this and explain what effect this will have on the minimum length of runway required.

Solution:

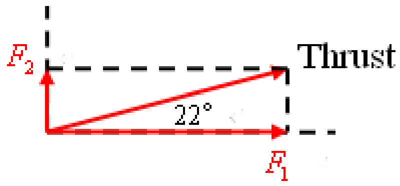
In practice, the air resistance increases with speed, hence the runway will be longer.

(c) After taking off, the aircraft climbs at an angle of 22° to the ground. The thrust from the engines remains at $1.9\times10^{5}N$. Calculate

(i) The horizontal component of the thrust,

(ii) The vertical component of the thrust.

Solution:



The thrust $T=1.9\times10^{5}N$

The horizontal component of the thrust is given by

$F_1=T\cos 22^0=1.76\times10^{5}N$

The vertical component of the thrust is given by

$F_2=T\sin 22^0=0.71\times10^{5}N$

13. Figure 13.1 shows how the velocity, v, of a car varies with time, t.

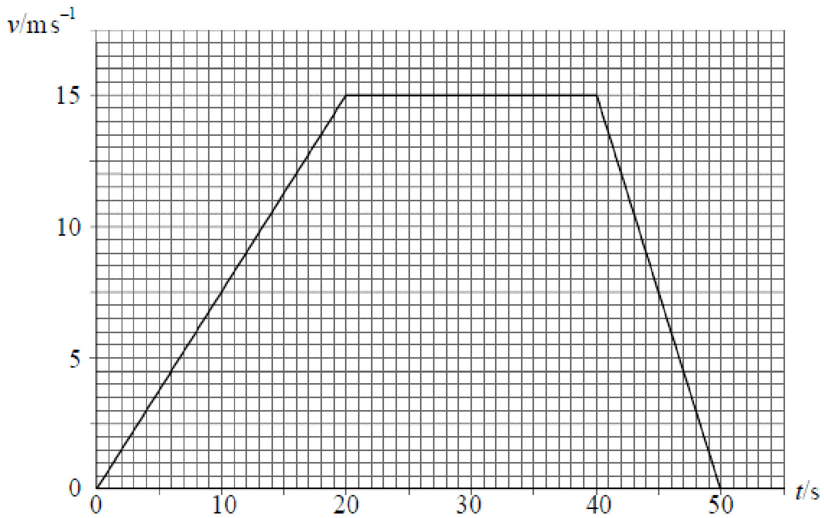


Fig. 13.1 velocity—time graph

(a) Describe the motion of the car for the 50 s period.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

Solution:

0—20 s: the car uniformly accelerates to a velocity of 15 m/s.

20—40 s: the car moves with constant velocity 15 m/s.

40—50 s: the car uniformly decelerates from a velocity of 15 m/s to 0 m/s.

(b) The mass of the car is 1200 kg. Calculate for the first 20 s of motion, (b)
(i) the change in momentum of the car,
(b) (ii) the rate of change of momentum,
(b) (iii) the distance travelled.

Solution: for the first 20 s of motion

(i) At t = o s, the initial velocity is u = 0 m/s; at t = 20 s, the final velocity is v = 15 m/s. thus the change in momentum of the car is given by

Therefore,

$$\Delta p = mv - mu = (1200kg) \times 15m/s - 0 = 1.8 \times 10^4 kg \cdot m/s$$

(ii)

$$\text{The rate of change of momentum} = \frac{change\ \ in\ \ momentum}{time\ taken} = \frac{1.8 \times 10^4 kg \cdot m/s}{20} = 0.9 \times 10^3 kg \cdot m/s^2$$

(iii) The area under a velocity—time graph measures the displacement traveled.

Thus the area for the first 20 s is given by

$$A = \frac{1}{2} \times 20 \times 15 = 150$$

Therefore the distance traveled is 150 m.

14. A car is travelling on a level road at a speed of 15.0 m s^{-1} towards a set of traffic lights when the lights turn red. The driver applies the brakes 0.5 s after seeing the lights turn red and stops the car at the traffic lights. **Table 14.1** shows how the speed of the car changes from when the traffic lights turn red.

Table 14.1

Time/s	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Speed/ms^{-1}	15.0	15.0	12.5	10.0	7.5	5.0	2.5	0.0

(a) Draw a graph of speed on the y-axis against time on the x-axis on the grid provided (**Fig. 14.1**).

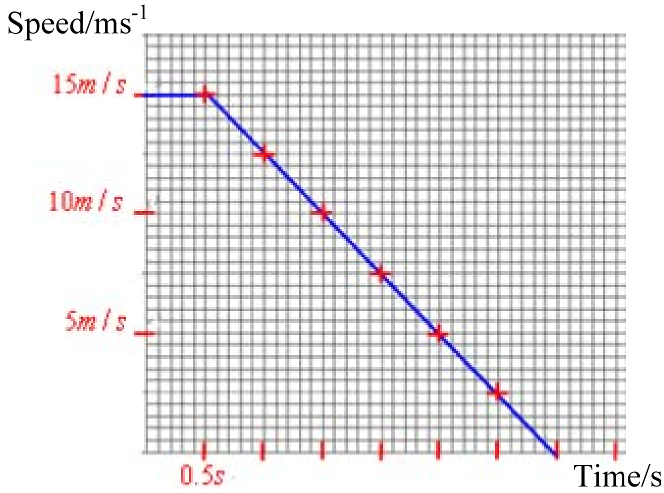


Fig. 14.1 speed—time graph

(b) (i) State and explain what feature of the graph shows that the car's deceleration was uniform.

Solution:

Deceleration is uniform because the graph is a decreasing straight line. And the gradient of the line represents the deceleration.

(b) (ii) Use your graph to calculate the distance the car travelled after the lights turned red to when it stopped.

Solution:

Distance traveled = area under the line (0s to 3.5s).

$$\text{Area} = \frac{1}{2} \times (0.5 + 3.5) \times 15 = 30$$

Therefore, distance traveled = 30 m.

15. Galileo used an inclined plane, similar to the one shown in **Fig. 15.1**, to investigate the motion of falling objects.

(a) Explain why using an inclined plane rather than free fall would produce data which is valid when investigating the motion of a falling object.

Solution:

Freefall is too quick; Galileo had no accurate method to time freefall.

(b) In a demonstration of Galileo's investigation, the number of swings of a pendulum was used to time a trolley after it was released from rest. A block was positioned to mark the distance that the trolley had travelled after a chosen whole number of swings.

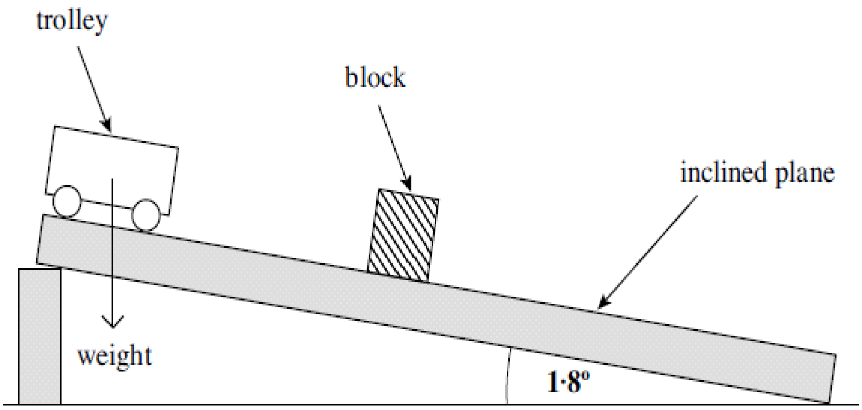


Fig. 15.1

The mass of the trolley in **Fig. 15.1** is 0.20 kg and the slope is at an angle of 1.8° to the horizontal.

(b) (i) Show that the component of the weight acting along the slope is about 0.06 N.

Solution:

The component of weight acting along the slope is given by

$W_1 = W \sin 1.8^0 = 0.2 \times 9.81 \times 0.031 = 0.06N$

(b) (ii) Calculate the initial acceleration down the slope.

Solution:

The initial resultant force along the slope equals to W_1, thus

$$a = \frac{W_1}{m} = \frac{0.06}{0.2} = 0.3 m/s^{-2}$$

(c) In this experiment, the following data was obtained. A graph of the data (**Fig. 15.2**) is shown below it.

Time/pendulum swings	Distance travelled/m
1	0.29
2	1.22
3	2.70
4	4.85

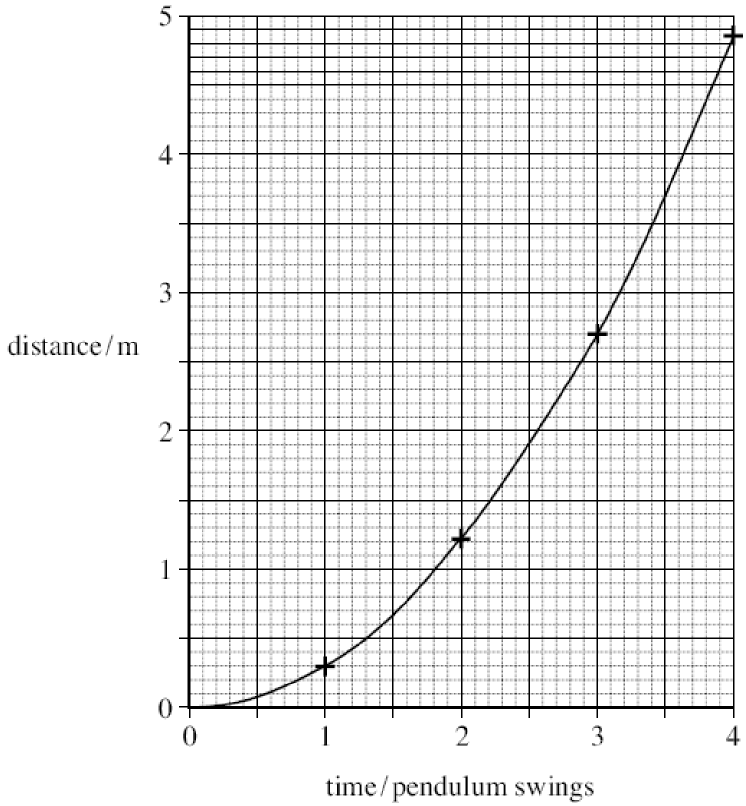


Fig. 15.2 distance—time graph

(c) From **Fig. 15.2**, state what you would conclude about the motion of the trolley?

Give a reason for your answer.

Solution:

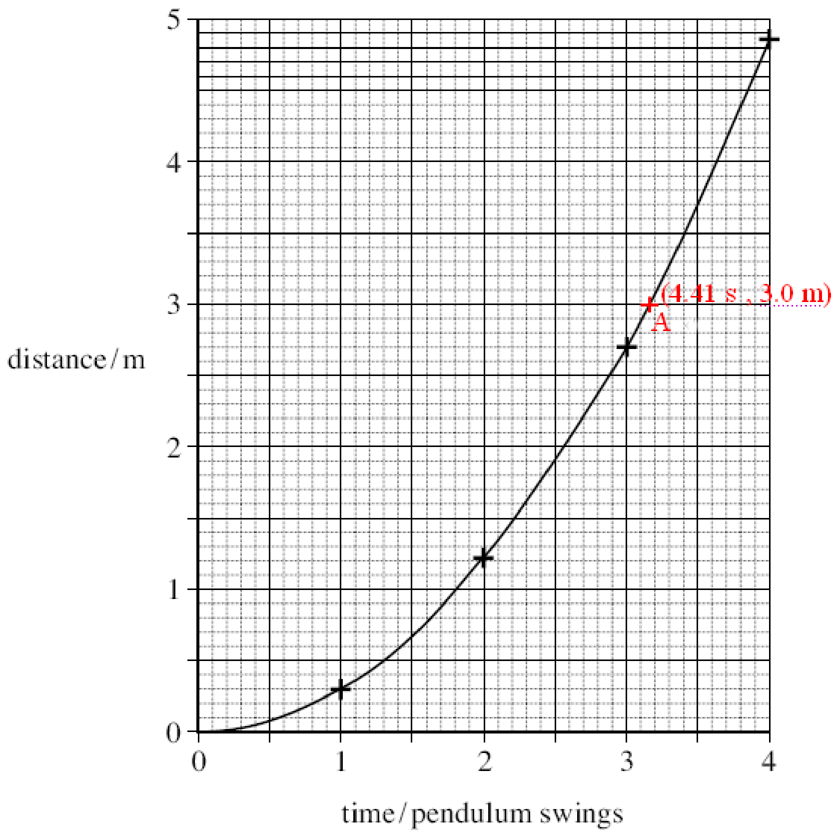
The gradient of the curve increases as time increasing. Thus the speed of the trolley is increasing.

