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1 Five boxes weigh 5 kg, 2 kg, 4 kg, 3 kg and 8 kg. They are stacked, in the order given, with the first box at the top of the stack. The boxes are to be packed into bins that can each hold up to 10 kg.

(i) Use the first-fit method to put the boxes into bins. Show clearly which boxes are packed in which bins. [2]

(ii) Use the first-fit decreasing method to put the boxes into bins. You do not need to use an algorithm for sorting. Show clearly which boxes are packed in which bins. [2]

(iii) Why might the first-fit decreasing method not be practical? [1]

(iv) Show that if the bins can only hold up to 8 kg each it is still possible to pack the boxes into three bins. [1]

2 A puzzle involves a 3 by 3 grid of squares, numbered 1 to 9, as shown in Fig. 1a below. Eight of the squares are covered by blank tiles. Fig. 1b shows the puzzle with all of the squares covered except for square 4. This arrangement of tiles will be called position 4.

1	2	3
4	5	6
7	8	9

Fig. 1a

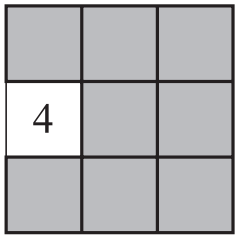


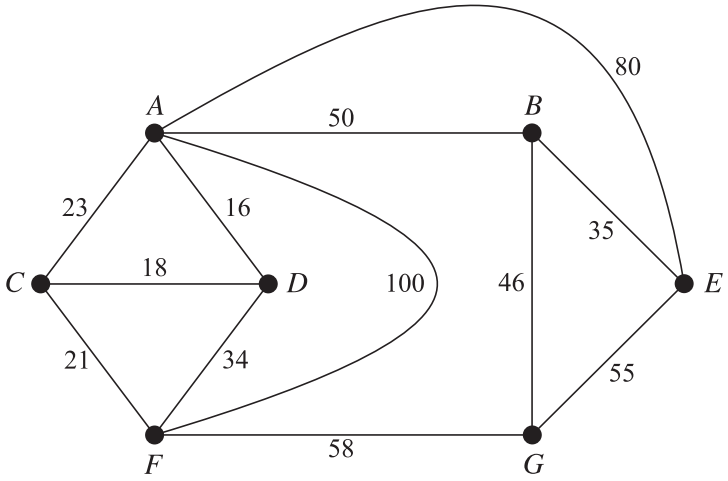
Fig. 1b

A move consists of sliding a tile into the empty space. From position 4, the next move will result in position 1, position 5 or position 7.

(i) Draw a graph with nine vertices to represent the nine positions and arcs that show which positions can be reached from one another in one move. What is the least number of moves needed to get from position 1 to position 9? [3]

(ii) State whether the graph from part **(i)** is Eulerian, semi-Eulerian or neither. Explain how you know which it is. [2]

3 Answer this question on the insert provided.



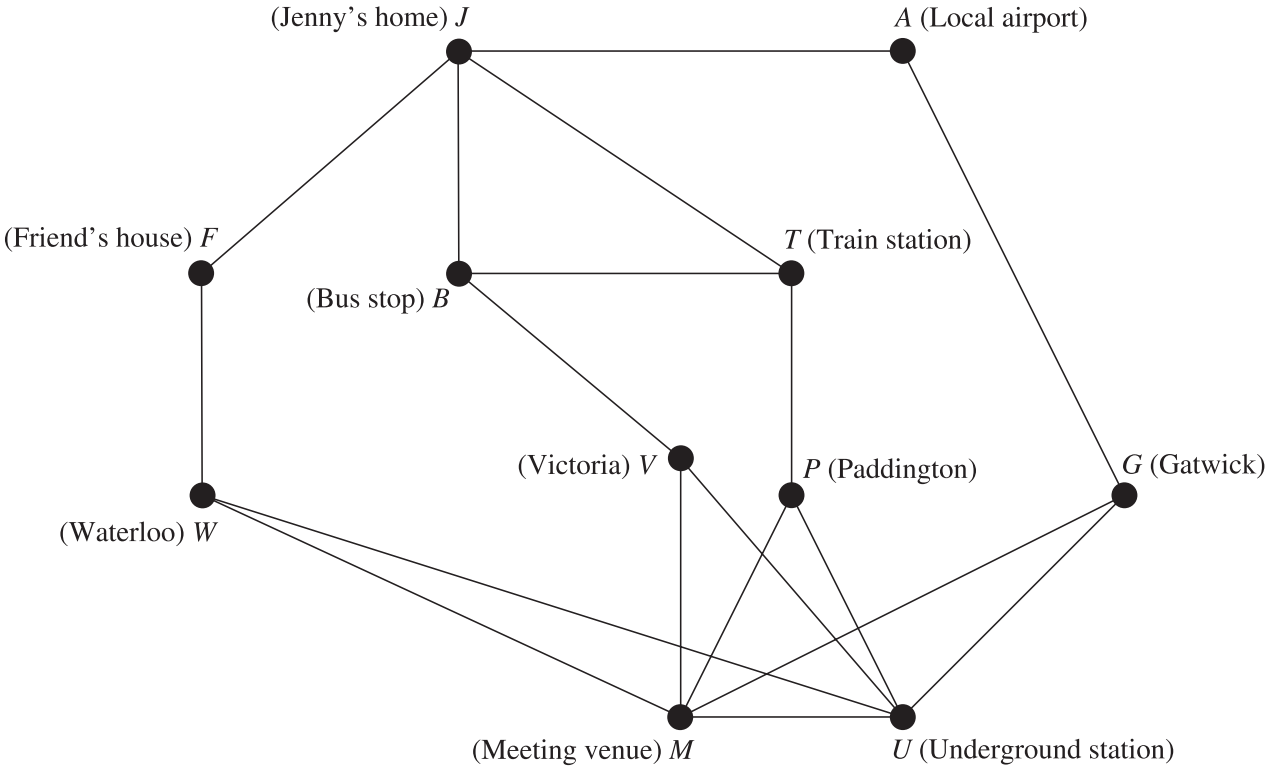
(i) This diagram shows a network. The insert has a copy of this network together with a list of the arcs, sorted into increasing order of weight. Use Kruskal's algorithm on the insert to find a minimum spanning tree for this network. Draw your tree and give its total weight. [5]

(ii) Use your answer to part **(i)** to find the weight of a minimum spanning tree for the network with vertex *G*, and all the arcs joined to *G*, removed. Hence find a lower bound for the travelling salesperson problem on the original network. [3]

(iii) Apply the nearest neighbour method, starting from vertex *A*, to find an upper bound for the travelling salesperson problem on the original network. [3]

4 Answer this question on the insert provided.

Jenny needs to travel to London to arrive in time for a morning meeting. The graph below represents the various travel options that are available to her.



It takes Jenny 120 minutes to drive from her home to the local airport and check in (arc JA). The journey from the local airport to Gatwick takes 80 minutes. From Gatwick to the underground station takes 60 minutes, and walking from the underground station to the meeting venue takes 15 minutes. Alternatively, Jenny could get a taxi from Gatwick to the meeting venue; this takes 80 minutes.

It takes Jenny 15 minutes to drive from her house to the train station. Alternatively, she can walk to the bus stop, which takes 5 minutes, and then get a bus to the train station, taking another 20 minutes. From the train station to Paddington takes 300 minutes, and from Paddington to the underground station takes a further 20 minutes. Alternatively, Jenny could walk from Paddington to the meeting venue, taking 30 minutes.

Jenny can catch a coach from her local bus stop to Victoria, taking 400 minutes. From Victoria she can either travel to the underground station, which takes 10 minutes, or she can walk to the meeting venue, which takes 15 minutes.

The final option available to Jenny is to drive to a friend's house, taking 240 minutes, and then continue the journey into London by train. The journey from her friend's house to Waterloo takes Jenny 30 minutes. From here she can either go to the underground station, which takes 20 minutes, or walk to the meeting venue, which takes 40 minutes.

(i) Weight the arcs on the graph in the insert to show these times. Apply Dijkstra's algorithm, starting from J, to give a permanent label and order of becoming permanent at each vertex. Stop when you have assigned a permanent label to vertex M. Write down the route of the shortest path from J to M. [9]

(ii) What does the value of the permanent label at M represent? [1]

(iii) Give two reasons why Jenny might choose to use a different route from J to M. [2]

5 Mark wants to decorate the walls of his study. The total wall area is $24\,m^2$. Mark can cover the walls using any combination of three materials: panelling, paint and pinboard. He wants at least $2\,m^2$ of pinboard and at least $10\,m^2$ of panelling.

Panelling costs £8 per m^2 and it will take Mark 15 minutes to put up $1\,m^2$ of panelling. Paint costs £4 per m^2 and it will take Mark 30 minutes to paint $1\,m^2$. Pinboard costs £10 per m^2 and it will take Mark 20 minutes to put up $1\,m^2$ of pinboard. He has all the equipment that he will need for the decorating jobs.

Mark is able to spend up to £150 on the materials for the decorating. He wants to know what area should be covered with each material to enable him to complete the whole job in the shortest time possible.

Mark models the problem as an LP with five constraints. His constraints are:

$$\begin{aligned} &x + y + z = 24, \\ &4x + 2y + 5z \leqslant 75, \\ &x \geqslant 10, \\ &y \geqslant 0, \\ &z \geqslant 2. \end{aligned}$$

(i) Identify the meaning of each of the variables x, y and z. [2]

(ii) Show how the constraint $4x + 2y + 5z \leqslant 75$ was formed. [2]

(iii) Write down an objective function, to be minimised. [1]

Mark rewrites the first constraint as $z = 24 - x - y$ and uses this to eliminate z from the problem.

(iv) Rewrite and simplify the objective and the remaining four constraints as functions of x and y only. [3]

(v) Represent your constraints from part **(iv)** graphically and identify the feasible region. Your graph should show x and y values from 9 to 15 only. [4]

6 **(i)** Represent the linear programming problem below by an initial Simplex tableau. [2]

$$\begin{aligned} &\text{Maximise} && P = 25x + 14y - 32z, \\ &\text{subject to} && 6x - 4y + 3z \leqslant 24, \\ &&& 5x - 3y + 10z \leqslant 15, \\ &\text{and} && x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{aligned}$$

(ii) Explain how you know that the first iteration will use a pivot from the x column. Show the calculations used to find the pivot element. [3]

(iii) Perform **one** iteration of the Simplex algorithm. Show how each row was calculated and write down the values of x, y, z and P that result from this iteration. [7]

(iv) Explain why the Simplex algorithm cannot be used to find the optimal value of P for this problem. [1]

7 In this question, the function INT(X) is the largest integer less than or equal to X. For example,

$$\text{INT}(3.6) = 3,$$
$$\text{INT}(3) = 3,$$
$$\text{INT}(-3.6) = -4.$$

Consider the following algorithm.