(d) Each complete pendulum swing had a period of 1.4 s. Use the distance-time graph above to find the speed of the trolley after it had travelled 3.0 m.

Solution:

From **Fig. 15.2**, the time taken for traveling 3.0 m is given by

$$t = 3\times1.4 + 1.5\times\frac{1.4}{10} = 4.41s$$



And initial speed $u = 0m/s$, thus

$s = \frac{u+v}{2} \times t = \frac{vt}{2}$, gives

Speed, $v = \frac{2s}{t} = \frac{2 \times 3.0m}{4.41s} = 1.36m/s$

Module 2: Forces in action

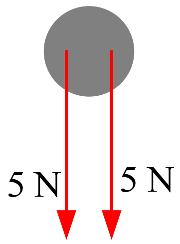
Chapter 1 Force and Nonlinear motion

1-1 Force definition

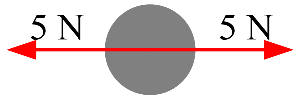
Interaction of objects is called Force. Force is a vector; the SI unit is the **Newton (N)**.

If two or more forces act on something, their combined effect is called the **net (resultant) force.**

Two simple examples are shown below:



Resultant force = 10 N downwards



Resultant force = 0 N

Newton definition:

1 Newton (N), as the amount of force that will give an object of mass 1 kg an acceleration of $1ms^{-2}$.

1-2 Weight and g

On Earth, everything feels the downward force of gravity. This gravitational force is called weight. As for other forces, its SI unit is the Newton (N).

Near the Earth's surface, the gravitational force on each kg is about 10 N; the gravitational field strength is 10 N kg $^{-1}$. This is represented by the symbol g.

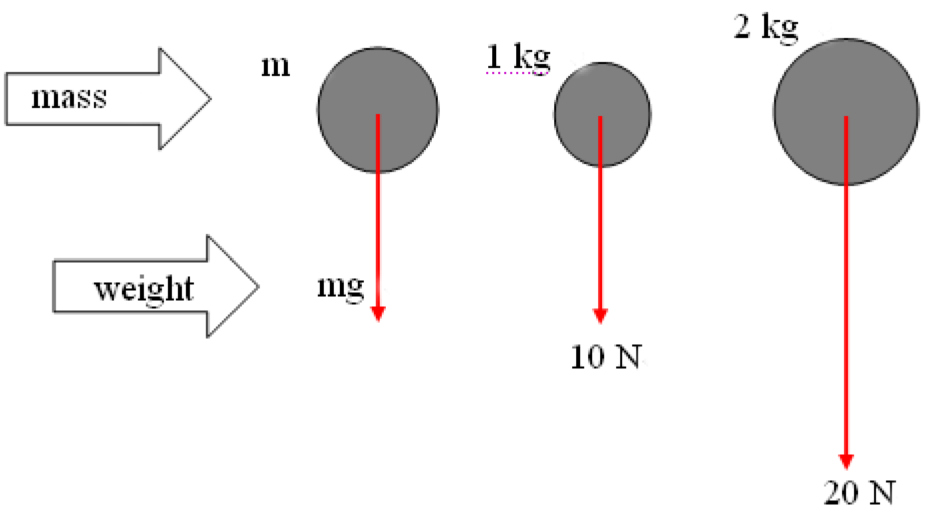
So weight = mass × gravitational field strength

In symbol

$W = mg$

For example, in the diagram below, all the masses are falling freely (gravity is the only force acting). From $F = ma$, it follows that all the masses have the same downward acceleration, g. this is the acceleration of free fall.

$$acceleration = \frac{weight}{mass} = 10ms^{-2} = g$$



Note: you can think of g:

Either as a gravitational field strength of 10 N kg^{-1}

Or as an acceleration of free fall of 10 m s^{-2}

In more accurate calculations, the value of g is normally taken to be 9.81, rather than 10.

1-3 Newton's first law of motion

If there is no resultant force acting:

① A stationary object will stay at rest,

② A moving object will maintain a constant velocity (a steady speed in a straight line).

From Newton's first law, it follows that if an object is at rest or moving at constant velocity, then the forces on it must be balanced.

Note: the more mass an object has, the more it resists any change in motion (because more force is needed for any given acceleration). Newton called this resistance to change in motion inertia.

Momentum:

The product of an object's mass m and velocity v is called its momentum:

$$momentum = mv$$

Momentum is measured in $kg \cdot ms^{-1}$. It is a vector.

1-4 Newton's second law

The rate of change of momentum of an object is proportional to the resultant force acting.

This can be written in the following form:

$$resultant\ force = \frac{change\ in\ momentum}{time\ taken}$$

In symbol:

$$F = \frac{mv - mu}{t} \cdots\cdots \quad ①$$

Where v is final velocity, u is initial velocity of an object.

Equation ① can be rewritten $F = \frac{m(v-u)}{t}$

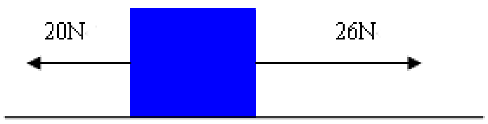
And acceleration, $a = \frac{v-u}{t}$. So

$$F = ma \cdots\cdots \quad ②$$

Note:

1. Equation ① and ② are therefore different versions of the same principle.
2. ***F* = *ma*** cannot be used for a particle traveling at very high speeds because its mass increases.
3. When using equations ① and ②, remember that *F* is the **net (resultant) force** acting. For example, for the figure below, the **net (resultant) force** is $26 - 20 = 6N$ to the right. The acceleration *a* can be worked out as follows:

$$a = \frac{F}{m} = \frac{6}{2} = 3ms^{-2}$$



Impulse:

As $F = \frac{m(v-u)}{t}$ can be rewritten $Ft = mv - mu$

In words force × time = change in momentum.

The quantity 'force × time' is called an impulse.

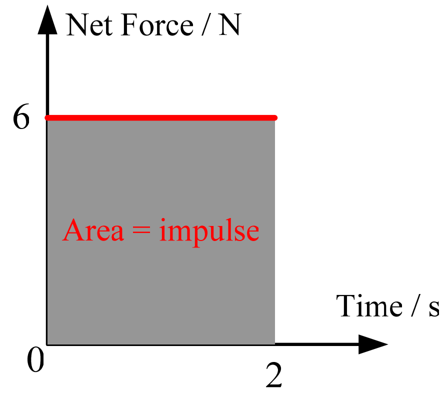
A given impulse always produces the same change in momentum, irrespective of the mass. For example, if a resultant force of 6 N acts for 2s, the impulse delivered is 6×2=12Ns.

This will produce a momentum change of $12kgms^{-1}$

So a 4 kg mass will gain $3ms^{-1}$ of velocity

Or a 2 kg mass will gain $6ms^{-1}$ of velocity, and so on.

The graph below is for a uniform net force of 6 N. in 2s, the impulse delivered is 12 Ns. numerically, and this is equal to the area of the graph between the 0 and 2 s points.



1-5 Newton's third law of motion

If A is exerting a force on B, then B is exerting an equal but opposite force on A.

The law is sometimes expressed as follows:

To every action, there is an equal but opposite reaction.

Note:

·It does not matter which force you call the action and which the reaction. One cannot exist without the other.

·the action and reaction do not cancel each other out because they are acting on different objects.

1-6 drag force and terminal speed

Any object moving through a fluid experiences a force that drags on it due to the fluid. The drag force depends on:

(i) The shape of the object

(ii) Its speed

(iii) The viscosity of the fluid which is a measure of how easily the fluid flows past a surface.

Note: the faster an object travels in a fluid, the greater the drag force on it.

1. Drag force in air

Considering an object released from rest in air, and then the speed of the object increases as it falls, so the drag force on it due to the fluid increases. The resultant force on the object is the difference between the force of gravity on it (its weight) and the drag force. As the drag force increases, the resultant force decreases, so the acceleration becomes less as it falls. If it continues falling, it attains terminal speed, when the drag force on it is equal and opposite to its weight. Its acceleration is then zero and its speed remains constant as it falls.

And at any instant, the resultant force F = mg − D, where m is the mass of the object and D is the drag force.

Therefore, the acceleration of the object, $a = \frac{mg - D}{m} = g - \frac{D}{m}$

Note:

(i) The initial acceleration = g because the speed is zero, and therefore the drag force is zero; at the instant the object is released.

(ii) At the terminal speed, the potential energy lost by the object is converted, as it falls, to internal energy of the fluid by the drag force.

2. Drag force in liquid

An object moving through a fluid experiences a ***resistive force, or drag***, that is proportional to the **viscosity** of the fluid. If the object is moving *slowly enough*, ***the drag force is proportional to its speed v***. if the object is a sphere of radius *r*, the force is

$$F = 6\pi\eta r v$$

Where η is again the coefficient of viscosity. This equation is known as **Stokes's law**. **Stokes's law** can be used to relate the speed of a sphere falling in a liquid to the viscosity of that liquid.

Consider a solid sphere of radius r dropped into the top of a column of liquid (**Fig. 1.1**). At the top of the column, the sphere accelerates downward under the influence of gravity. However there are two additional forces, both acting upward: the constant buoyant force and a speed-dependent retarding force given by Stokes's law. When the sum of the upward forces is equal to the gravitational force, the sphere travels with a constant speed v_t, called the ***terminal speed***. To determine this speed, we write the equation for the equilibrium of forces:

$$F_{grav} = F_{buoyant} + F_{drag}$$

We can express the gravitational force in terms of the density ρ of the sphere, its volume $\frac{4}{3}\pi r^3$, and g:

$$F_{grav} = \frac{4}{3}\pi r^3 \rho g$$

The buoyant force is equal to the weight of the displaced liquid, which has a density ρ':

$$F_{buoyant} = \frac{4}{3}\pi r^3 \rho' g$$

The retarding force is expressed by Stokes's law with the speed v_t:

$$F_{drag} = 6\pi\eta r v_t$$

Combining these equations, we get an expression for the terminal speed:

$$v_t = \frac{2r^2 g}{9\eta}(\rho - \rho')$$

The terminal speed is also called the sedimentation speed by biologists and geologists.

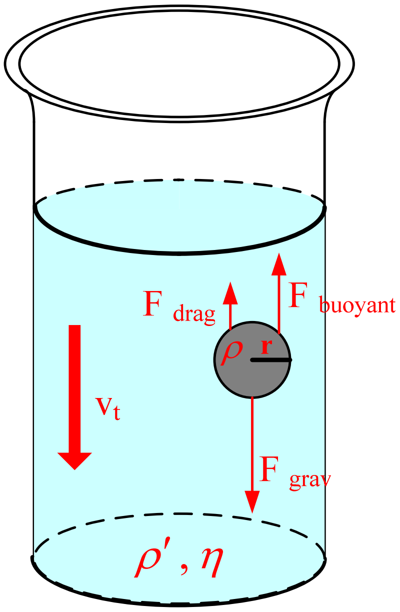


Fig. 1.1 A sphere falling in a viscous liquid reaches a terminal speed v_t that depends upon the radius and density of the sphere and the density and viscosity of the liquid.

Note: *Stokes's law* applies for situations in which the fluid flow is laminar, but not when the flow becomes turbulent.