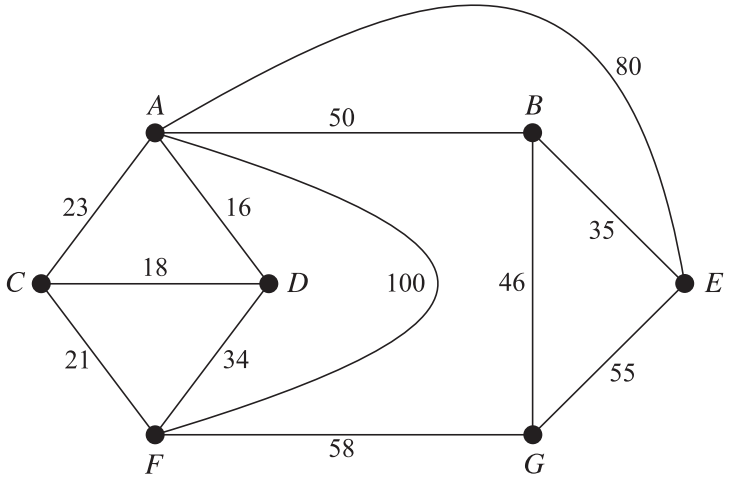
Step 1	Input B
Step 2	Input N
Step 3	Calculate $F = N \div B$
Step 4	Let $G = \text{INT}(F)$
Step 5	Calculate $H = B \times G$
Step 6	Calculate $C = N - H$
Step 7	Output C
Step 8	Replace N by the value of G
Step 9	If $N = 0$ then stop, otherwise go back to Step 3

(i) Apply the algorithm with the inputs $B = 2$ and $N = 5$. Record the values of F, G, H, C and N each time Step 9 is reached. [5]

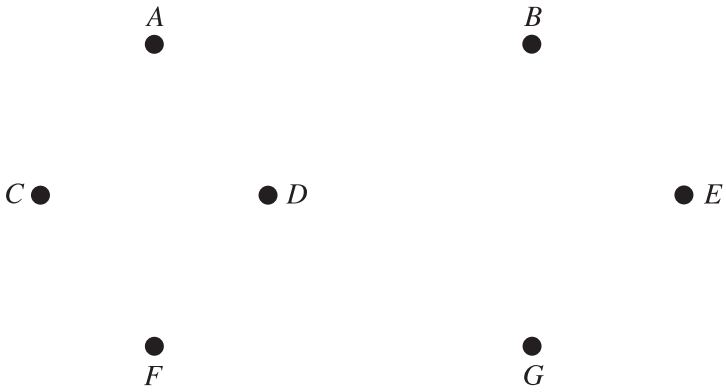
(ii) Explain what happens when the algorithm is applied with the inputs $B = 2$ and $N = -5$. [4]

(iii) Apply the algorithm with the inputs $B = 10$ and $N = 37$. Record the values of F, G, H, C and N each time Step 9 is reached. What are the output values when $B = 10$ and N is any positive integer? [4]

3 (i)



$AD = 16$
$CD = 18$
$CF = 21$
$AC = 23$
$DF = 34$
$BE = 35$
$BG = 46$
$AB = 50$
$EG = 55$
$FG = 58$
$AE = 80$
$AF = 100$



Total weight of arcs in minimum spanning tree = ...

(ii) .....

.....

Weight of spanning tree for the network with vertex *G* removed = ...

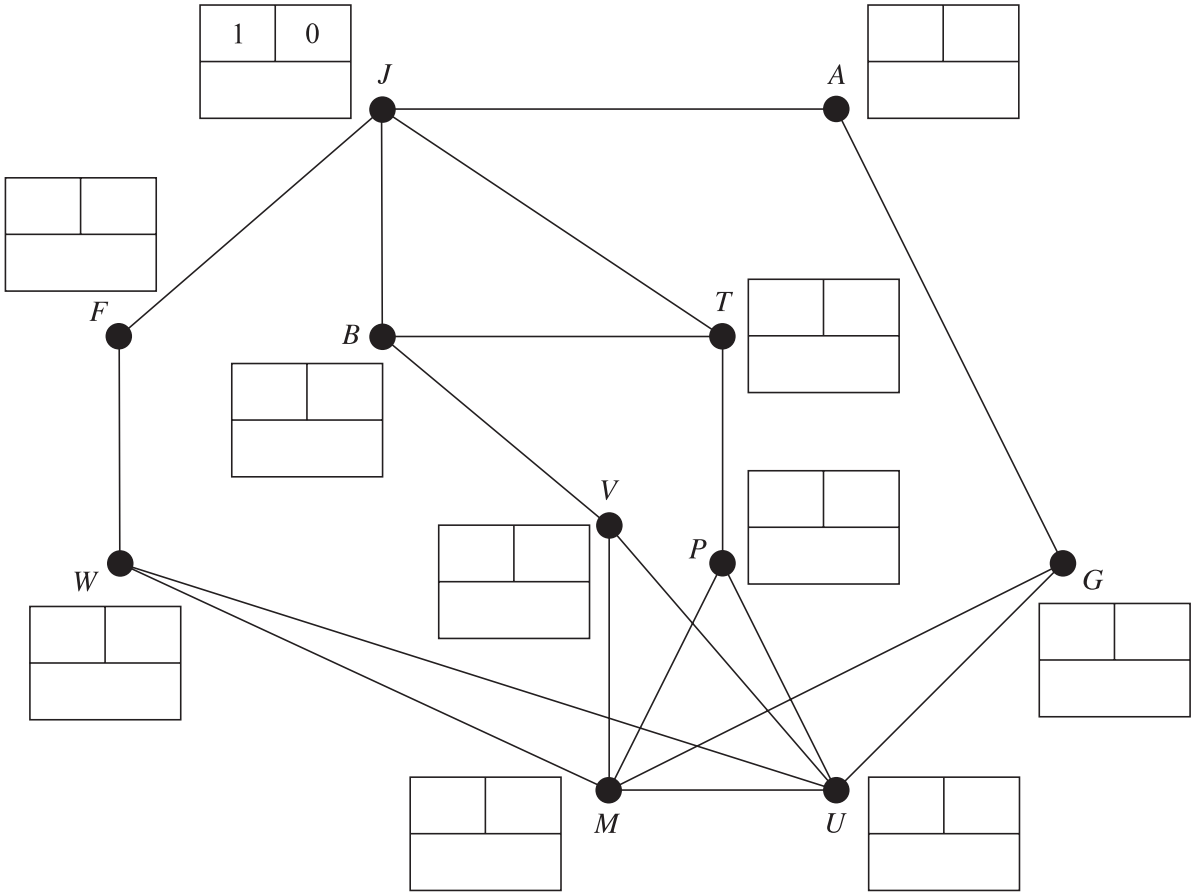
.....

Lower bound for travelling salesperson problem on original network = ...

(iii) .....

Upper bound for travelling salesperson problem on original network = ...

4 **(i)**



Route of shortest path from *J* to *M*: .....

(ii) .....

.....

(iii) .....

.....

.....

mock papers 2

1.

29	52	73	87	74	47	38	61	41

The numbers in the list represent the lengths in minutes of nine radio programmes. They are to be recorded onto tapes which each store up to 100 minutes of programmes.

(a) Obtain a lower bound for the number of tapes needed to store the nine programmes.

(2)

(b) Use the first-fit bin packing algorithm to fit the programmes onto the tapes.

(3)

(c) Use the first-fit decreasing bin packing algorithm to fit the programmes onto the tapes.

(3)

(Total 8 marks)

2.

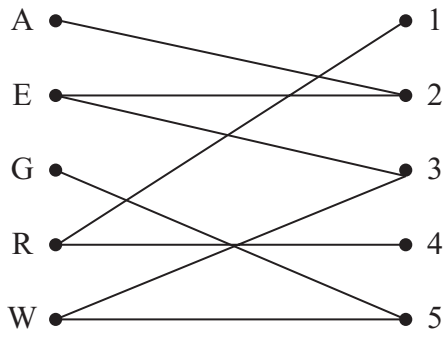


Figure 1

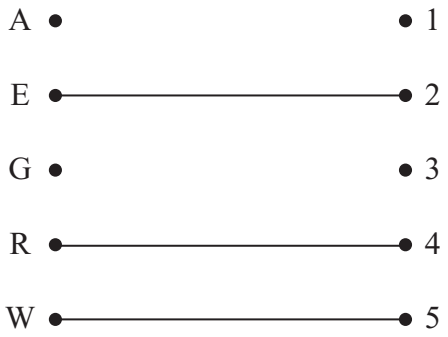


Figure 2

Five tour guides, Alice, Emily, George, Rose and Weidi, need to be assigned to five coach trips, 1, 2, 3, 4 and 5. A bipartite graph showing their preferences is given in Figure 1 and an initial matching is given in Figure 2.

(a) Use the maximum matching algorithm, starting with vertex G, to increase the number of matchings. State the alternating path you used.

(2)

(b) List the improved matching you found in (a).

(1)

(c) Explain why a complete matching is not possible.

(2)

Weidi agrees to be assigned to coach trip 3, 4 or 5.

(d) **Starting with your current maximal matching**, use the maximum matching algorithm to obtain a complete matching.

(3)

(Total 8 marks)

3.

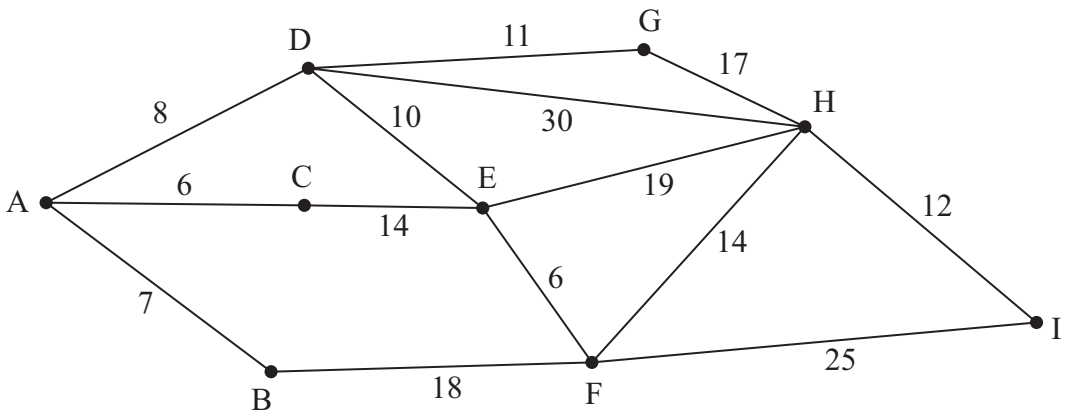


Figure 3

Figure 3 shows a network of roads. The number on each arc represents the length, in km, of that road.

(a) Use Dijkstra's algorithm to find the shortest route from A to I. State your shortest route and its length.

(5)

Sam has been asked to inspect the network and assess the condition of the roads. He must travel along each road at least once, starting and finishing at A.

(b) Use an appropriate algorithm to determine the length of the shortest route Sam can travel. State a shortest route.

(4)

(The total weight of the network is 197km)

(Total 9 marks)

4.

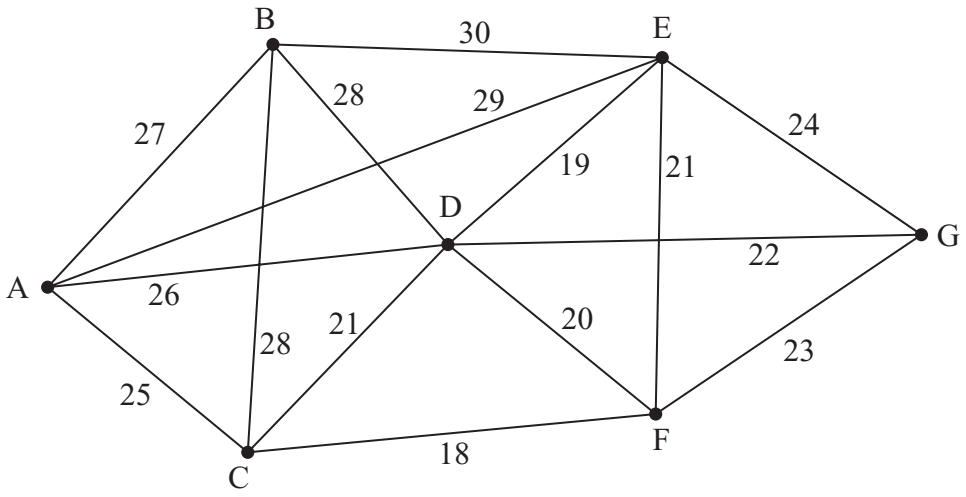


Figure 4

(a) State two differences between Kruskal's algorithm and Prim's algorithm for finding a minimum spanning tree.

(2)

(b) Listing the arcs in the order that you consider them, find a minimum spanning tree for the network in Figure 4, using

(i) Prim's algorithm,

(ii) Kruskal's algorithm.

(6)

(Total 8 marks)

5.

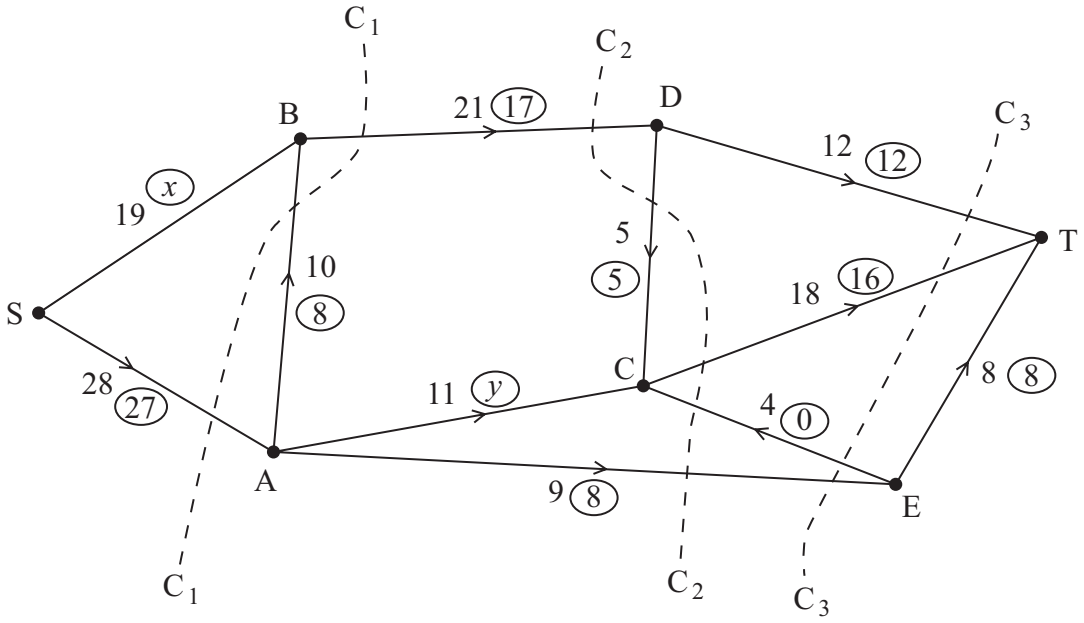


Figure 5

Figure 5 shows a capacitated, directed network of pipes. The number on each arc represents the capacity of that pipe. The numbers in circles represent a feasible flow.

(a) State the values of x and y.

(2)

(b) List the saturated arcs.

(2)

(c) State the value of the feasible flow.

(1)

(d) State the capacities of the cuts C_1, C_2, and C_3.

(3)

(e) By inspection, find a flow-augmenting route to increase the flow by one unit. You must state your route.

(1)

(f) Prove that the new flow is maximal.

(2)

(Total 11 marks)

6. The tableau below is the initial tableau for a maximising linear programming problem in x, y and z.

Basic variable	x	y	z	r	s	t	Value
r	4	$\frac{7}{3}$	$\frac{5}{2}$	1	0	0	64
s	1	3	0	0	1	0	16
t	4	2	2	0	0	1	60
P	–5	$\frac{-7}{2}$	–4	0	0	0	0

(a) Taking the most negative number in the profit row to indicate the pivot column at each stage, perform two complete iterations of the simplex algorithm. State the row operations you use.

(9)

(b) Explain how you know that your solution is not optimal.

(1)

(Total 10 marks)

7.

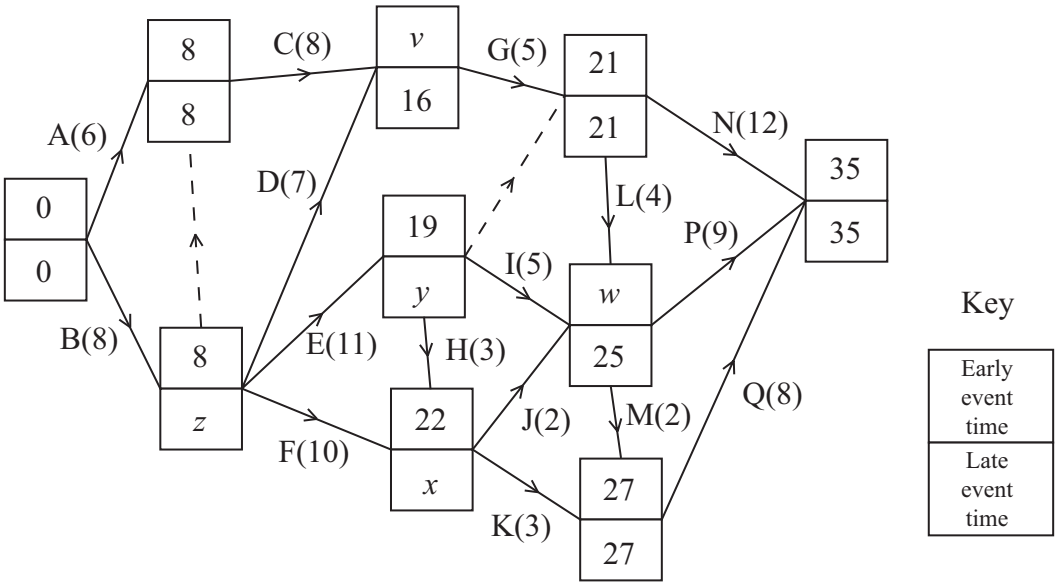


Figure 6

The network in Figure 6 shows the activities that need to be undertaken to complete a building project. Each activity is represented by an arc. The number in brackets is the duration of the activity in days. The early and late event times are shown at each vertex.

(a) Find the values of *v, w, x, y* and *z*.

(3)

(b) List the critical activities.

(1)

(c) Calculate the total float on each of activities H and J.

(2)

(d) Draw a cascade (Gantt) chart for the project.

(4)

The engineer in charge of the project visits the site at midday on day 8 and sees that activity E has not yet been started.

(e) Determine if the project can still be completed on time. You must explain your answer.

(2)

Given that each activity requires one worker and that the project must be completed in 35 days,

(f) use your cascade chart to determine a lower bound for the number of workers needed. You must justify your answer.

(2)

(Total 14 marks)

8. Class 8B has decided to sell apples and bananas at morning break this week to raise money for charity. The profit on each apple is 20p, the profit on each banana is 15p. They have done some market research and formed the following constraints.

- They will sell at most 800 items of fruit during the week.
- They will sell at least twice as many apples as bananas.
- They will sell between 50 and 100 bananas.

Assuming they will sell all their fruit, formulate the above information as a linear programming problem, letting a represent the number of apples they sell and b represent the number of bananas they sell.

Write your constraints as inequalities.

(Total 7 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer book for mock papers 2

1.

Q1

(Total 8 marks)

Turn over

2.

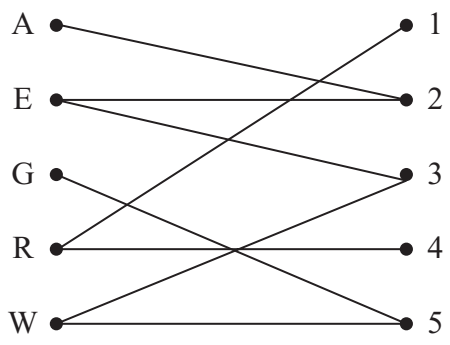


Figure 1

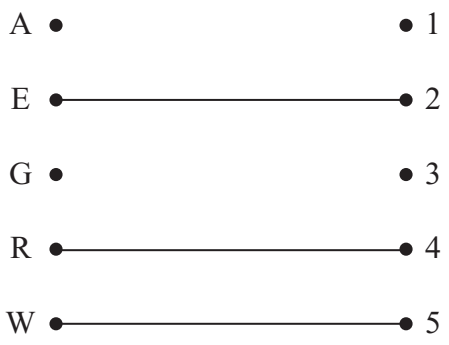


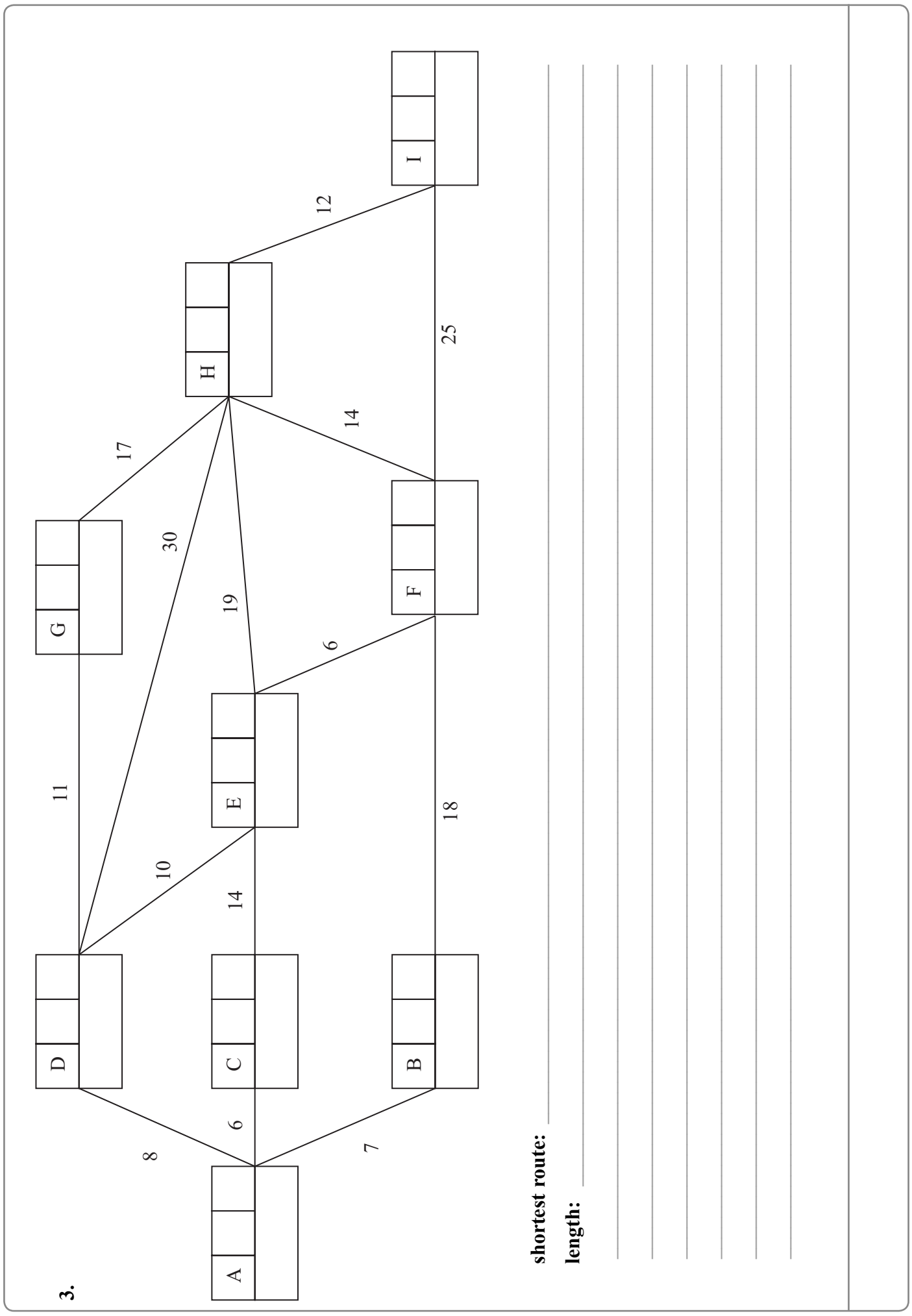
Figure 2

A •	• 1
E •	• 2
G •	• 3
R •	• 4
W •	• 5

Q2

(Total 8 marks)

Turn over



Q3

(Total 9 marks)

4.

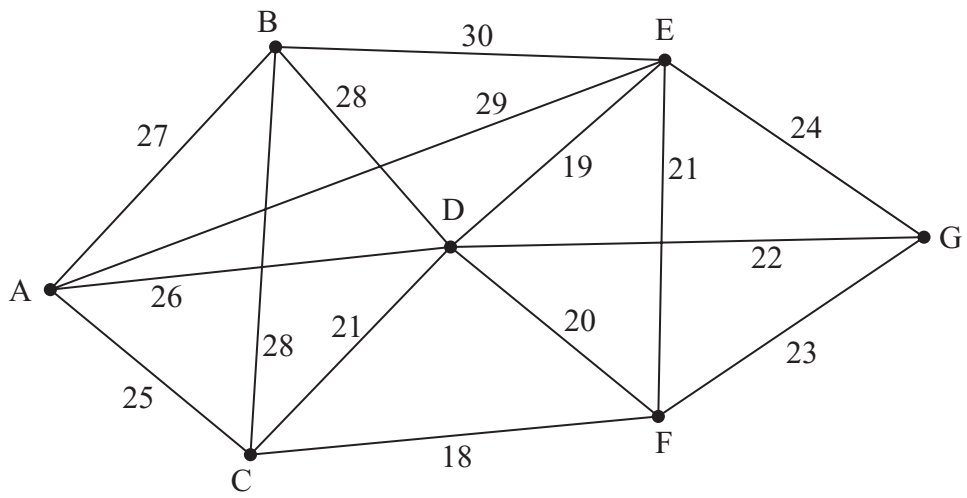


Figure 4

Q4

(Total 8 marks)

5.