But whenever an object moves rapidly enough, the retarding force F depends not on the speed (*Stokes's law*), but on the square of the speed:

$$F = bv^2$$

Where b is a constant determined for each different case.
An object falling from rest through the air falls with increasing speed until, at the terminal speed v_t, the retarding force of the air is equal in magnitude to the gravitational force:

$$mg = bv_t^2$$

Thus, the terminal speed can be written as

$$v_t = \sqrt{\frac{mg}{b}}$$

Where the constant b depends on the density ρ of the air and the area A of the body presented to the air flow. Then the equation for the terminal speed is

$$v_t = \sqrt{\frac{mg}{C_D \frac{\rho}{2} A}}$$

Where **C_D** is called the **drag coefficient**. This equation also holds for objects moving horizontally through the air at any speed if mg is replaced by the retarding, or drag, force on the object. Thus, the aerodynamic drag on a moving object, such as a car, becomes approximately

$$F_{drag} = 0.65 C_D A v^2$$

1-7 9 Worked examples

1. A rocket engine ejects 100kg of exhaust gas per second at a velocity (relative to the rocket) of 200m/s (**Fig. 1.1**). What is the forward thrust (force) on the rocket?

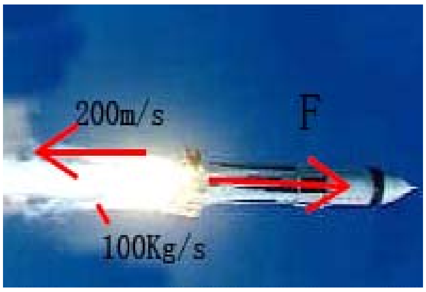


Fig. 1.1

By Newton's third law, the forward force on the rocket is equal to the backward force pushing out the exhaust gas. By Newton's second law, this force F is equal to the momentum gained per second by the gas, so it can be calculated using equation $F = \frac{m(v-u)}{t}$ with the following values:

$m = 100kg \quad t = 1s \quad u = 0 \quad v = 200ms^{-1}$

So, $F = \frac{m(v-u)}{t} = \frac{100 \times (200-0)}{1} = 20000N$.

2. A block of mass 2kg is pushed along a table with a constant velocity by a force of 5N. When the push is increased to 9N, what is

a. the resultant force,

b. the acceleration?

Solution: when the block moves with constant velocity the forces acting on it are balanced. The force of friction opposing its motion must therefore be 5N.

a. When the push is increased to 9N the resultant force F on the block is (9-5) N=4N, (since the frictional force is still 5N).

b. The acceleration a is obtained from $F = ma$ where F=4N and m=2kg.

So $a = \frac{F}{m} = \frac{4N}{2kg} = \frac{4kgms^{-2}}{2kg} = 2ms^{-2}$

3. A car of mass 1200kg traveling at 72km/h is brought to rest in 4s. Find

a. the average deceleration,

b. the average braking force,

c. The distance moved during the deceleration.

Solution:

a. The deceleration is found from $v = u + at$ where $v = 0$.

$u = 72km/h = \frac{72 \times 1000}{60 \times 60} = 20ms^{-1}$

And $t = 4s$

Hence $0 = 20 + a \times 4$

So $a = -5ms^{-2}$

The deceleration is $5ms^{-2}$

b. The average braking force F is given by $F = ma$, where $m = 1200kg$ and $a = -5ms^{-2}$. Therefore

$F = 1200 \times (-5) = -6000N$

'—' represents the direction of the braking force is opposite to the motion of the car.

So the braking force is 6000N.

c. To find the distance moved, we used

$s = \frac{u+v}{2}t = \frac{20+0}{2} \times 4 = 40m$

4. (a). what resultant force produces an acceleration of $5ms^{-2}$ in a car of mass 1000kg.

(b). what acceleration is produced in a mass of 2kg by a resultant force of 30N.

Solution:

a. use $F = ma = 1000 \times 5 = 5000N$

b. $F = ma \Rightarrow a = \frac{F}{m} = \frac{30}{2} = 15ms^{-2}$

5. A rocket has a mass of 500kg.

a. What is its weight on earth where g=10N/kg.

b. At lift-off the rocket engine exerts an upward force of 25000N. What is the resultant force on the rocket? What is its initial acceleration?

Solution:

a. weight=mass × gravitational field strength

weight $= 500 \times 10 = 5000N$

b. resultant force = upward force - weight=25000 – 5000 = 20000N

So initial acceleration, $a = \frac{resultant\ \ force}{mass} = \frac{20000}{500} = 40ms^{-2}$

6. An athlete trains by dragging a heavy load across a rough horizontal surface (**Fig. 6.1**).

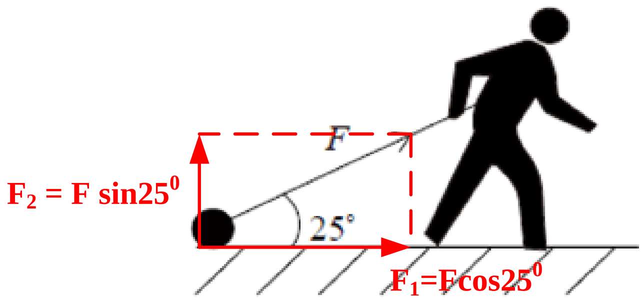


Fig. 6.1

The athlete exerts a force of magnitude F on the load at an angle of 25^0 to the horizontal.

(a) Once the load is moving at a steady speed, the average horizontal frictional force acting on the load is 470 N.

Calculate the average value of F that will enable the load to move at constant speed.

Solution:

The load is moving at constant speed, from Newton's first law, the resultant force is equal to zero. Thus

$F_1 = F\cos25^0 = \text{frictional force} = f = 470\text{ N}$

The average value of **F** is given by

$$F = \frac{470N}{\cos 25^0} = 519N$$

(b) The load is moved a horizontal distance of 2.5 km in 1.2 hours.

Calculate

(i) The work done on the load by the force F.

Solution:

Work done=force × distance moved in direction of force.

$$W = F_1 \times S = (F\cos 25^0 N) \times (2.5 \times 10^3 m) = 470 \times 2.5 \times 10^3 = 1175kJ$$

(ii) The minimum average power required to move the load.

Solution:

$$power = \frac{work\ \ done}{time\ taken} = \frac{1175 \times 10^3 J}{1.2 \times 60 \times 60s} = 272W$$

(c) The athlete pulls the load uphill at the same speed as in part (a).

Explain, in terms of energy changes, why the minimum average power required is greater than in (b)(ii).

Solution:

When the load is pulled uphill, some of the work need to be done to increase the gravitational potential energy.

7. An aluminum sphere of radius 1.0 mm is dropped into a bottle of glycerin at 20 ℃. What is the terminal speed of the sphere?

Solution:

We calculate the terminal speed directly by using the equation

$$v_t = \frac{2r^2 g}{9\eta}(\rho - \rho')$$

The radius in meters is $1.0\times10^{-3} m$. The densities are given by $\rho = 2.7\times10^3 kg/m^3$ and $\rho' = 1.26\times10^3 kg/m^3$. The viscosity is given by $R = 1.49 Pa\cdot s$. Thus,

$$v_t = 2.1\times10^{-3} m/s$$

8. A steel ball of mass 0.15kg released from rest in a liquid, falls a distance of 0.20m in 5.0s. Assuming the ball reaches terminal speed within a fraction of a second, calculate

(i) Its terminal speed,

(ii) The drag force on it when it falls at terminal speed.

Strategy: as the ball reaches terminal speed within a fraction of a second, so the ball falls a distance of 0.20m in 5.0s with the constant terminal speed, let the terminal speed V.

So (i) $s = Vt \Rightarrow 0.2 = V\times5$

$V = 0.04 \text{ m s}^{-1}$

(ii) When the ball falls at terminal speed, the drag force on it is equal and opposite to its weight.

So drag force, F = weight = mg = 0.15×9.8 = 1.47 N

9. Explain why a raindrop falling vertically through still air reaches a constant velocity.

Answers: Because as the falling of the raindrop, its speed is increasing; and the air resistance of the raindrop is increasing with the increasing speed, so the resultant force of the raindrop decreases, by the Newton's second law, $F = ma$, its acceleration decreases. So when the speed reaches to a certain value, the resultant force is equal to zero, then the raindrop reaches a constant velocity.

Chapter 2 Equilibrium

2-1 balanced forces

When forces act on a point object, the object is in equilibrium means the resultant force is zero (the object keeps at rest or moving at constant speed). In other words, when a point object keeps at rest or moving at constant speed, means it is in equilibrium and the resultant forces on it is zero.

Conditions for equilibrium for two or three coplanar forces acting at a point:

(i) When two forces act on a point object, the object is in equilibrium (at rest or moving at constant velocity) only if the two forces are equal and opposite of each other. The resultant of the two forces is therefore zero. The two forces are said to be balanced.

(ii) When three forces act on a point object, the object is in equilibrium (at rest or moving at constant velocity) only if the resultant of any two of the forces is equal and opposite to the third force.

- resolve each force along the same parallel and perpendicular lines
- balance the components along each line.

2-2 moments

The moment of a force about any point is defined as the force × the perpendicular distance from the line of action of the force to the point.

That is:

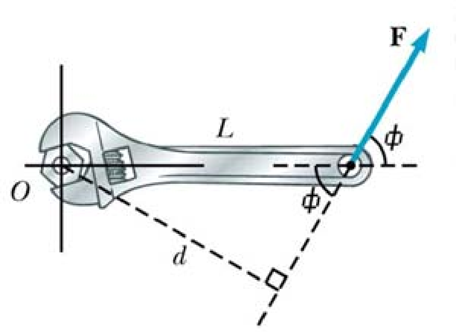
The moment of the force = F × d

Note: d is the line of action of the force to the point.

Unit of the moment of the force: **Newton metre (Nm)**

For examples

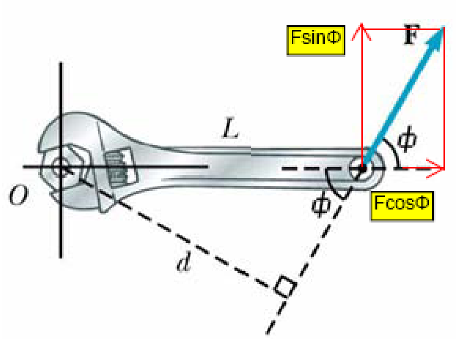
(i)



$d = L\sin\theta$

The moment of the force = F × d = $FL\sin\theta$

We can also resolve the force F



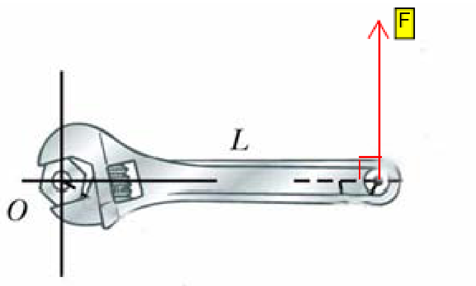
The force $F\cos\phi$ does not cause turning effect, Moment of the force $F\cos\phi$ is zero. (d = 0)

And the perpendicular distance from the line of action of the force $F\sin\theta$ to the point is L.

The moment of the force:

F × d = $F\cos\phi \times 0$ + $F\sin\theta \cdot d = F\sin\theta L = FL\sin\theta$

(ii)



$d = L$

The moment of the force = F × d = FL

2-3 Couples and torque of a couple

When a driver turn s a steering wheel (**Fig. 2.1**), he exerts two equal but opposite forces on it. The two forces form a couple. The turning effect of a couple is the sum of moment of the two forces. The moment of a couple is called a torque.

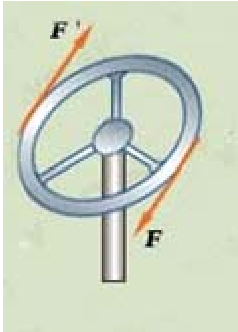


Fig. 2.1

So

(i) A couple consists of two forces, equal in magnitude but opposite in direction whose lines of action do not coincide.

(ii) The torque of a couple is the product of one of the forces and the perpendicular distance between the forces.

For example

Calculate the torque produced by two forces, each of magnitude 30 N, acting in opposite directions with their lines of action separated by a distance of 25cm.

Answers:

Torque = force × separation of forces

$= 30 \times 25 \times 10^{-2}$

$= 7.5$ N m

2-4 the principle of moments

Condition for equilibrium:

1. the net external force must be zero

$$\sum \vec{F} = 0$$

$$\sum F_x = 0 \ \ and \ \sum F_y = 0$$

2. the net external torque must be zero

$$\sum \tau = 0$$

Then considering the moments of the forces about any point, for equilibrium, The sum of the clockwise moments = the sum of the anticlockwise moments; This statement is known as the principle of moments.