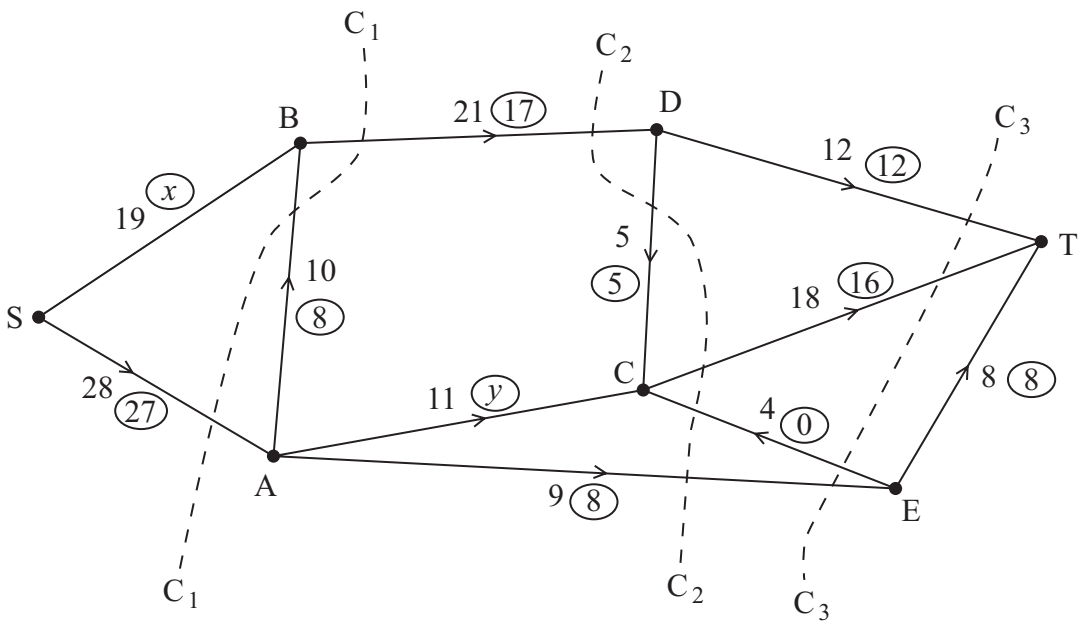


Figure 5

Q5

(Total 11 marks)

6.

Basic variable	x	y	z	r	s	t	Value
r	4	$\frac{7}{3}$	$\frac{5}{2}$	1	0	0	64
s	1	3	0	0	1	0	16
t	4	2	2	0	0	1	60
P	–5	$\frac{-7}{2}$	–4	0	0	0	0

You may not need to use all of these tableaux

b.v.	x	y	z	r	s	t	Value	Row operations
P								

b.v.	x	y	z	r	s	t	Value	Row operations
P								

b.v.	x	y	z	r	s	t	Value	Row operations
P								

b.v.	x	y	z	r	s	t	Value	Row operations
P								

b.v.	x	y	z	r	s	t	Value	Row operations
P								

(Total 10 marks)

Q6

Turn over

7. (a) v = _____, w = _____, x = _____,

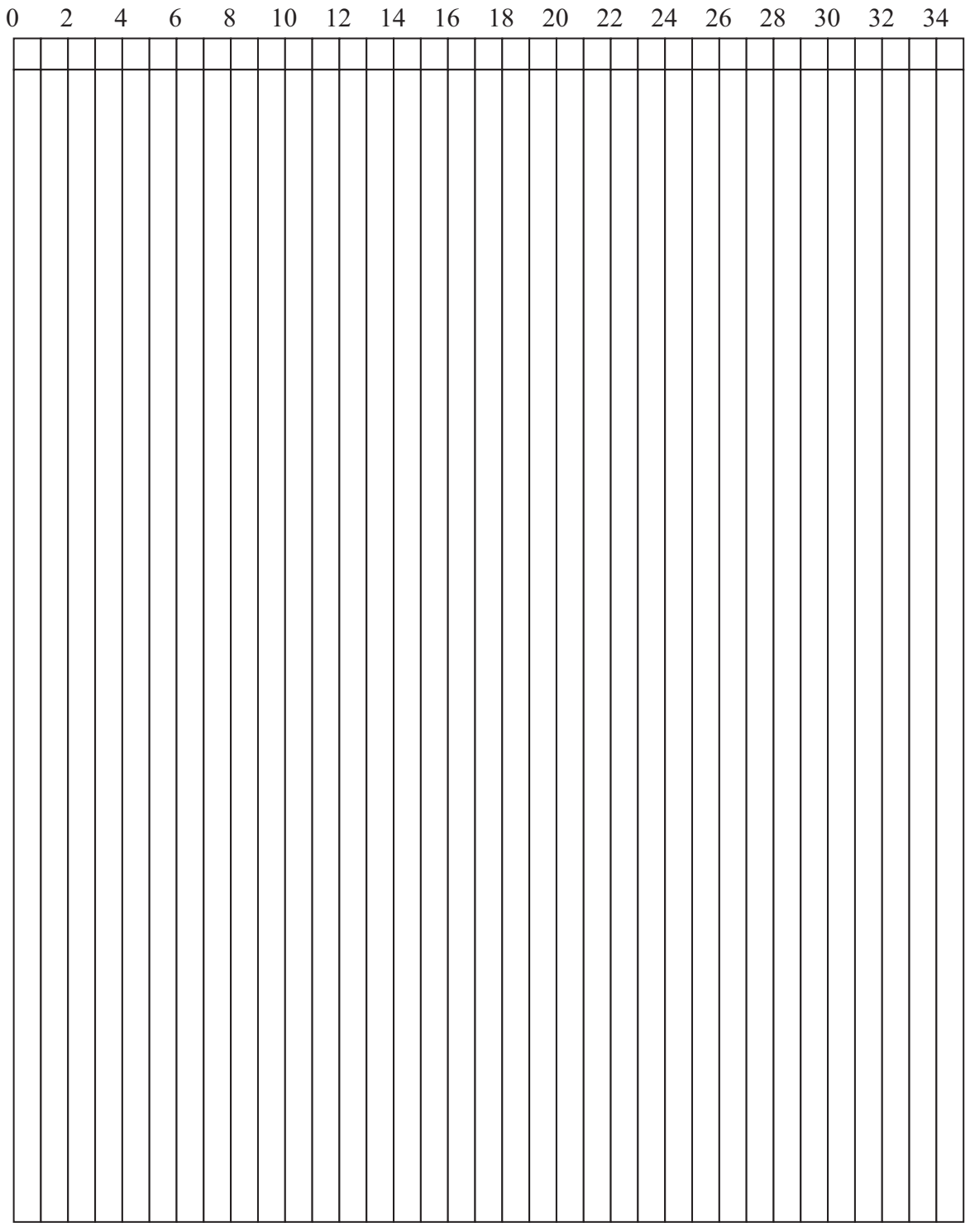
y = _____, z = _____,

(b) Critical activities _____

(c) Total float on activity H _____

Total float on activity J _____

(d)



(e)

(f)

Q7

(Total 14 marks)

Turn over

8.

Q8

(Total 7 marks)

TOTAL FOR PAPER: 75 MARKS

END

1 This question is about using bubble sort to sort a list of numbers into **increasing** order.

(i) Which numbers, if any, can be guaranteed to be in their correct final position after the first pass? [1]

Suppose now that the original, unsorted list was 3, 2, 1, 5, 4.

(ii) Write down the list that results after one pass through bubble sort. How many comparisons and how many swaps were used in this pass? [2]

(iii) Write down the list that results after a second pass through bubble sort. How many more passes will be required until the algorithm terminates? [2]

Bubble sort is a quadratic order algorithm.

(iv) A computer takes 0.2 seconds to sort a list of 500 numbers using bubble sort. Approximately how long will it take to sort a list of 3000 numbers? [2]

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

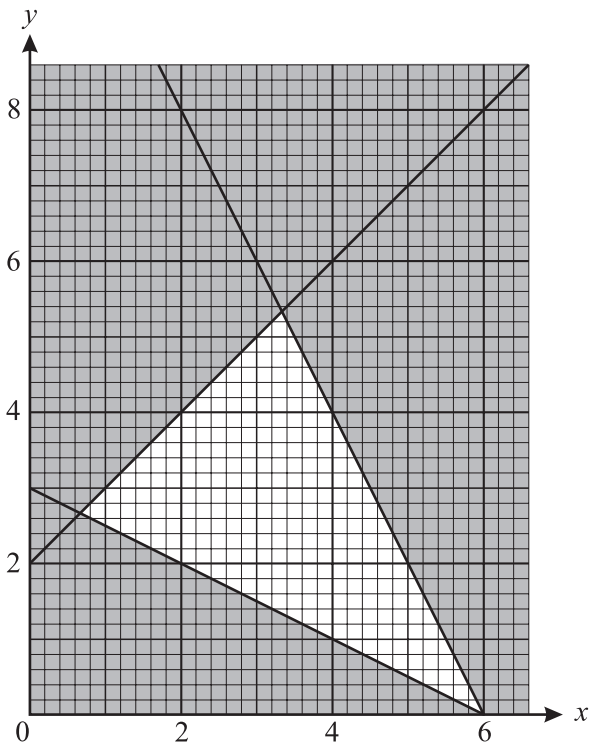
A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(i) Draw an Eulerian graph with four vertices, of orders 2, 2, 4 and 4, and no others. Explain why your graph is not simply connected. [3]

(ii) Draw a non-Eulerian graph with four vertices, of orders 2, 2, 4 and 4, and no others. Explain why your graph is non-Eulerian even though its vertices are all of even order. [3]

3 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Write down the inequalities that define the feasible region. [4]

(ii) Calculate the coordinates of the three vertices of the feasible region. [4]

The objective is to maximise $5x + 3y$.

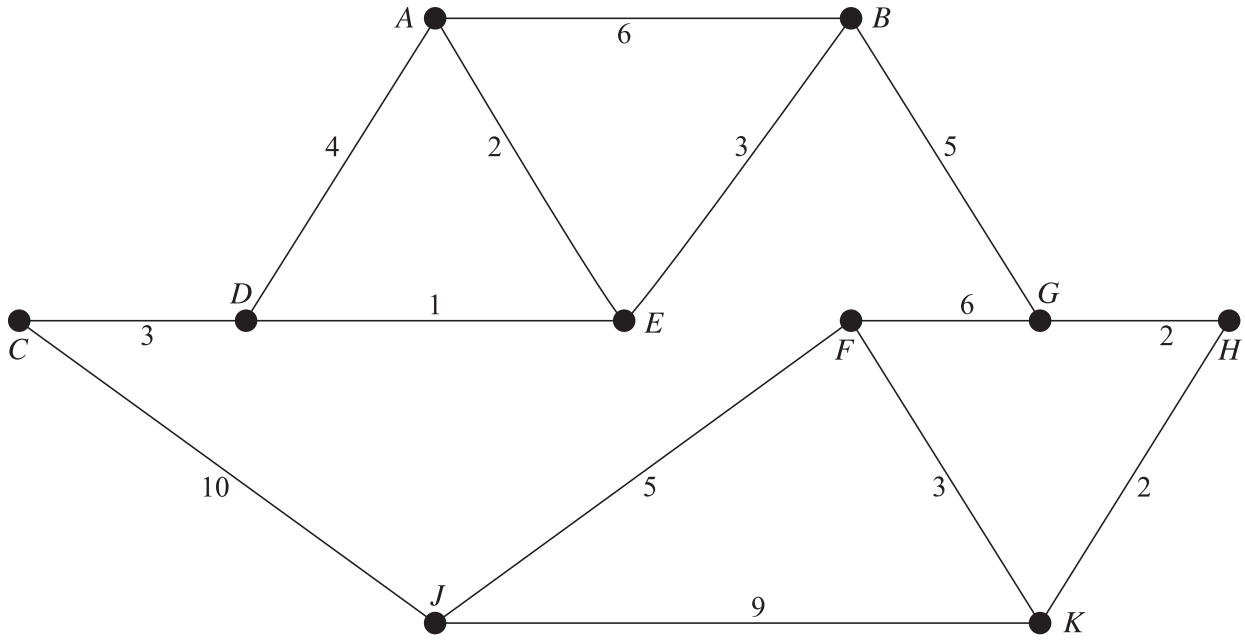
(iii) Find the values of x and y at the optimal point, and the corresponding maximum value of $5x + 3y$. [3]

The objective is changed to maximise $5x + ky$, where k is positive.

(iv) Find the range of values of k for which the optimal point is the same as in part **(iii)**. [3]

4 Answer this question on the insert provided.

The vertices in the network below represent the rooms in a house. The arcs represent routes between rooms, and the weights on the arcs represent distances in metres.



(i) On the diagram in the insert, use Dijkstra's algorithm to find the shortest path from A to K. You must show your working, including temporary labels, permanent labels and the order in which permanent labels are assigned. Write down the route of the shortest path from A to K and give its length in metres. [7]

A locked door blocks the route CJ, so this arc cannot be used.

(ii) Use your answer to part **(i)** to find the route of the shortest path from A to J and its length in metres. [2]

(iii) Alterations mean that the length of route FJ changes from its current value of 5 metres. By how much would it have to change if the route of the shortest path from A to J, not using CJ, changes from that found in part **(ii)**? [2]

5 Laura is booking buses to transport students home from a college party. She wants to book four buses to travel to Easton and five buses to travel to Weston. She contacts the local bus companies to ask about availability and cost. This information is summarised in the table below.

Company	Number of buses available	Cost per bus to Easton	Cost per bus to Weston
Anywhere Autos (*A*)	3	£250	£250
Busy Buses (*B*)	3	£200	£140
County Coaches (*C*)	3	£300	£280

Suppose that Laura books x buses to travel to Easton from company A and y buses to travel to Easton from company B.

(i) **Copy and complete** the following table to show, in terms of x and y, how many buses Laura books from each company to each town and show that the total cost is £$(2090 - 20x + 40y)$. [5]

	E	W
A	x	
B	y	
C		$x + y - 1$

(ii) Laura wants to spend no more than £2150 on the buses.

Show that this leads to the constraint $-x + 2y \leqslant 3$. [1]

When Laura looks at the times that the companies could run the buses, she realises that she will need at least one bus from C to E. This leads to the constraint $x + y \leqslant 3$.

Each bus from A can carry 50 students, each bus from B can carry 40 students and each bus from C can carry 60 students. Laura wants to maximise the number of students who can travel to W.

(iii) Show that this leads to needing to maximise the objective function $x + 2y$. [2]

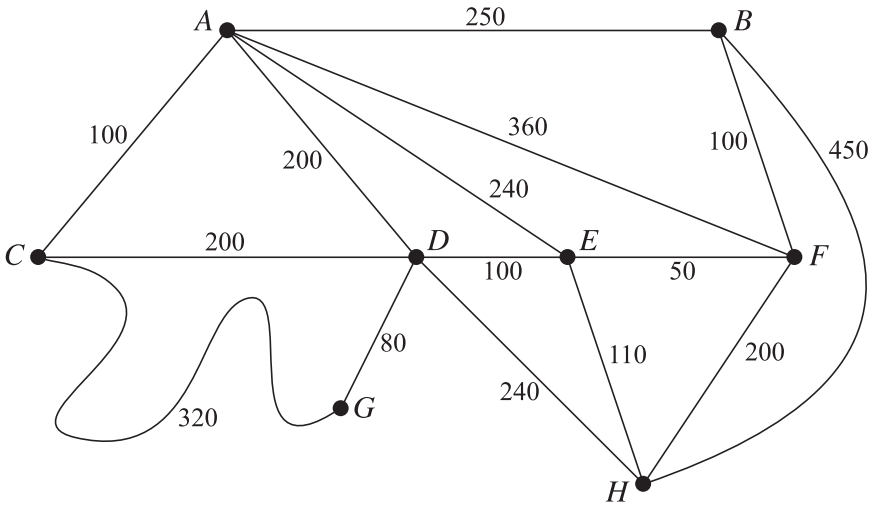
Laura's problem gives the linear programming problem:

$$\begin{aligned} &\text{Maximise} && P = x + 2y, \\ &\text{subject to} && -x + 2y \leqslant 3, \\ &&& x + y \leqslant 3, \\ &\text{and} && x \geqslant 0,\ y \geqslant 0, \quad \text{with } x \text{ and } y \text{ both integers.} \end{aligned}$$

(iv) Represent this problem as an initial Simplex tableau. [2]

(v) Use the Simplex algorithm, pivoting first on a value chosen from the y column, to find the values of x and y at the optimum point. [6]

6 The network below represents a simplified map of a forest. The nodes represent locations in the forest and the arcs represent footpaths. The weights on the arcs represent distances, in metres.



(a) Woody the forest ranger wants to start from rangers' hut (*H*) and walk along every footpath at least once using the shortest possible total distance.

(i) Which standard network problem does Woody need to solve to find the shortest route that covers every arc? [1]

The total length of all the footpaths shown is 3000 metres.

(ii) Use an appropriate algorithm to find the length of the shortest route that Woody can use. Show all your working. (You may find the lengths of shortest paths between nodes by inspection.) [4]

Suppose that, instead, Woody wants to start from the car park (*C*) and walk along every footpath at least once using the shortest possible total distance.

(iii) What is the length of the shortest route that Woody can use if he starts from the car park? At which node does this route end? [3]

(b) There is a nesting box at each node of the network.

Cyril the squirrel lives in the forest. He wants to start from his drey (*D*) and check each nesting box, to see whether he has stored any nuts there, before returning to his drey. Cyril is a vain squirrel, so he wants to use the footpaths so that people can see him. However he is also a very lazy squirrel, so he would like to check the boxes in the shortest distance possible.

(i) Apply the nearest neighbour method starting at node *D* to find a tour through all the nodes that starts and ends at *D*. Calculate the total weight of this tour. Explain why the nearest neighbour method fails if you start at node *A*. [3]

(ii) Construct a minimum spanning tree by using Prim's algorithm on the reduced network formed by deleting node *A* and all the arcs that are directly joined to node *A*. **Start building your tree at node *B***. (You do *not* need to represent the network as a matrix.) Give the order in which nodes are added to your tree and draw a diagram to show the arcs in your tree. Calculate the total weight of your tree. [5]

(iii) From your previous answers, what can you say about the shortest possible distance that Cyril must travel to visit each nesting box and return home to his drey? [2]

mock papers 4

1.

Max Lauren John Hannah Kieran Tara Richard Imogen

(a) Use a quick sort to produce a list of these names in ascending alphabetical order. You must make your pivots clear.

(5)

(b) Use the binary search algorithm on your list from part (a) to try to locate the name 'Hugo'.

(4)

(Total 9 marks)

2.

	A	**B**	**C**	**D**	**E**	**F**
A	-	24	-	-	23	22
B	24	-	18	19	17	20
C	-	18	-	11	14	-
D	-	19	11	-	13	-
E	23	17	14	13	-	21
F	22	20	-	-	21	-

The table shows the distances, in metres, between six vertices, **A**, **B**, **C**, **D**, **E** and **F**, in a network.

(a) Draw the weighted network using the vertices given in Diagram 1 in the answer booklet.

(3)

(b) Use Kruskal's algorithm to find a minimum spanning tree. You should list the edges in the order that you consider them and state whether you are adding them to your minimum spanning tree.

(3)

(c) Draw your tree on Diagram 2 in the answer booklet and find its total weight.

(2)

(Total 8 marks)

---

3. (a) Draw the activity network described in this precedence table, using activity on arc and exactly two dummies.

(5)

Activity	Immediately preceding activities
A	-
B	-
C	-
D	**B**
E	**B, C**
F	**B, C**
G	**F**
H	**F**
I	**G, H**
J	**I**

(b) Explain why each of the two dummies is necessary.

(2)

(Total 7 marks)

4.

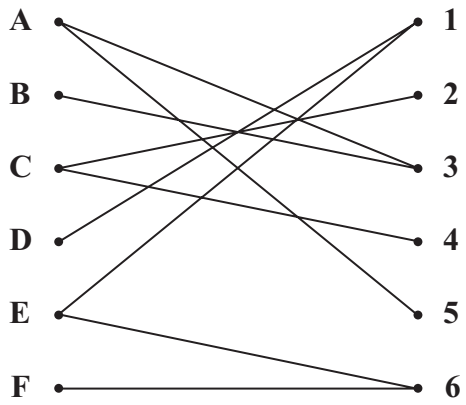


Figure 1

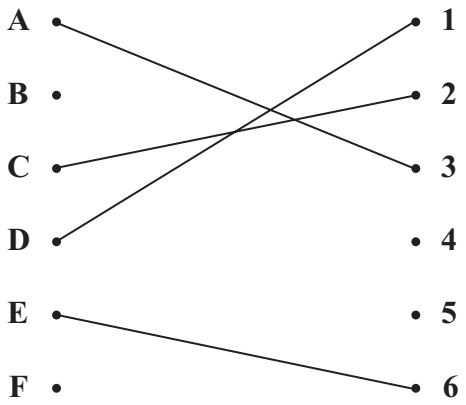


Figure 2

Figure 1 shows the possible allocations of six people, A, B, C, D, E and F, to six tasks, 1, 2, 3, 4, 5 and 6.
Figure 2 shows an initial matching.

(a) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You must list the alternating path used, and your improved matching.

(3)

(b) Explain why it is not possible to find a complete matching.

(2)

D now has task 2 added to their possible allocation.

(c) Using the improved matching found in part (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. You must list the alternating path used and your complete matching.

(3)

(Total 8 marks)

5.

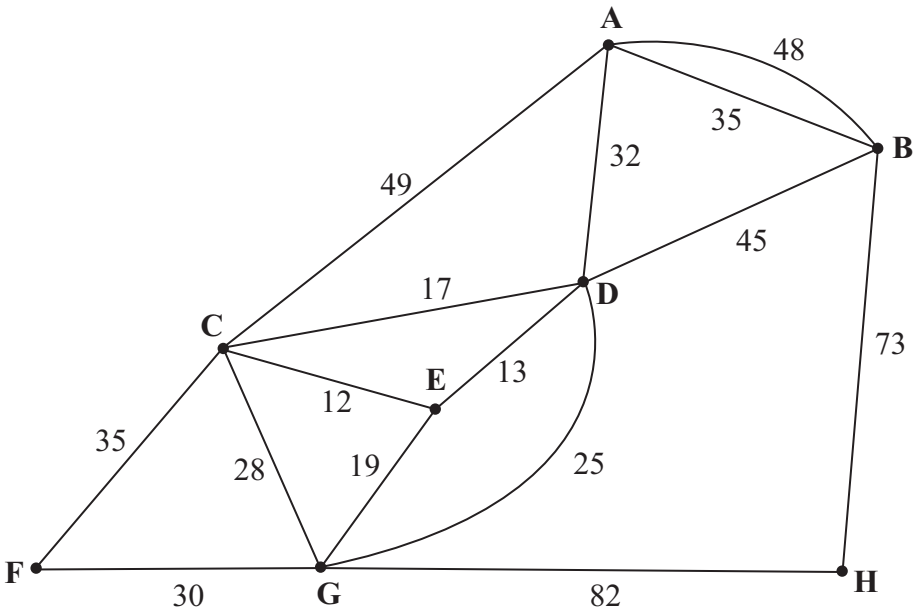


Figure 3

(The total weight of the network in Figure 3 is 543 km.)

Figure 3 models a network of railway tracks that have to be inspected. The number on each arc is the length, in km, of that section of railway track.
Each track must be traversed at least once and the length of the inspection route must be minimised.
The inspection route must start and finish at the same vertex.

(a) Use an appropriate algorithm to find the length of the shortest inspection route. You should make your method and working clear.

(5)

It is now permitted to start and finish the inspection at two distinct vertices.

(b) State which two vertices should be chosen to minimise the length of the new route. Give a reason for your answer.

(3)

(Total 8 marks)

6.