2-5 Centre of gravity and Determination of Centre of Gravity (c.g.) of irregular lamina using the plumb line method

(i) An object may be made to balance at a particular point. When it is balanced at this point, the object does not turn and so all the weight on one side of the pivot is balanced by the weight on the other side. Supporting the object at the pivot means that the only force which has to be applied at the pivot is one to stop the object falling—that is, a force equal to the weight of the object. Although all parts of the object have weight, the whole weight of the object appears to act at his balance point. This point is called the centre of gravity of the object.

Center of Gravity: The point on the object that no turning effect produced by the force of the gravity.

Note: for a uniform body such as a ruler, the centre of gravity is at the geometrical centre.

(ii) Determination of Centre of Gravity (c.g.) of irregular lamina using the plumb line method:

Suppose we have to find the c.g. of an irregularly shaped lamina of cardboard (**Fig. 2.2**).

Make a hole A in the lamina and hang it so that it can swing freely on a nail clamped in a stand. It will come to rest with its c.g. vertically below A. to locate the vertical line through A tie a plumb line to the nail, the figure below, and mark its position AB on the lamina. The c.g. lies on AB.

Hang the lamina from another position C and mark the plumb line position CD. The c.g. lies on CD and must be at the point of intersection of AB and CD. Check this by hanging the lamina from a third hole. Also try balancing it at its c.g. on the rip of your fore-finger.

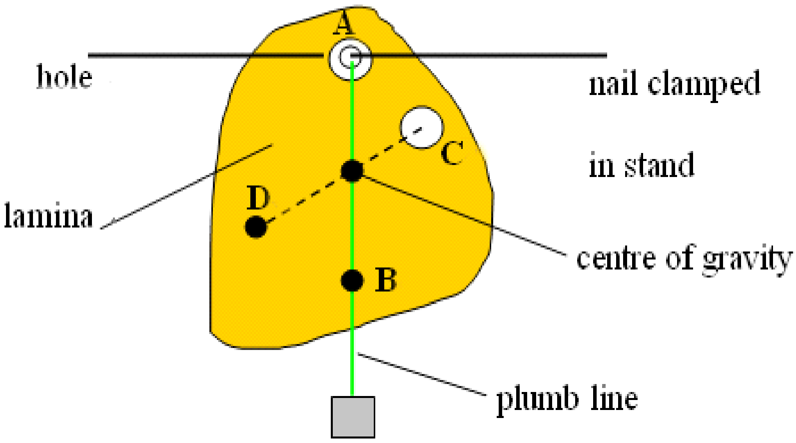


Fig. 2.2 c.g. of an irregularly shaped lamina of cardboard

2-6 Density and Pressure

1. Weight density

The quantity which relates a body's weight to its volume is known as its *weight density*.

The **weight density** *D* of a body is defined as the ratio of its weight *W* to its volume *V*. the SI unit is the Newton per cubic meter (N/m^3).

$$D = \frac{W}{V} \qquad W = DV$$

2. Mass density

Since the weight of a body is not constant but varies according to location, a more useful relation for density takes advantage of the face that mass is a universal constant, independent of gravity.

The **mass density** ρ of a body is defined as the ratio of its mass *m* to its volume *V*.

$$\rho = \frac{m}{V} \qquad m = \rho V$$

SI unit of ***mass density*** is kilograms per cubic meter (kg/m^3).

Note:

The relation between weight density and mass density is found by recalling that $W = mg$, thus

$$D = \frac{W}{V} = \frac{mg}{V} = \rho g$$

3. Pressure

To make sense of some effects in which a force acts on a body we have to consider not only the force but also the area on which it acts. For example, wearing skis prevents you sinking into soft snow because your weight is spread over a greater area. We say the pressure is less.

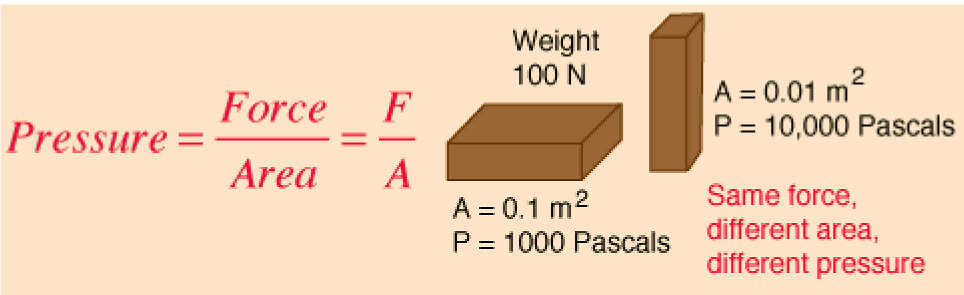
Pressure is the *normal force* acting on *unit area* and is calculated from

$$Pressure = \frac{Force}{Area} = \frac{F}{A}$$

Where A is the area over which the perpendicular force F is applied.

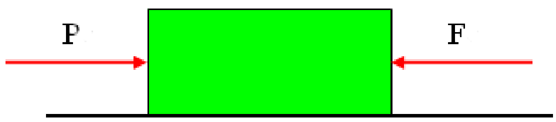
The unit of pressure is the Pascal (Pa); it equals 1 Newton per square metre (N/m^2) and is quite a small pressure.

The greater the area over which a force acts, the less the pressure.



2-7 23 Worked examples

1. For the figure below, if P is a force of 20N and the object moves with constant velocity. What is the value of the opposing force F?



Solution:

By the Newton's first law of motion, the object is moving with constant velocity, its resultant force is zero, that is $P - F = 0$

So $F = P = 20N$

2. An object resting on a horizontal surface (**Fig. 2.1**), the resultant force is zero.

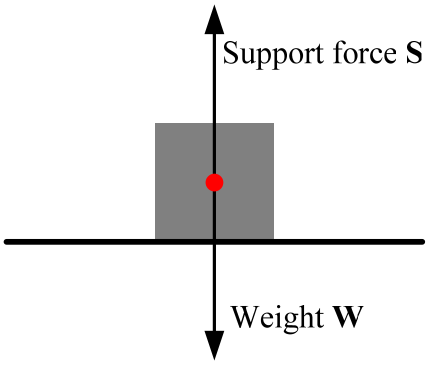


Fig. 2.1 An object resting on a horizontal surface

Then $W = S$

3. An object of weight $\boldsymbol{W} = 5\text{N}$ is moving along a rough slope that is at an angle of $\theta = 30°$ to the horizontal with a constant speed, the object is acted by a frictional force ***F*** and a support force ***S***, as shown in **Fig. 3.1:**
Calculate the frictional force ***F*** and the support force ***S***.

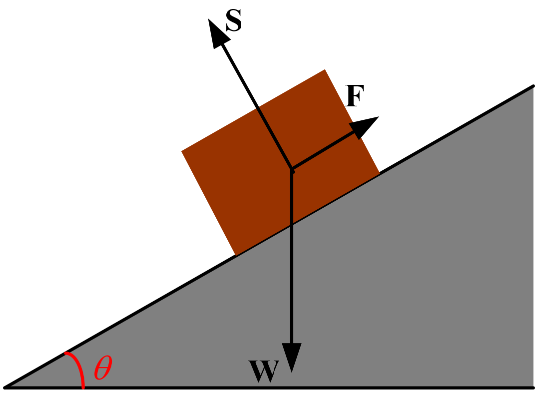


Fig. 3.1

Strategy:
The object moving down the slope with a constant speed means it keeping in equilibrium, that is the resultant of the three forces W, F, S is zero. Therefore resolve the forces along the slope and vertically to the slope (**Fig. 3.2**).

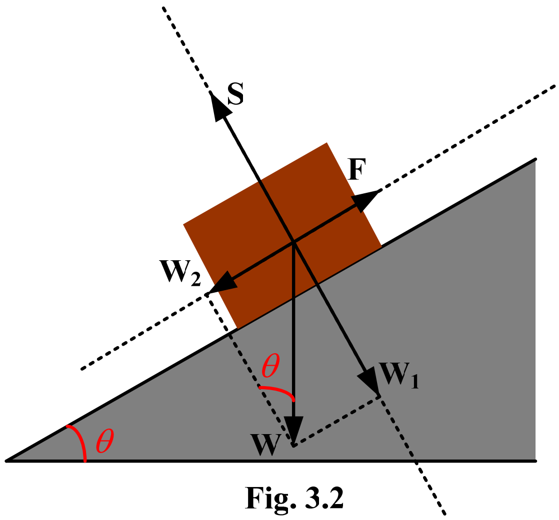


Fig. 3.2

From the figure, by the equilibrium condition,

$W_2 = F$ $W_1 = S$

And $W_2 = W \sin\theta = 5\sin 30^o = 2.5N$

$W_1 = W \cos\theta = 5\cos 30^o = 4.3N$

Frictional force $F = W_2 = 2.5$N

Support force $S = W_1 = 4.3$N

4. Fig. 4.1 shows a stationary metal block hanging from the middle of a stretched wire which is suspended from a horizontal beam. The tension in each half of the wire is 15 N.

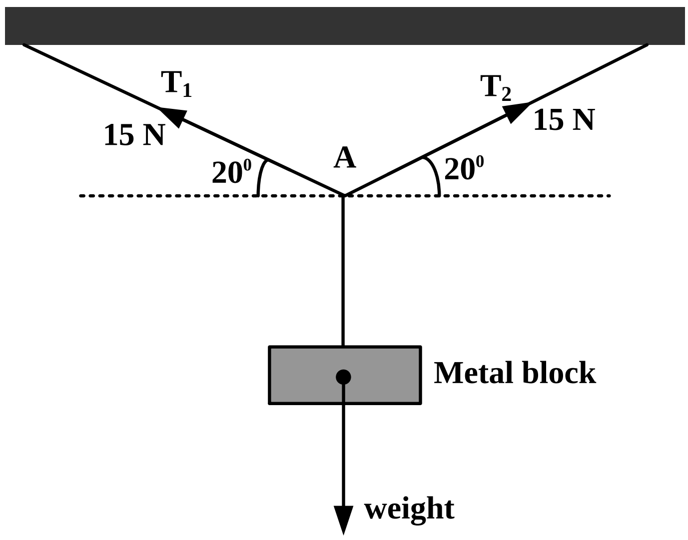


Fig. 4.1

(a) Calculate for the wire at A,

(i) The resultant horizontal component of the tension forces, The resultant horizontal component of the tension forces is equal to

$T_1 \cos 20 - T_2 \cos 20 = 0$

(ii) The resultant vertical component of the tension forces.

The resultant vertical component of the tension forces,

$T = T_1 \sin 20 + T_2 \sin 20 = 10.3\,N$

(b) (i) State the weight of the metal block.

Strategy: the metal block is at a stationary state,

So weight of the metal block, $W = T = 10.3\,N$

(ii) Explain how you arrived at your answer, with reference to an appropriate law of motion.

Strategy: From Newton's first law, it follows that if an object is at rest or moving at constant velocity, then the forces on it must be balanced.

5. Fig. 5.1 shows a sledge moving down a slope at constant velocity.
The angle of the slope is 22°.

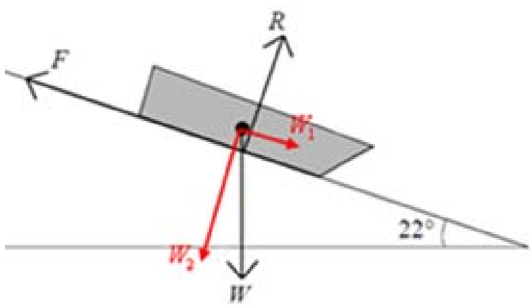


Fig. 5.1

The three forces acting on the sledge are weight, W, friction, F, and the normal reaction force, R, of the ground on the sledge.

(a) With reference to an appropriate law of motion, explain why the sledge is moving at constant velocity.

Solution:

Because the sledge is moving at constant velocity, the resultant force must be zero.

(b) The mass of the sledge is 4.5 kg. Calculate the component of W,

(b) (i) parallel to the slope,

(b) (ii) perpendicular to the slope,

Solution:

(i) parallel to the slope:

$$W_1 = W\sin 22^0 = mg\sin 22^0 = (4.5kg)\cdot(9.81N/kg)\sin 22^0 = 16.5N$$

(ii) Perpendicular to the slope

$$W_2 = W\cos 22^0 = mg\sin 22^0 = (4.5kg)\cdot(9.81N/kg)\cos 22^0 = 41N$$

(c) State the values of F and R.