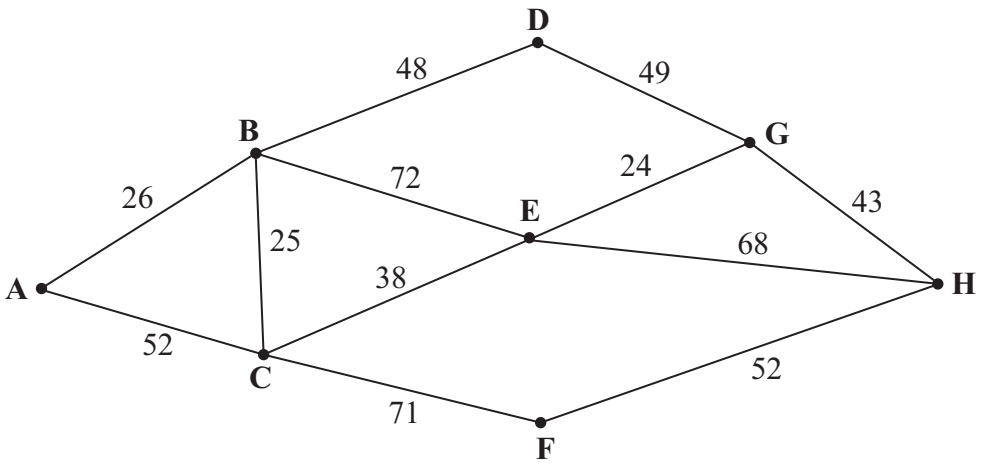


Figure 4

Figure 4 shows a network of roads through eight villages, A, B, C, D, E, F, G and H. The number on each arc is the length of that road in km.

(a) Use Dijkstra's algorithm to find the shortest route from A to H. State your shortest route and its length.

(5)

There is a fair in village C and you cannot drive through the village. A shortest route from A to H which avoids C needs to be found.

(b) State this new minimal route and its length.

(2)

(Total 7 marks)

7. A linear programming problem is modelled by the following constraints

$$8x + 3y \leqslant 480$$
$$8x + 7y \geqslant 560$$
$$y \geqslant 4x$$
$$x, y \geqslant 0$$

(a) Use the grid provided in your answer book to represent these inequalities graphically. Hence determine the feasible region and label it R.

(6)

The objective function, F, is given by

$$F = 3x + y$$

(b) Making your method clear, determine

(i) the minimum value of the function F and the coordinates of the optimal point,

(ii) the maximum value of the function F and the coordinates of the optimal point.

(6)

(Total 12 marks)

8.

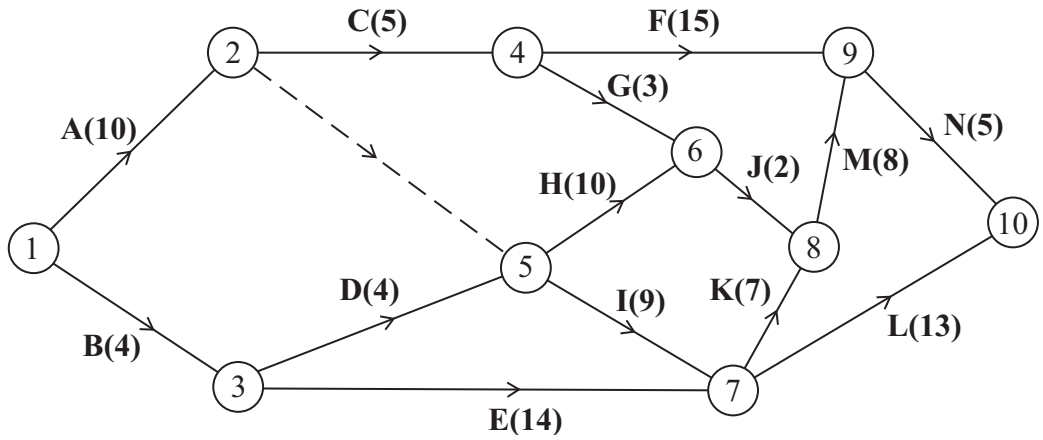


Figure 5

The network in Figure 5 shows the activities involved in a process. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, taken to complete the activity.

(a) Calculate the early time and the late time for each event, showing them on the diagram in the answer book.

(4)

(b) Determine the critical activities and the length of the critical path.

(3)

(c) Calculate the total float on activities F and G. You **must** make the numbers you used in your calculation clear.

(3)

(d) On the grid in the answer book, draw a cascade (Gantt) chart for the process.

(4)

Given that each task requires just one worker,

(e) use your cascade chart to determine the minimum number of workers required to complete the process in the minimum time. Explain your reasoning clearly.

(2)

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer book for mock papers 4

1.

(Question 1 continued)

Q1

(Total 9 marks)

2.

	A	**B**	**C**	**D**	**E**	**F**
A	-	24	-	-	23	22
B	24	-	18	19	17	20
C	-	18	-	11	14	-
D	-	19	11	-	13	-
E	23	17	14	13	-	21
F	22	20	-	-	21	-

(a)

Diagram 1

(Question 2 continued)

(b)

(c)

Diagram 2

Total weight of tree _____

(Total 8 marks)

Q2

Turn over

3. (a)

(Question 3 continued)

(b)

Q3

(Total 7 marks)

4.

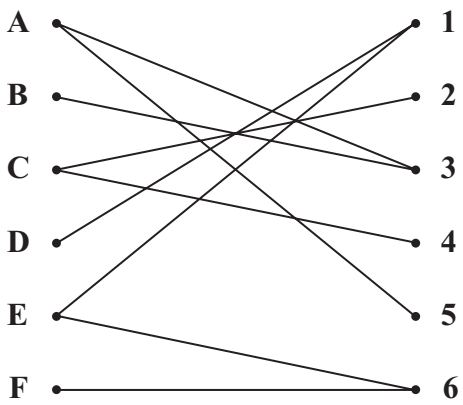


Figure 1

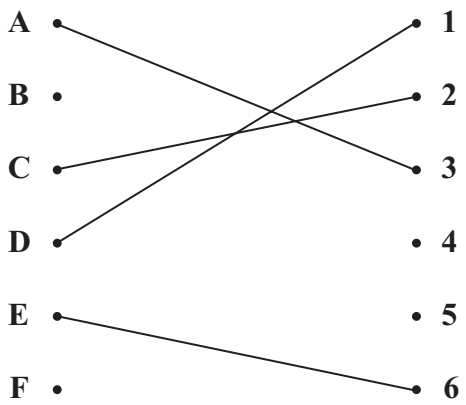


Figure 2

A •	• 1	A •	• 1
B •	• 2	B •	• 2
C •	• 3	C •	• 3
D •	• 4	D •	• 4
E •	• 5	E •	• 5
F •	• 6	F •	• 6

(Total 8 marks)

Q4

5.

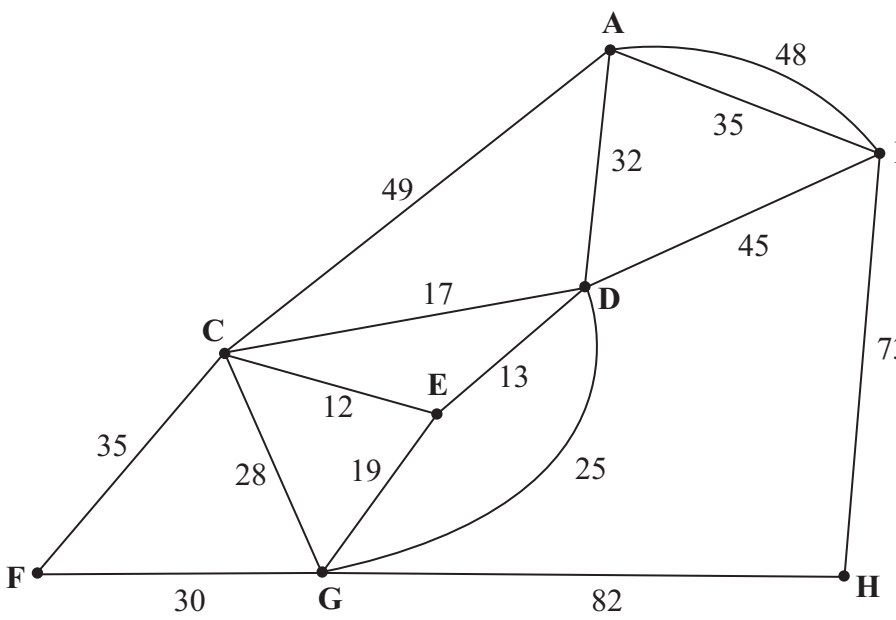


Figure 3

(The total weight of the network in Figure 3 is 543 km.)

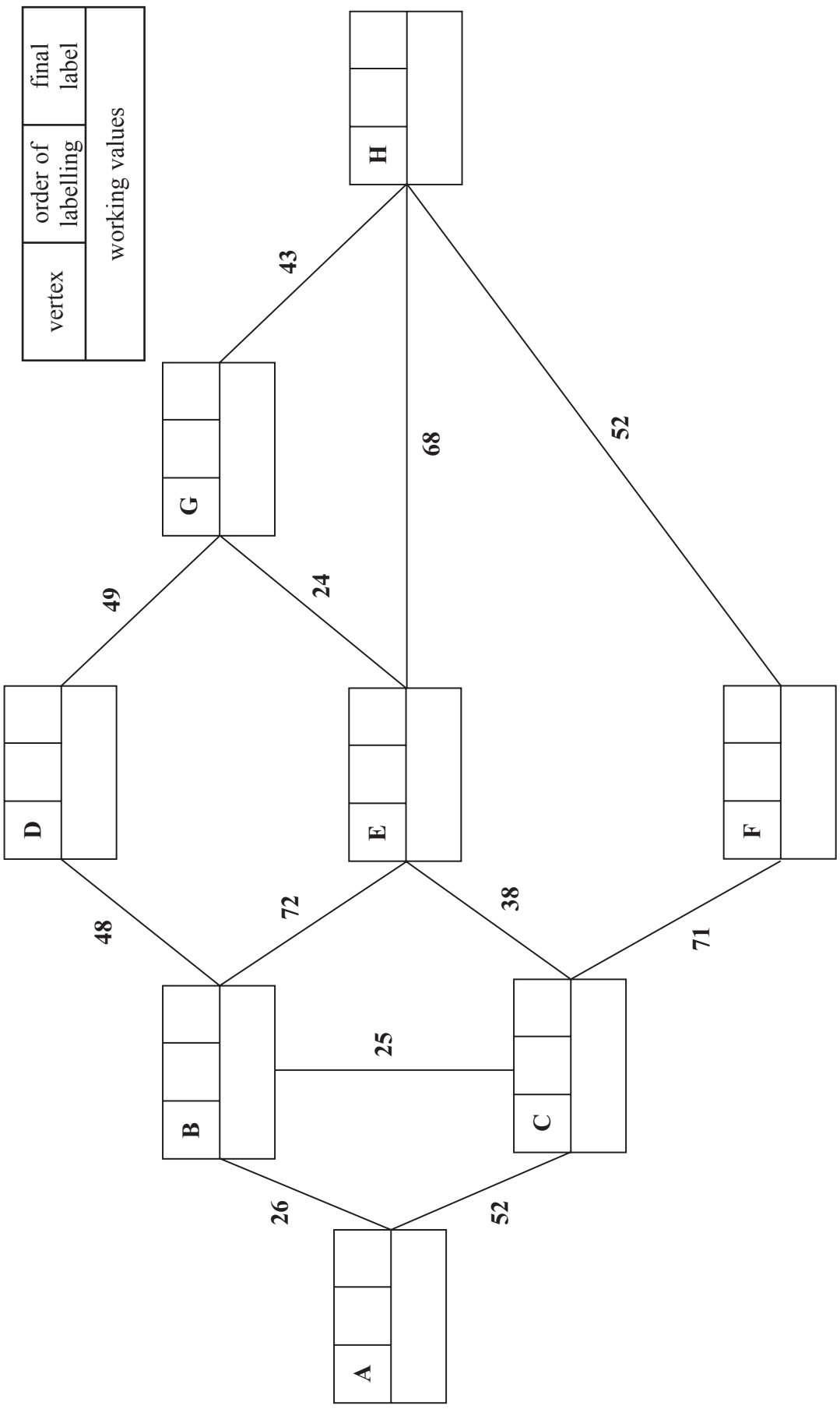
(Question 5 continued)

Q5

(Total 8 marks)

Turn over

6. (a)