Solution: The sledge is in equilibrium (moving with constant velocity), thus the resultant force is zero.

Therefore

$$F = W_1 = 16.5N$$

$$R = W_2 = 41N$$

6. Considering a uniform metre rule balanced on a pivot at its centre, supporting weights W_1 *and* W_2 suspended from the rule on either side of the pivot (**Fig. 6.1**).

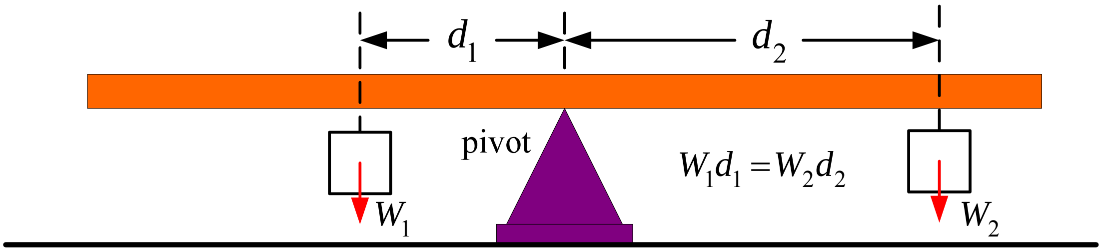


Fig. 6.1 a uniform metre rule balanced on a pivot at its centre

Weight W_1 provides an anticlockwise moment about the pivot = W_1d_1.

Weight W_2 provides an anticlockwise moment about the pivot = W_2d_2.

For equilibrium, applying the principle of moments:

$$W_1d_1 = W_2d_2$$

If now adding a third weight W_3 on the same side of the weight W_2 at the distance d_3 to the pivot, and then adjust the distance d_1 to d_1' to keep the rule rebalanced (**Fig. 6.2**).

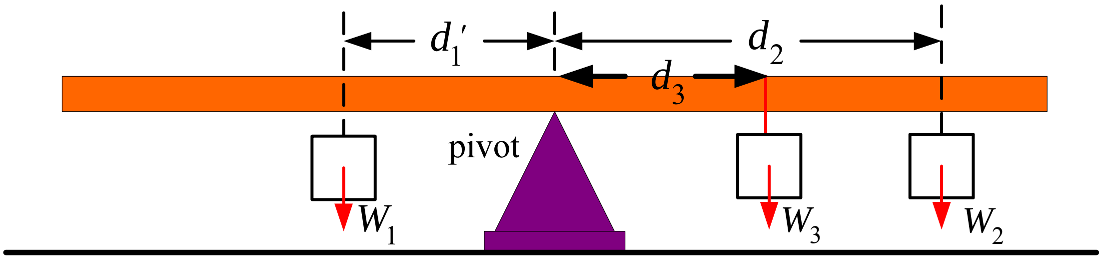


Fig. 6.2

Then

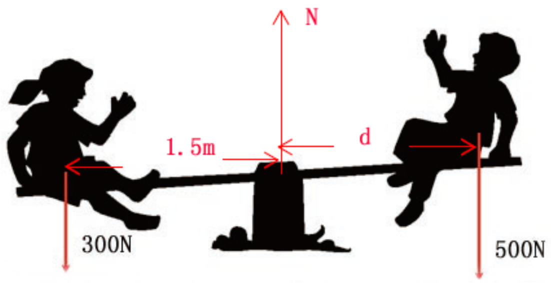
The sum of the clockwise moments = $W_3d_3 + W_2d_2$

The sum of the anticlockwise moments = $W_1d_1^{'}$

Applying the principle of moments:

$W_1d_1^{'} = W_3d_3 + W_2d_2$

7. A child of weight 300N sits on a seesaw at a distance 1.5m from the pivot at the centre. The seesaw is balanced by a second child of weight 500 N, calculate the distance of the second child from the pivot, and the support force N by the pivot.



Strategy:

For equilibrium,

The sum of the clockwise moments = the sum of the anticlockwise moments

Answers:

The sum of the clockwise moments = 500d

The sum of the anticlockwise moments = 300 × 1.5 = 450Nm

And 500d = 450

d = 0.9m

The net external force must be zero

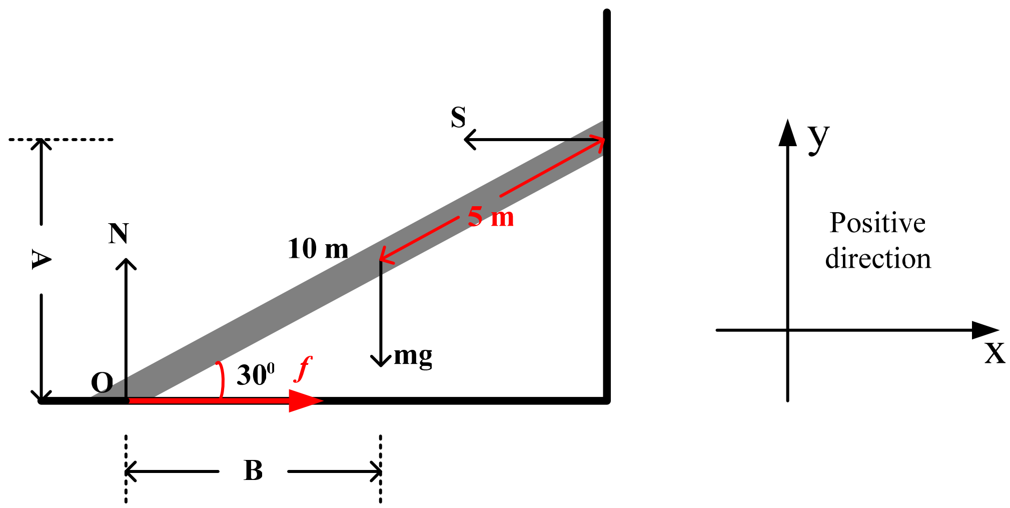
$$\sum \vec{F} = 0$$

Choose the upwards as the positive direction

Then, N + (-300) + (-500) = 0

N=800N

8. A ladder of length of 10m is placed against the wall at an angle of 30°, weight of the ladder is 100N, the ladder is in equilibrium, calculate the frictional force f and the support force N and S?



Strategy: the ladder is in equilibrium, then

1. the net external force must be zero

$$\sum \vec{F} = 0$$
$$\sum F_X = 0 \ \ and \ \ \sum F_y = 0$$

2. the net external torque must be zero

$$\sum \tau = 0$$

Solutions:

$$\sum \vec{F} = 0$$
$$\sum F_X = 0 \ \ and \ \ \sum F_y = 0$$

Gives

$f + (-S) = 0 \ \Rightarrow f = S$

$N + (-mg) = 0 \Rightarrow N = mg = 100N$

Choose O as the axis of rotation

And $\sum \tau = 0$, so

The sum of the clockwise moments = the sum of the anticlockwise moments

N and f do not produce turning effect about O, so

The sum of the clockwise moments = mg × B

$B = 5\cos 30^o$ =4.3m

The sum of the clockwise moments = mg × B = 100×4.3 = 430Nm

The sum of the anticlockwise moments = S × A

$A = 10\sin 30^o = 5m$

The sum of the anticlockwise moments = S × A = 5S

So

5S=430Nm S=86N

Then $f = S = 86N$

9. A uniform metre rule supports a 2N weight at its 50mm mark. The rule is balanced horizontally on a horizontal knife-edge at its 400mm mark (**Fig. 9.1**). Sketch the arrangement and calculate the weight of the rule.

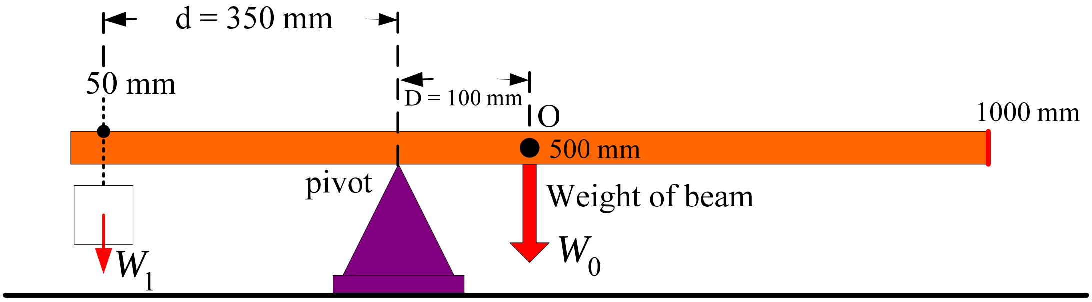


Fig. 9.1

O: is the centre of the gravity.

Strategy:

For equilibrium,

The sum of the clockwise moments = the sum of the anticlockwise moments

The sum of the clockwise moments = $W_0 \cdot D = W_0 \times 100 \times 10^{-3} = 0.1W_0$

The sum of the anticlockwise moments = $W_1 \cdot d = 2 \times 350 \times 10^{-3} = 0.7Nm$

And $0.1W_0 = 0.7 \Rightarrow W_0 = 7N$

10. A uniform beam of weight 230N and of length 10m rests horizontally on the tops of two brick walls, 8.5m apart, such that a length of 1.0m projects beyond one wall and 0.5m projects beyond the other wall (**Fig. 10.1**).

Calculate:

a: the support force of each wall on the beam

b: the force of the beam on each wall.

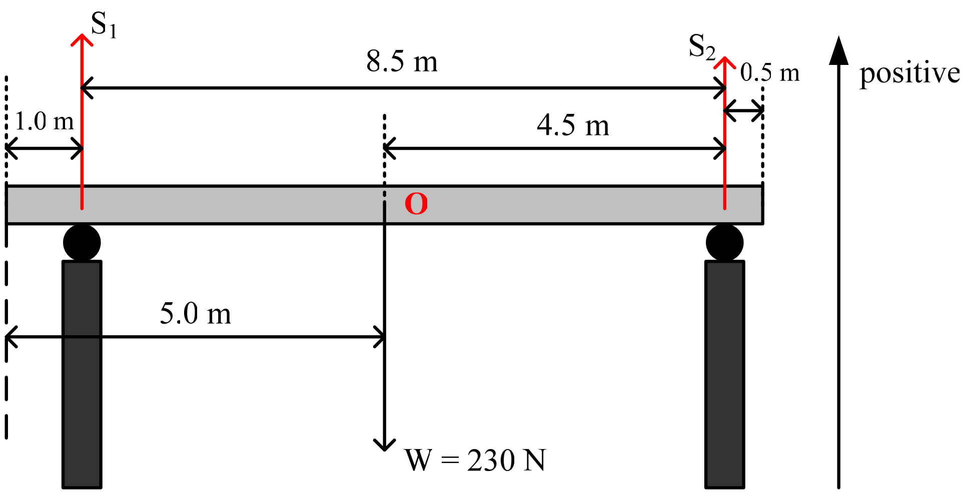


Fig. 10.1

Strategy: the beam keeps rest on the wall, for equilibrium

1. the net external force must be zero

$$\sum \vec{F} = 0$$

$$\sum F_x = 0 \ \ and \ \sum F_y = 0$$

2. the net external torque must be zero

$$\sum \tau = 0$$

Solutions:

$$\sum \vec{F} = 0$$

So, $S_1 + S_2 + (-W) = 0$, rearranging: $S_1 + S_2 = W = 230N$……①

$$\sum \tau = 0$$

Choose the point O as the axis, then

$4S_1 = 4.5S_2$ …… ②

From equation ② we get $S_1 = 1.125\ S_2$

And $S_1 + S_2 = 230N$

So, it can be gained that:

$1.125\ S_2 + S_2 = 230N$ $\quad$ $S_2 = 108N$

$S_1 = 230 – 108 = 122N$

b: 122N at the 1.0m end and 108N at the other end, both vertically downwards.

11. (a) State the principle of moments.