(Question 6 continued)

shortest route: _____ length: _____

(b) _____

_____

_____

_____

_____

_____

_____

shortest route (avoiding C): _____

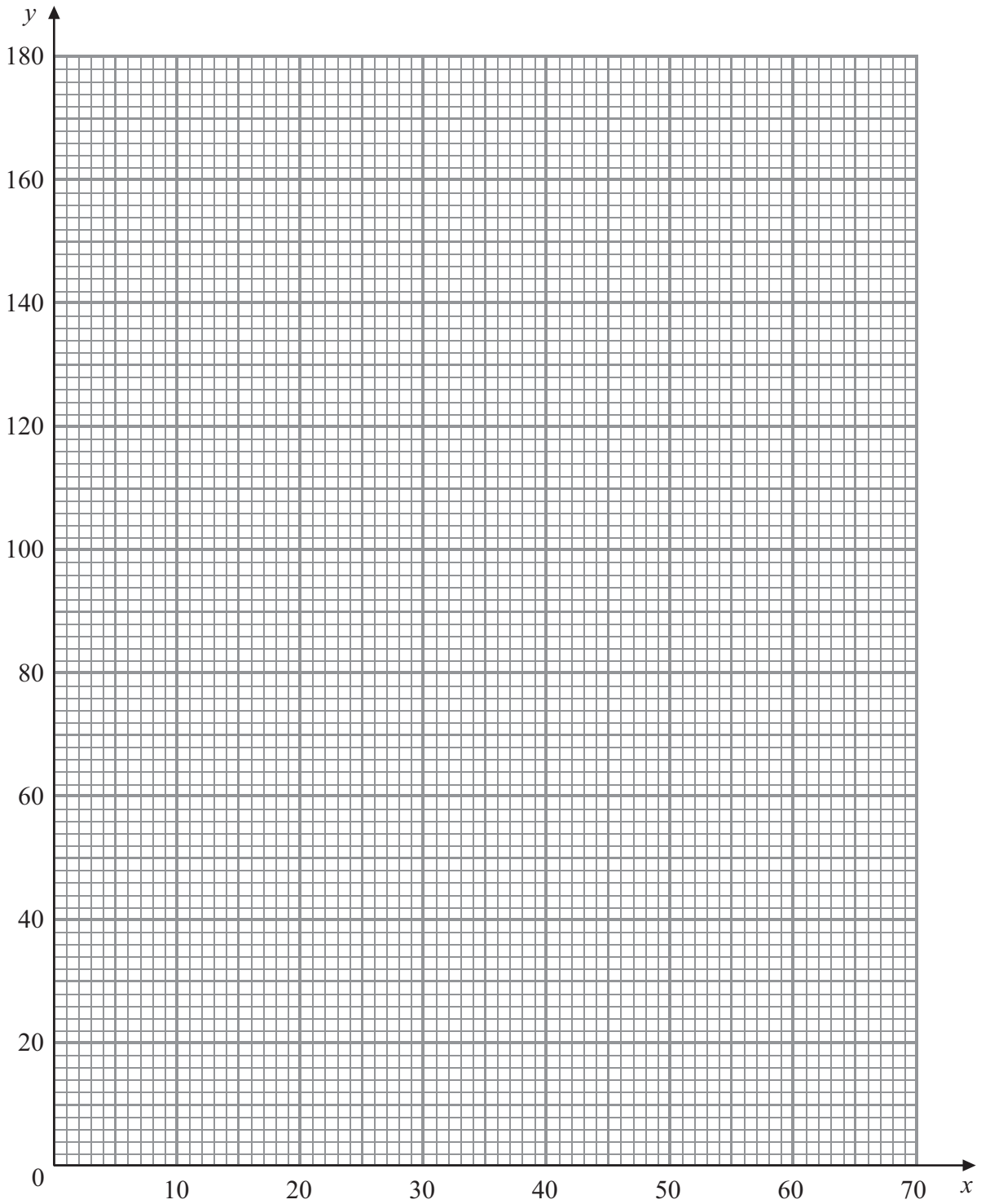
length: _____

(Total 7 marks)

Q6

Turn over

7.

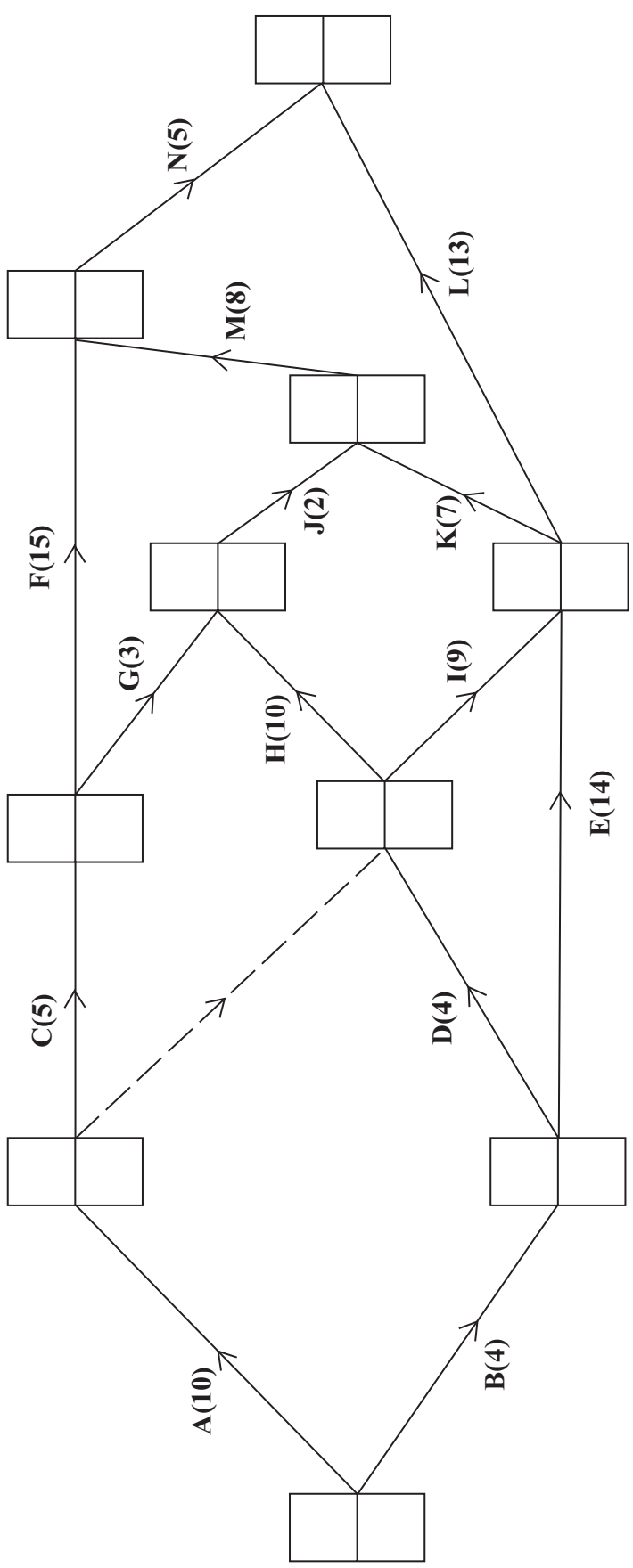


(Question 7 continued)

(Total 12 marks)

Q7

8. (a)



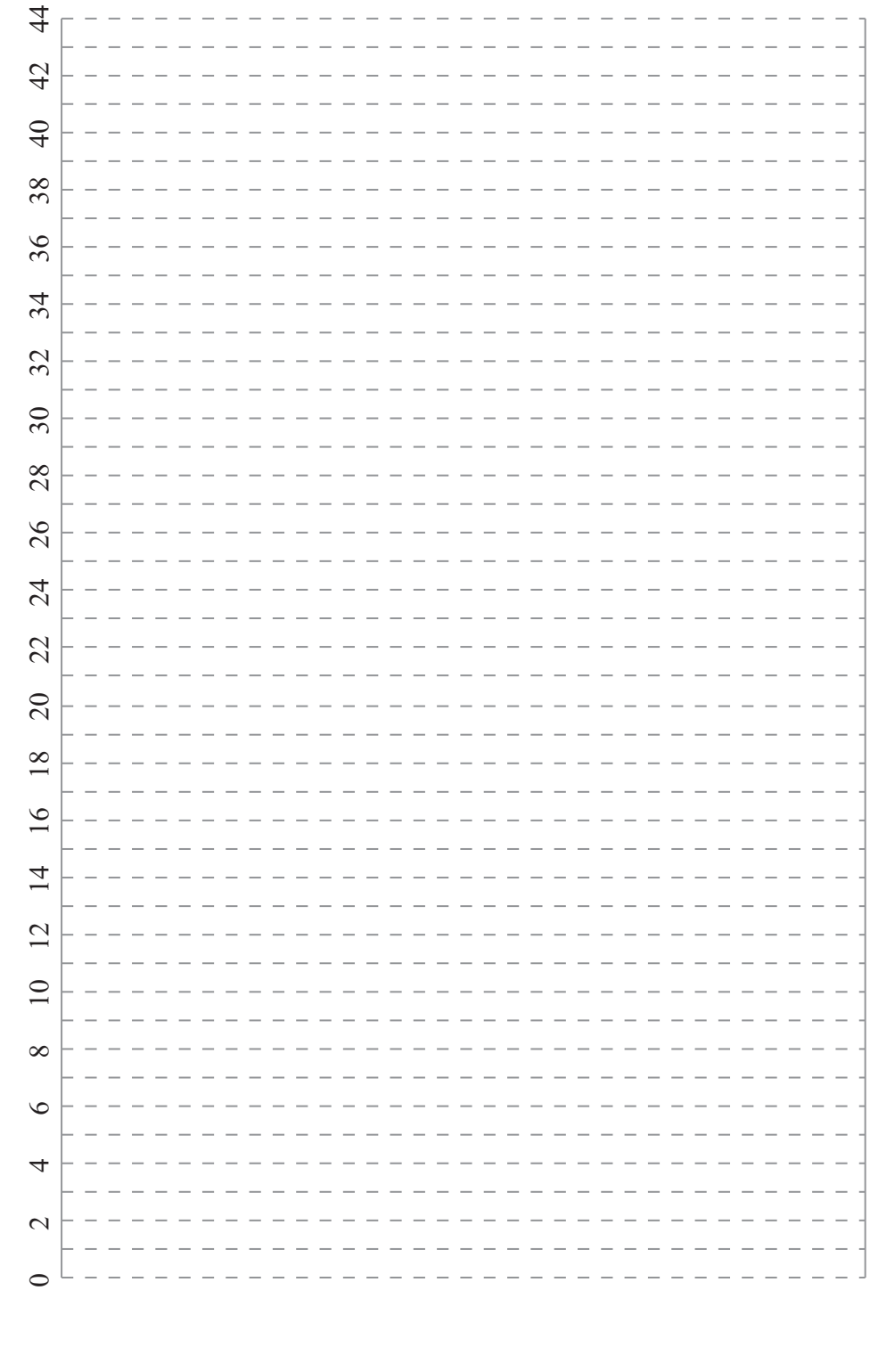
(b) Critical activities: _____

Length of critical path: _____

(c) Total float on activity F: _____

Total float on activity G: _____

(Question 8 continued)



(d)

(e)

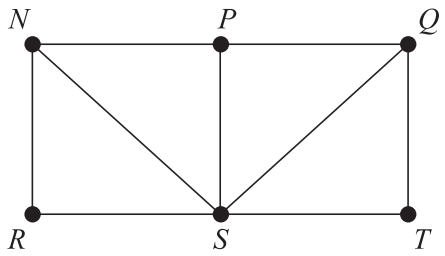
Q8

(Total 16 marks)

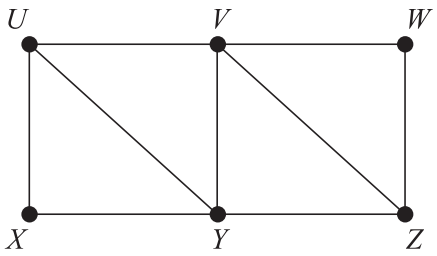
TOTAL FOR PAPER: 75 MARKS

END

1 Two graphs A and B are shown below.



Graph A



Graph B

(i) Write down an example of a cycle on graph A. [1]

(ii) Why is U–Y–V–Z–Y–X not a path on graph B? [1]

(iii) How many arcs would there be in a spanning tree for graph A? [1]

(iv) For each graph state whether it is Eulerian, semi-Eulerian or neither. [2]

(v) The graphs show designs to be etched on metal plates. The etching tool is positioned at a starting point and follows a route without repeating any arcs. It may be lifted off and positioned at a new starting point. What is the smallest number of times that the etching tool must be positioned, including the initial position, to draw each graph? [2]

An arc is drawn connecting Q to U, so that the two graphs become one. The resulting graph is not Eulerian.