(b) (i) Draw a labeled diagram of the apparatus you would use to verify the

principle of moments.
(ii) Describe the procedure that would be used and state what measurements are taken.
You may be awarded marks for the quality of written communication in your answer.
(iii) Explain how the results would be used to verify the principle of moments.
Memos and answers:
Considering the moments of the forces about any point, for equilibrium,
The sum of the clockwise moments = the sum of the anticlockwise moments;
This statement is known as the principle of moments.
Diagram to verify:
Considering a uniform metre rule balanced on a pivot at its centre, supporting weights W_1 *and* W_2 suspended from the rule on either side of the pivot (**Fig. 11.1**).

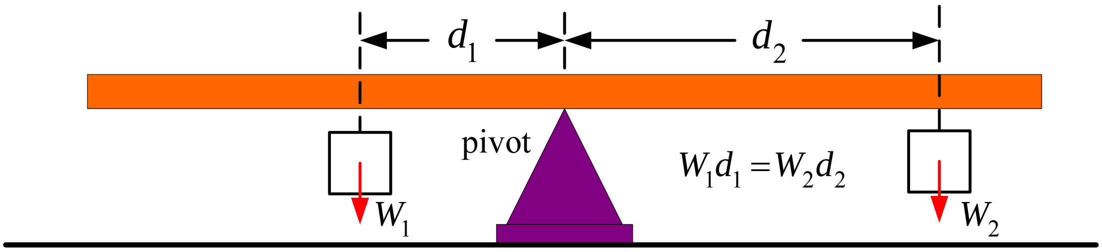


Fig. 11.1

Weight W_1 provides an anticlockwise moment about the pivot = W_1d_1.
Weight W_2 provides an anticlockwise moment about the pivot = W_2d_2.
Move W_1 and W_2 till the metre is in equilibrium, measure the distance d_1 and d_2, change the weight, repeat the process above.
For equilibrium, applying the principle of moments:
$W_1d_1 = W_2d_2$

12. (a) Define the moment of a force.
Answers: The moment of a force about any point is defined as the force × the perpendicular distance from the line of action of the force to the point. Thus, the moment of the force = F × d
(b) **Fig. 12.1** shows a uniform diving board of weight, **W**, that is fixed at A. The diving board is supported by a cylinder at C, which exerts an upward

force, P, on the board.

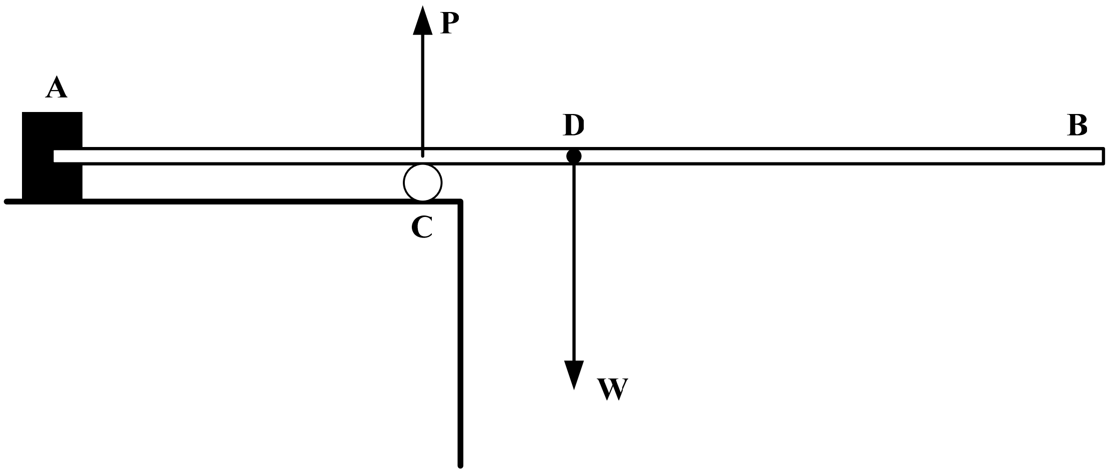


Fig. 12.1

(i)

By considering moments about A, explain why the force P must be greater than the weight of the board, W.

Answers: if the board is in balance, then the sum of the clockwise moments (W × AD) = the sum of the anticlockwise moments (P × AC). Because AC is less than AD, the force P must be greater than the weight of the board, W.

(ii) State and explain what would be the effect on the force P of a girl walking along the board from A to B.

Answers: the force P must increase, since the moment of the girl's weight about the pivot A increases as the distance increases from A to B. so when the girl at A the force P is a minimum value, and at B, the force has a maximum value.

13. (a) Define the moment of a force about a point.

The moment of a force about any point is defined as the force × the perpendicular distance from the line of action of the force to the point.

That is:

The moment of the force = F × d

Note: d is the line of action of the force to the point.

Unit of the moment of the force: Newton metre (Nm)

(b) **Fig. 13.1** shows a trailer attached to the towbar of a stationary car. The weight of the trailer is 1800 N and is shown acting through its centre of gravity. ***F*** is the force exerted by the towbar on the trailer. F_R is the total normal reaction force experienced by the trailer. When stationary all forces acting on the trailer are vertical.

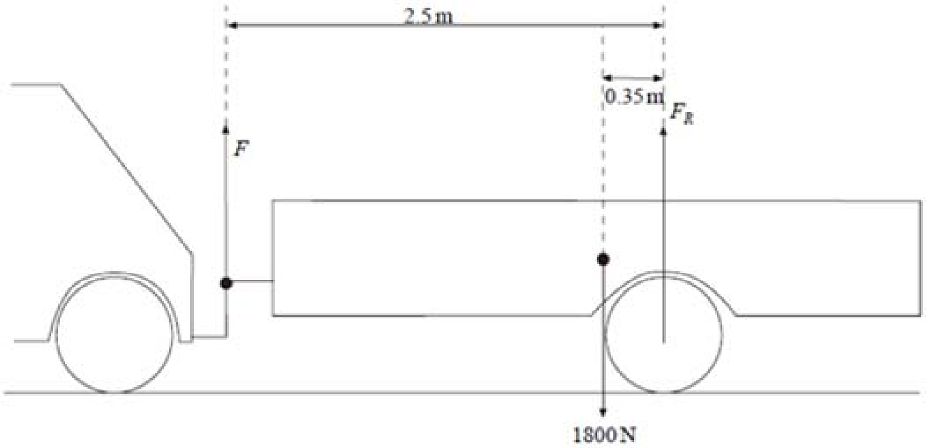


Fig. 13.1

(i) Explain what is meant by centre of gravity.

Center of Gravity:

The point on the object that no turning effect produced by the force of the gravity.

(ii) Calculate the force, F, exerted by the towbar on the trailer.

Strategy: the system is in equilibrium,

1. the net external force must be zero

$\sum \vec{F} = 0$

$\sum F_x = 0$ and $\sum F_y = 0$

2. the net external torque must be zero

$\sum \tau = 0$

So

$F_R + F = 1800$

And choose the centre of gravity as the pivot, $F \times (2.5 - 0.35) = 0.35 F_R$

From the above two equation, $F_R = 1548\,N$ $F = 252\,N$

(iii) Calculate F_R.

From (ii) $F_R = 1548\,N$

(c) The car starts to move forwards. State and explain what happens to the magnitude and direction of force, F.

You may be awarded marks for the quality of written communication in your answer.

Answers: the force must have a horizontal component, so the force increases in magnitude and act at an angle to the vertical.

14. Fig. 14.1 shows a supermarket trolley.

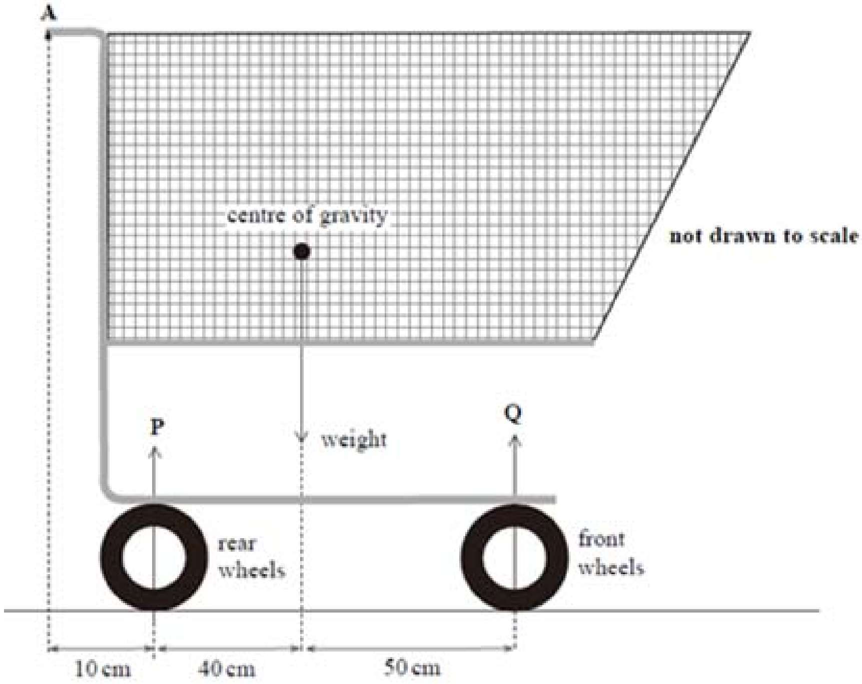


Fig. 14.1

The weight of the trolley and its contents is 160 N.

(a) Explain what is meant by centre of gravity.

Center of Gravity:

The point on the object that no turning effect produced by the force of the gravity.

(b) P and Q are the resultant forces that the ground exerts on the rear wheels and front wheels respectively. Calculate the magnitude of

(i) Force P,

(ii) Force Q.

Strategy: Condition for equilibrium:

1. the net external force must be zero

$$\sum \vec{F} = 0$$

$$\sum F_x = 0 \ \ and \ \ \sum F_y = 0$$

2. the net external torque must be zero

$\sum \tau = 0$

So P + Q = weight = 160 N … equation ①

And choose the centre of the gravity as the pivot,

P × 40 = Q × 50 … equation ②

From equation ① and ②, P = 71 N, Q = 89 N.

(c) Calculate the minimum force that needs to be applied vertically at A to lift the front wheels off the ground.

Strategy: the minimum force to lift the front wheels off the ground means the force Q is equal to zero.

So let the force applied at A as R, and choose the force P as the pivot,

R × 10 = weight × 40 = 160×40, so R = 640 N.

(d) State and explain, without calculation, how the minimum force that needs to be applied vertically at A to lift the rear wheels off the ground compares to the force you calculated in part (c).

You may be awarded marks for the quality of written communication in your answer.

Answers: The force now needed is less than that to lift the front wheels, because the distance to the pivot increases now. Then smaller force causes larger moment.

15. Figure 15.1 shows an apparatus used to locate the centre of gravity of a non-uniform metal rod.

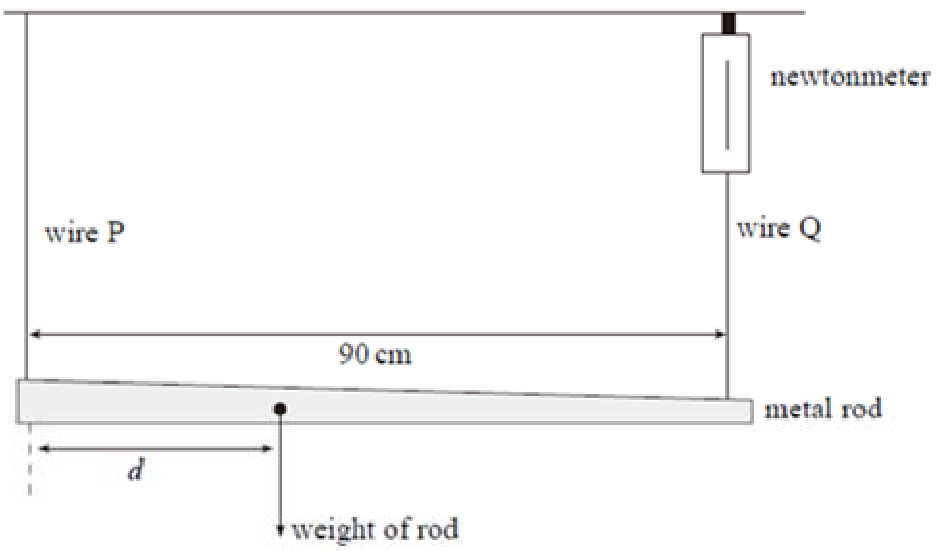


Fig. 15.1

The rod is supported horizontally by two wires, P and Q and is in equilibrium.
(a) State two conditions that must be satisfied for the rod to be in equilibrium.
Solution:
Condition for equilibrium:

1. the net external force must be zero

$$\sum \vec{F} = 0$$

$$\sum F_x = 0 \ \ and \ \sum F_y = 0$$

2. the net external torque must be zero

$$\sum \tau = 0$$

(b) Wire Q is attached to a newtonmeter so that the force the wire exerts on the rod can be measured. The reading on the newtonmeter is 2.0 N and the weight of the rod is 5.0 N.
Calculate
(i) The force that wire P exerts on the rod,
Solution:
The net external force must be zero, thus P = 5.0 – 2.0 = 3.0 N.
(ii) The distance d.
Solution:
Choose the centre of the rod as the pivot, therefore
$P \times d = Q \times (90 - d)$ gives
$3 \times d = 2 \times (90 - d)$
d = 36 cm

16. (a) Define the moment of a force.
Solution:
The moment of a force about any point is defined as the force × the perpendicular distance from the line of action of the force to the point.
That is:

The moment of the force = **F** × d

Note: d is the line of action of the force to the point.
Unit of the moment of the force: Newton metre (Nm)

(b) **Fig. 16.1** shows the force, **F**, acting on a bicycle pedal.

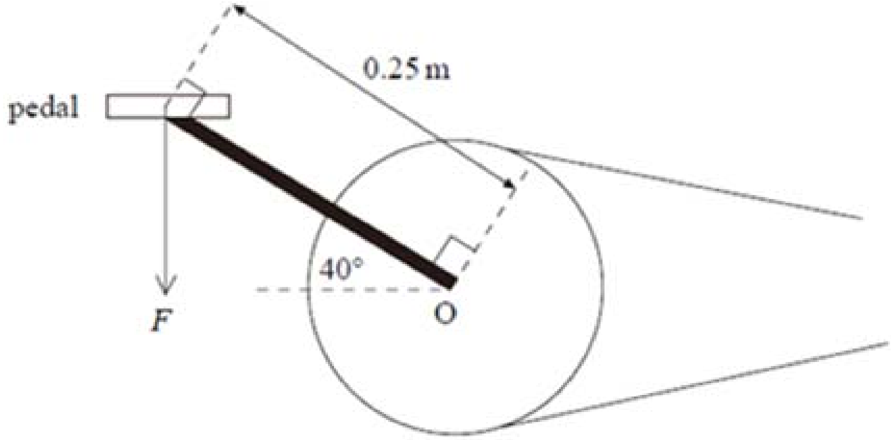


Fig. 16.1

(i) The moment of the force about O is 46 Nm in the position shown.

Calculate the value of the force, F.

Solution:

It can be calculated that

$F \times 0.25 \times \cos 40^0 = 46$

Gives

$F = \frac{46}{0.25 \times \cos 40^0} = 240N$

(ii) Force, F, is constant in magnitude and direction while the pedal is moving downwards. State and explain how the moment of F changes as the pedal moves through 80°, from the position shown.

Solution:

The angle decreases to zero, and then increasing from zero to 40° .

Thus, the perpendicular distance increases to a maximum value, and then decreasing again. At the same time, the force is constant in magnitude and direction.

Therefore, the moment increases to a maximum value when shaft is horizontal and then decreases.

17. Figure 17.1 shows two of the forces acting on a uniform ladder resting against a smooth vertical wall.

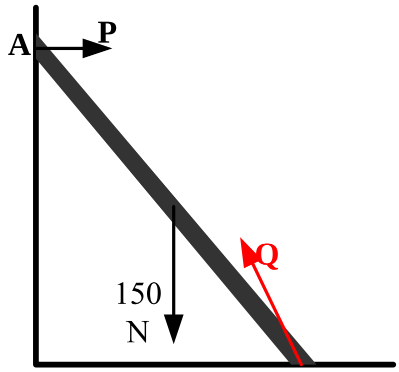


Fig. 17.1

The ladder is 6.0 m long and has a weight of 150 N. The horizontal force, P, exerted on the ladder by the wall is 43 N. Force Q (not shown) is the force the ground exerts on the ladder at B.

(a) Explain why the force, Q must have

(i) a vertical component,

(ii) a horizontal component.

Solution:

The ladder is keeping rest, thus the resultant force acting on the ladder must be zero.

Therefore, there must be a vertical component force of Q to balance the weight, a horizontal component force of Q to balance the force P.

(b) Draw an arrow on the diagram to represent the force Q.

(c) State the

(i) Horizontal component of Q,

(ii) Vertical component of Q.

Solution:

(i) From (a), the horizontal component of Q equals to force P, whose value is 43 N.

(ii) The vertical component of Q equals to weight, whose value is 150 N.

(d) State and explain the effect on force Q if a person stands on the bottom of the ladder and the direction of P is unchanged.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

Solution:

Force Q increases in magnitude, since the vertical component of force increases. And the direction of Q moves closer to vertical.

18. (a) State the principle of moments.

Solution:

For a body in equilibrium,

The sum of the clockwise moments=the sum of the anticlockwise moments;

This statement is known as the principle of moments.

Fig. 18.1 shows a child's mobile in equilibrium.

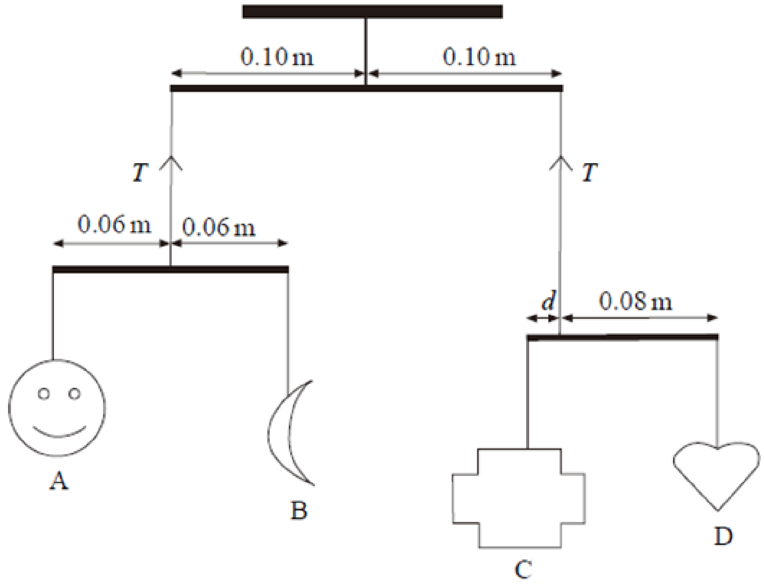


Fig. 18.1

A piece of cotton thread is attached to the rod supporting objects A and B and another piece of cotton thread supports the rod holding objects C and D. The tension in the cotton threads is ***T*** and all the rods are horizontal.

(b) (i) Complete the following table assuming the weights of the rods are negligible.

Weight of object A/N	Weight of object B/N	Weight of object C/N	Weight of object D/N
0.40			0.10

Solution:

By the principle of moments:

$W_A \times 0.06 = W_B \times 0.06$, gives $W_B = 0.40N$.

And

$W_A + W_B = W_C + W_D$

Thus $0.4 + 0.4 = 0.1 + W_C$, gives $W_C = 0.7N$.

(ii) Calculate the distance, d.

Solution:

By the principle of moments:

$(0.7N)d = (0.10N) \times (0.08m)$

Thus, d = 0.011 m.

(iii) Calculate the magnitude of T.

Solution:

$T = W_A + W_B = 0.4 + 0.4 = 0.8N$

(c) Object A becomes detached and falls to the ground. State and explain the initial effect on

(i) The rod holding objects A and B,

Solution:

The beam holding B turns clockwise.

(ii) The rod holding objects C and D,

Solution:

The rod falls.

(iii) The rod closest to the top of the mobile.

Solution:

The beam rotates clockwise due to because of the unbalanced moment.

19. (a) State the principle of moments for a body in equilibrium.

Solution:

Considering the moments of the forces about any point, for equilibrium,

The sum of the clockwise moments=the sum of the anticlockwise moments;

This statement is known as the principle of moments.

(b) **Fig. 19.1** shows a vertical force, ***F***, being applied to raise a wheelbarrow which has a total weight of 500 N.

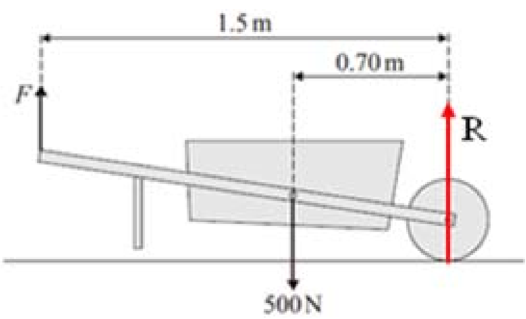


Fig. 19.1

(i) On **Fig. 19.1** draw an arrow to represent the position and direction of the force, R, exerted by the ground on the wheel.

(ii) Calculate the minimum value of the vertical force, F, needed to raise the legs of the wheelbarrow off the ground.

Solution:

(ii) From figure 1, when $F_{\min} \cdot (1.5m) = (500N) \times 0.7m$

The legs of the wheelbarrow are raised off the ground.

$$F_{\min} = \frac{(500N) \times 0.7m}{1.5m} = 233N$$

(iii) Calculate the magnitude of R when the legs of the wheelbarrow have just left the ground.

Solution:

For the wheelbarrow, the resultant force is zero now:

$$F_{\min} + R = 500N \Rightarrow R = 500 - 233 = 267N$$

20. (a) (i) State two conditions necessary for an object to be in equilibrium.

(a) (ii) For each condition state the consequence if the condition is not met.

(i) Condition for equilibrium:

1. the net external force must be zero

$$\sum \vec{F} = 0$$

$$\sum F_x = 0 \ \ and \ \sum F_y = 0$$

2. the net external torque must be zero

$$\sum \tau = 0$$

(ii) If the net external force is not zero, there will be acceleration.

If the net external torque is not zero, the object would rotate with angular acceleration.

Fig. 20. 1 shows a pole vaulter holding a uniform pole horizontally. He keeps the pole in equilibrium by exerting an upward force, U, with his leading hand, and a downward force, D, with his trailing hand.

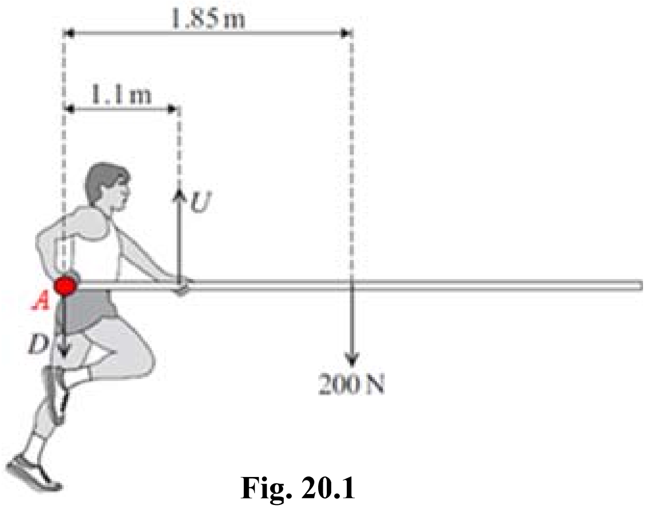


Fig. 20.1

Weight of pole = 200 N

Length of pole = 3.7 m

1 (b) Calculate for the situation shown in **Fig. 20.1**,

1 (b) (i) the force, U,

1 (b) (ii) the force, D.

Solutions:

The pole is in equilibrium, thus the net external force and the net external torque must be zero.

Therefore

$U = D + 200N$ …… (1)

Choose the point A as the pivot, then

$U \times 1.1m = (200N) \times 1.85m$ …… (2)

Form equation (1) and (2), we get

U = 336.4 N

D = 136.4 N

(c) Explain the effect on the magnitudes of U and D if the vaulter moves his leading hand closer to the centre of gravity of the pole and the pole is still in equilibrium.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

Solution:

When the vaulter moves his leading hand closer to the centre of gravity of the pole, the perpendicular distance to the pivot is increased. So the force U decreases. And $U = D + 200N$, thus the change in D is consistent with U.

21. Fig. 21.1 shows a motorcycle and rider. The motorcycle is in contact with the road at A and B.

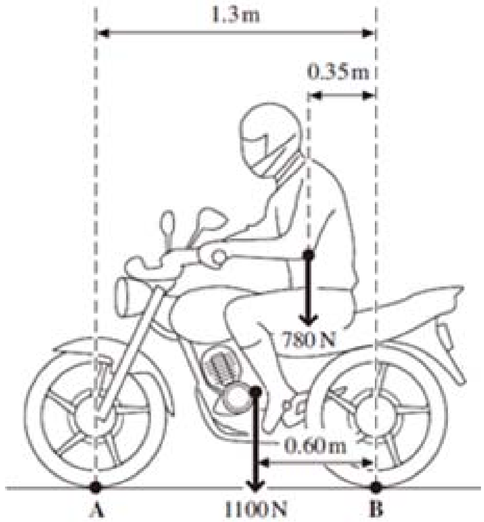


Fig. 21.1

The motorcycle has a weight of 1100 N and the rider's weight is 780 N.

(a) State the principle moments.

Solution:

For equilibrium,

The sum of the clockwise moments = the sum of the anticlockwise moments;

This statement is known as the principle of moments.

(b) Calculate the moment of the rider's weight about B. give an appropriate unit.

Solution:

The moment of a force about any point is defined as the force × the perpendicular distance from the line of action of the force to the point.

That is:

The moment of the force = F × d

Note: d is the line of action of the force to the point.

Unit of the moment of the force: Newton metre (Nm)

Therefore,

The moment of the weight about B = 780 × 0.35 = 273 Nm

(c) By taking the moments about B, calculate the vertical force that the road exerts on the front tyre at A. state your answer to an appropriate number of significant figures.

Solution:

The sum of the clockwise moments $= N_A \times 1.3$

The sum of the anticlockwise moments $= 1100 \times 0.60 + 780 \times 0.35 = 933 N \cdot m$

For equilibrium:

$N_A \times 1.3 = 933$

$N_A = \frac{933}{1.3} = 718 N$

(d) Calculate the vertical force that the road exerts on the rear tyre at B.

Solution:

The net external force must be zero.

Thus

$718 + N_B = 1100 + 780 = 1880$

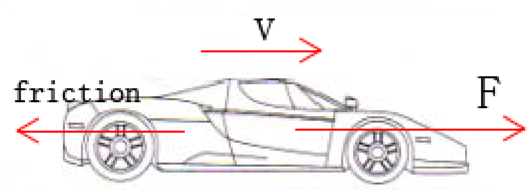
$N_B = 1880 - 718 = 1162 N$

(e) The maximum power of the motorcycle is 7.5 kW and it has a maximum speed of $26 m/s$, when travelling on a level road.

Calculate the total horizontal resistive force for this speed.

Solution:

Power and velocity



Above, the car's engine provides a forward force F which balances the total frictional force on the car. As a result, the car maintains a steady velocity v. the displacement of the car is s in time intervals Δt. P is the power being delivered to the wheels.

So the work done (by F) = Fs

$$power = P = \frac{work\ done}{time\ taken} = \frac{Fs}{\Delta t}$$

But $v = \frac{s}{\Delta t}$

So $P = Fv$

Therefore, at its maximum speed, the resistive force is given by

$$F = \frac{P}{v} = \frac{7.5\times10^3}{26} = 288.5N$$

22. A cylindrical tank for water ($\rho = 1000 kg/m^3$) is 3 m long and 1.5 m in diameter. How many kilograms of water will the tank hold?

Solution:

First we find the volume:

$$V = \pi r^2 h = 3.14\times\left(\frac{1.5}{2}\right)^2 \times 3 = 5.3m^3$$

Substituting the volume and mass density into $m = \rho V$, we obtain

$$m = \rho V = \left(1000kg/m^3\right)\left(5.3m^3\right) = 5.3\times10^3 kg$$

23. A lady of weight 495 N, standing on the ground with the contact area of 412 cm ^{2}.

(i) What is the pressure of her shoes to the ground?

(ii) now she stands on the ground on one foot, what is the pressure?

Strategy:

$$Pressure = \frac{Force}{Area} = \frac{F}{A}$$

(i) F = 495 N, S_1 = 412 cm ^{2} = 0.0412 m ^{2}

$$P_1 = \frac{495}{0.0412} = 1.2\times10^4 Pa$$

(ii) F = 495 N, S_2 = 412/2 cm ^{2} = 0.0206 m ^{2}

$$P_2 = \frac{495}{0.0206} = 2.4\times10^4 Pa$$

Chapter 3 Car safety

3.1. Stopping distance

Traffic accidents often happen because vehicles are being driven too fast and too close. A driver needs to maintain a safe distance between his or her own vehicle and the vehicle in front. If a vehicle suddenly brakes, the driver of the following vehicle needs to brake as well to avoid a crash.

Thinking distance: is the distance traveled by a vehicle in the time it takes the driver to react. So for a vehicle moving at constant speed v, the thinking distance S1 = speed × reaction time = u t， where t is the reaction time of the driver.

So the thinking distance is affected by the car's speed and the driver's reaction time; and the reaction time is affected by tiredness, alcohol and drugs and distractions.

Braking distance: is the distance traveled by a car in the time it takes to stop safely, from when the brakes are first applied. Assuming constant deceleration, a, and from initial speed u to zero speed.

So the braking distance, $S_2 = \frac{u^2}{2a}$

Braking distance is affected by the car's speed, road conditions (icy, wet) and car conditions (tyres, brakes).

Stopping distance:

Stopping distance = thinking distance + braking distance $= ut + \frac{u^2}{2a}$

3.2. Car safety

(i) Impact force

The effect of a collision on a vehicle can be measured in terms of the acceleration or deceleration of the vehicle. By expressing an acceleration or deceleration in terms of g, the acceleration due to gravity, the force of the impact can then easily be related to the weight of the vehicle. For example, suppose a vehicle hits a wall and its acceleration is $-30ms^{-2}$. In terms of g, the acceleration = －3g. So the impact force of the wall on the vehicle must have been 3 times its weight (= 3mg, where m is the mass of the vehicle). Such an impact is sometimes described as being ‘equal to 3g’. This statement, although technically wrong because the acceleration not he impact force is equal to 3g, is a convenient way of expressing the effect of an impact on a vehicle or a person.

(ii) Contact time and impact time

When objects collide and bounce off each other, they are in contact with each other for a certain time, which is the same for both objects. The shorter the contact time, the greater the impact force for the same initial velocities of the two objects. When two vehicles collide, they may or may not separate from each other after the collision. If they remain tangled together, they exert forces on each other until they are moving at the same velocity. The duration of the impact force is not the same as the contact time in this situation.

The impact time, *t*, (the duration of the impact force) can be worked out by applying the equation $s = \frac{1}{2}(u+v)t$ to one of the vehicles, where s is the distance moved by that vehicle during the impact, u is its initial velocity and v is its final velocity. If the vehicle mass is known, the impact force can also be calculated.

For a vehicle of mass m in time t,

$$\text{the impact time, } t = \frac{2s}{u+v}$$

$$\text{the acceleration, } a = \frac{v-u}{t}$$

$$\text{the impact force, } F = ma$$

For example, suppose a 1000kg vehicle moving at 20ms^{-1} slows down in a distance of 4.0 m to a velocity of $12ms^{-1}$, as a result of hitting a stationary vehicle. Rearranging the above equation gives

$$t = \frac{2s}{u+v} = \frac{2\times 4}{20+12} = 0.25s$$

The acceleration, $a = \frac{v-u}{t} = \frac{12-20}{0.25} = -32ms^{-2} = -3.3g$

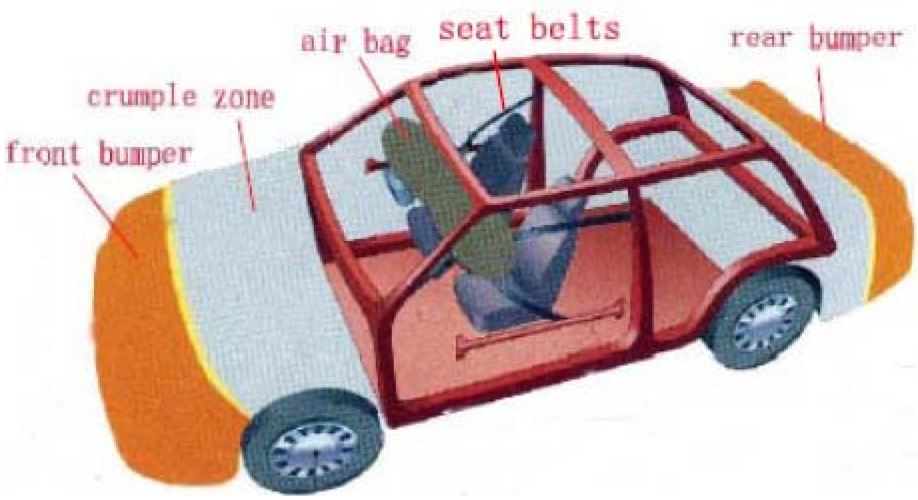
(Where $g = 9.8$ m s $^{-2}$)

The impact force F = ma = 1000 × (-32) = -32000N.

Note: the work done by the impact force F over an impact distance s (= Fs) is equal to the change of kinetic energy of the vehicle, the impact force can also be worked out using the equation

$$F = \frac{change\ of\ kinetic\ energy}{impact\ distance}$$

(iii) Car safety features



Vehicle bumpers: gives way a little in a low-speed impact and so increase the impact time. The impact force is therefore reduced as a result. If the initial speed of impact is too high, the bumper and/or the vehicle chassis are likely to be damaged.

Crumple zones: the engine compartment of a car is designed to give way in a front-end impact. If the engine compartment were rigid, the impact time would be very short, so the impact force would be very large. By designing the engine compartment so it crumples in a front-end impact, the impact time is increased and the impact force is therefore reduced.

Seat belts: in a front-end impact, a correctly-fitted seat belt restrains the wearer from crashing into the vehicle frame after the vehicle suddenly stops. The restraining force on the wearer is therefore much less than the impact force would be if the wearer hit the vehicle frame. With the seat belt on, the wearer is stopped more gradually than without it.

Air bags: an airbag reduces the force on a person, because the airbag acts as a cushion and increases the impact time on the person. More significantly, the force of the impact is spread over the contact area, which is greater than the contact area with a seat belt. So the pressure on the body is less.

3.3 2 Worked examples

1. A vehicle is traveling at a speed of $18ms^{-1}$ on a level road, when the driver sees a pedestrian stepping off the pavement into the road 45 m ahead. The driver reacts within 0.4 s and applies the brakes, causing the car to decelerate at $4.8ms^{-2}$.

a. calculate the thinking distance, braking distance and the stopping distance

b. how far does the driver stop from where the pedestrian stepped into the road?

Strategy: the thinking distance S_1 = speed × reaction time = u *t*, where *t* is the reaction time of the driver.

The braking distance, $S_2 = \frac{u^2}{2a}$

Stopping distance = thinking distance + braking distance $= ut + \frac{u^2}{2a}$

Answers:

a.

$$S_1 = 18 \times 0.4 = 7.2m$$

$$S_2 = \frac{u^2}{2a} = \frac{18^2}{2 \times 4.8} = 33.75m$$

Stopping distance, $S = S_1 + S_2 = 33.75 + 7.2 = 40.95$ m

b. the distance between the car and the pedestrian = 45 − 40.95 = 4.05 m

2. a car of mass 1200 kg traveling at a speed of 15 m s^{-1} is struck from behind by another vehicle, causing its speed to increase to 19 m s^{-1} in a time of 0.20 s. calculate:

a. the acceleration of the car, in terms of g.

b. the impact force on the car.

Strategy: *the acceleration,* $a = \frac{v-u}{t}$, *the impact force,* $F = ma$

Answers: $a = \frac{v-u}{t} = \frac{19-15}{0.2} = 20ms^{-2}$

$F = ma = 1200 \times 20 = 24000N$