(vi) Extra arcs are then added to make an Eulerian graph. What is the smallest number of extra arcs that need to be added? [2]

2 A landscape gardener is designing a garden. Part of the garden will be decking, part will be flowers and the rest will be grass. Let d be the area of decking, f be the area of flowers and g be the area of grass, all measured in m^2.

The total area of the garden is $120\,m^2$ of which at least $40\,m^2$ must be grass. The area of decking must not be greater than the area of flowers. Also, the area of grass must not be more than four times the area of decking.

Each square metre of grass will cost £5, each square metre of decking will cost £10 and each square metre of flowers will cost £20. These costs include labour. The landscape gardener has been instructed to come up with the design that will cost the least.

(i) Write down a constraint in d, f and g from the total area of the garden. [1]

(ii) Explain why the constraint $g \leqslant 4d$ is required. [1]

(iii) Write down a constraint from the requirement that the area of decking must not be greater than the area of flowers. [1]

(iv) Write down a constraint from the requirement that at least $40\,m^2$ of the garden must be grass and write down the minimum feasible values for each of d and f. [3]

(v) Write down the objective function to be minimised. [1]

(vi) Write down the resulting LP problem, using slack variables to express the constraints from parts **(ii)** and **(iii)** as equations. [3]

(You are **not** required to solve the resulting LP problem.)

3 **(i)** Use shuttle sort to sort the five numbers 8, 6, 9, 7, 5 into increasing order. Write down the list that results at the end of each pass. Calculate and record the number of comparisons and the number of swaps that are made in each pass. [6]

(ii) The algorithm below is **part** of another method for sorting a list into increasing order. Apply it to the list 8, 6, 9, 7, 5. Show the result of each step. [5]

Step 1: Input the original list and call it list A.

Step 2: Remove the first item in list A and call this item X.

Step 3: If the first item remaining in list A is less than X move it to list B, otherwise move it to list C.

Step 4: If the next item remaining in list A is less than X move it to become the next item in list B, otherwise move it to become the next item in list C.

Step 5: If there are still items in list A, repeat Step 4.

Step 6: Count the number of items in list B and call this N.

Step 7: Put the items in list B at positions 1 to N of list A, item X at position $N + 1$ of list A and the items in list C at positions $N + 2$ onwards of list A.

Step 8: Display list A.

4 Consider the linear programming problem:

$$\begin{aligned} &\text{maximise} && P = 3x - 5y, \\ &\text{subject to} && x + 5y \leqslant 12, \\ & && x - 5y \leqslant 10, \\ & && 3x + 10y \leqslant 45, \\ &\text{and} && x \geqslant 0,\ y \geqslant 0. \end{aligned}$$

(i) Represent the problem as an initial Simplex tableau. [3]

(ii) Identify the entry on which to pivot for the first iteration of the Simplex algorithm. Explain how you made your choice of column and row. [2]

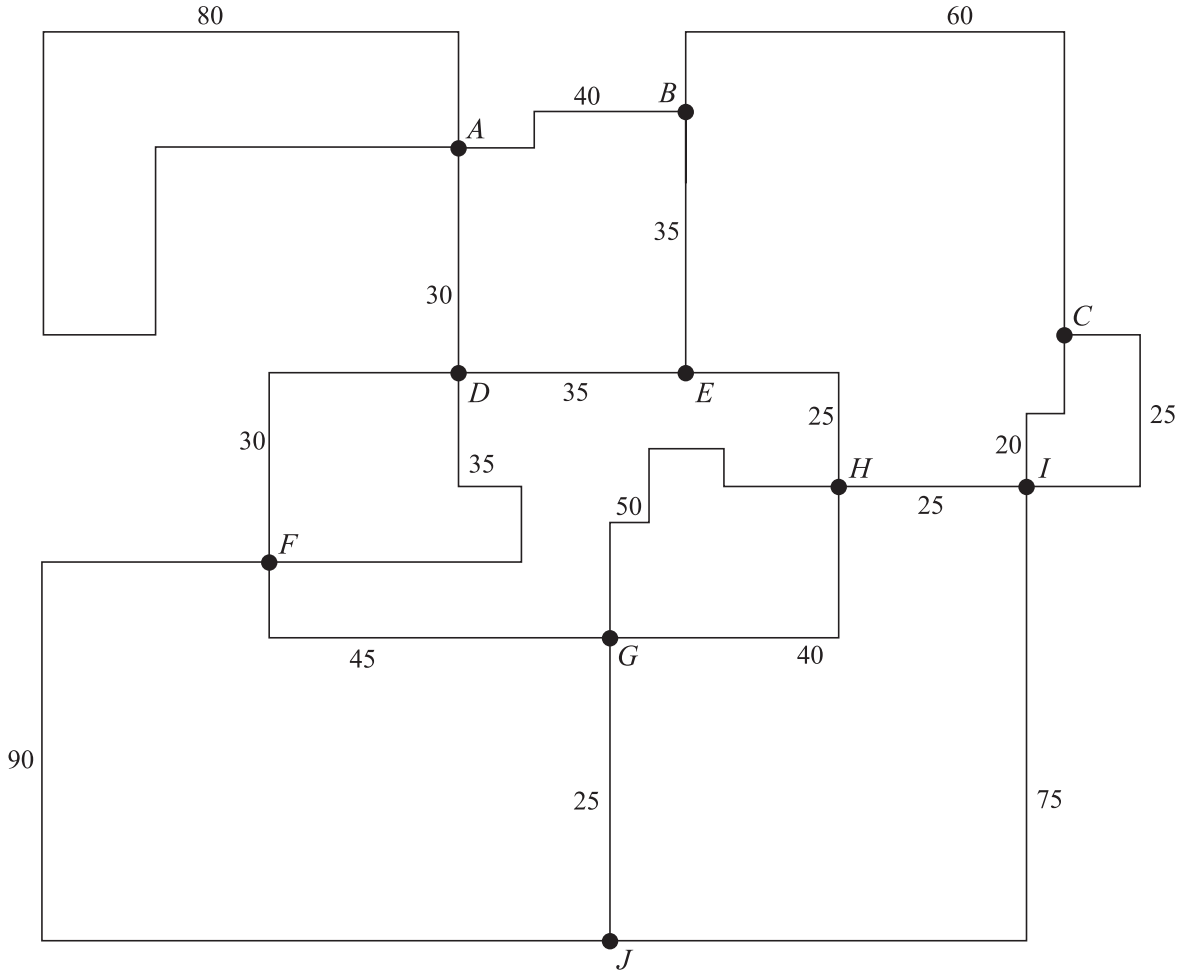
(iii) Perform **one** iteration of the Simplex algorithm. Write down the values of x, y and P after this iteration. [6]

(iv) Show that $x = 11$, $y = 0.2$ is a feasible solution and that it gives a bigger value of P than that in part **(iii)**. [2]

5 Answer this question on the insert provided.

The network below represents a simplified map of a building. The arcs represent corridors and the weights on the arcs represent the lengths of the corridors, in metres.

The sum of the weights on the arcs is 765 metres.



(i) Janice is the cleaning supervisor in the building. She is at the position marked as J when she is called to attend a cleaning emergency at B. On the network in the insert, use Dijkstra's algorithm, starting from vertex J and continuing until B is given a permanent label, to find the shortest path from J to B and the length of this path. [7]

(ii) In her job Janice has to walk along each of the corridors represented on the network. This requires finding a route that covers every arc at least once, starting and ending at J. Showing all your working, find the shortest distance that Janice must walk to check all the corridors. [5]

The labelled vertices represent 'cleaning stations'. Janice wants to visit every cleaning station using the shortest possible route. She produces a simplified network with no repeated arcs and no arc that joins a vertex to itself.

(iii) On the insert, complete Janice's simplified network. Which standard network problem does Janice need to solve to find the shortest distance that she must travel? [4]

6 Answer this question on the insert provided.

The table shows the distances, in miles, along the direct roads between six villages, A to F. A dash (−) indicates that there is no direct road linking the villages.

	A	B	C	D	E	F
A	–	6	3	–	–	–
B	6	–	5	6	–	14
C	3	5	–	8	4	10
D	–	6	8	–	3	8
E	–	–	4	3	–	–
F	–	14	10	8	–	–

(i) On the table in the insert, use Prim's algorithm to find a minimum spanning tree. Start by crossing out row A. Show which entries in the table are chosen and indicate the order in which the rows are deleted. Draw your minimum spanning tree and state its total weight. [6]

(ii) By deleting vertex B and the arcs joined to vertex B, calculate a lower bound for the length of the shortest cycle through all the vertices. [3]

(iii) Apply the nearest neighbour method to the table above, starting from F, to find a cycle that passes through every vertex and use this to write down an upper bound for the length of the shortest cycle through all the vertices. [4]

1.

	A	B	C	D	E	F
A	-	135	180	70	95	225
B	135	-	215	125	205	240
C	180	215	-	150	165	155
D	70	125	150	-	100	195
E	95	205	165	100	-	215
F	225	240	155	195	215	-

The table shows the lengths, in km, of potential rail routes between six towns, A, B, C, D, E and F.

(a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for this table. You must list the **arcs** that form your tree **in the order that they are selected**.

(3)

(b) Draw your tree using the vertices given in Diagram 1 in the answer book.

(1)

(c) State the total weight of your tree.

(1)

(Total 5 marks)

2.

32 45 17 23 38 28 16 9 12 10

The numbers in the list above represent the lengths, in metres, of ten lengths of fabric. They are to be cut from rolls of fabric of length 60m.

(a) Calculate a lower bound for the number of rolls needed.

(2)

(b) Use the first-fit bin packing algorithm to determine how these ten lengths can be cut from rolls of length 60m.

(4)

(c) Use full bins to find an optimal solution that uses the minimum number of rolls.

(3)

(Total 9 marks)

3.

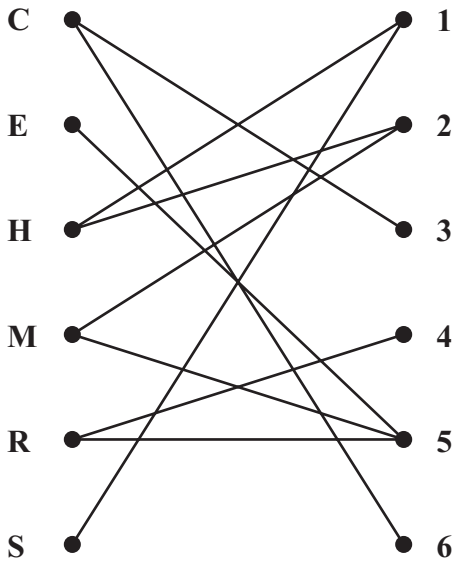


Figure 1

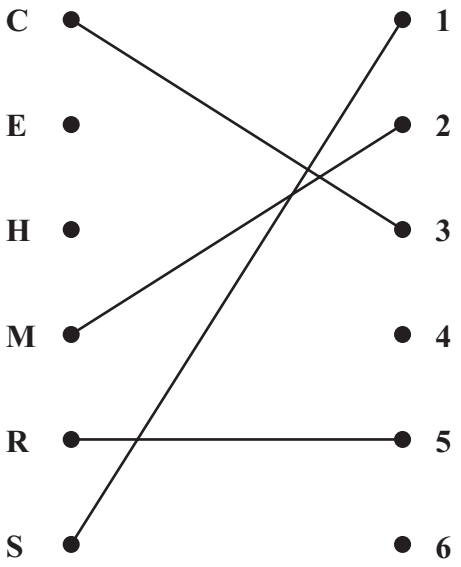


Figure 2

Figure 1 shows the possible allocations of six workers, Charlotte (C), Eleanor (E), Harry (H), Matt (M), Rachel (R) and Simon (S) to six tasks, 1, 2, 3, 4, 5 and 6.
Figure 2 shows an initial matching.

(a) List an alternating path, starting at H and ending at 4. Use your path to find an improved matching. List your improved matching.

(3)

(b) Explain why it is not possible to find a complete matching.

(1)

Simon (S) now has task 3 added to his possible allocation.

(c) Taking the improved matching found in (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. List clearly the alternating path you use and your complete matching.

.

(3)

(Total 7 marks)

4.

Miri
Jessie
Edward
Katie
Hegg
Beth
Louis
Philip
Natsuko
Dylan

(a) Use the quick sort algorithm to sort the above list into alphabetical order.

(5)

(b) Use the binary search algorithm to locate the name Louis.

(4)

(Total 9 marks)

5.

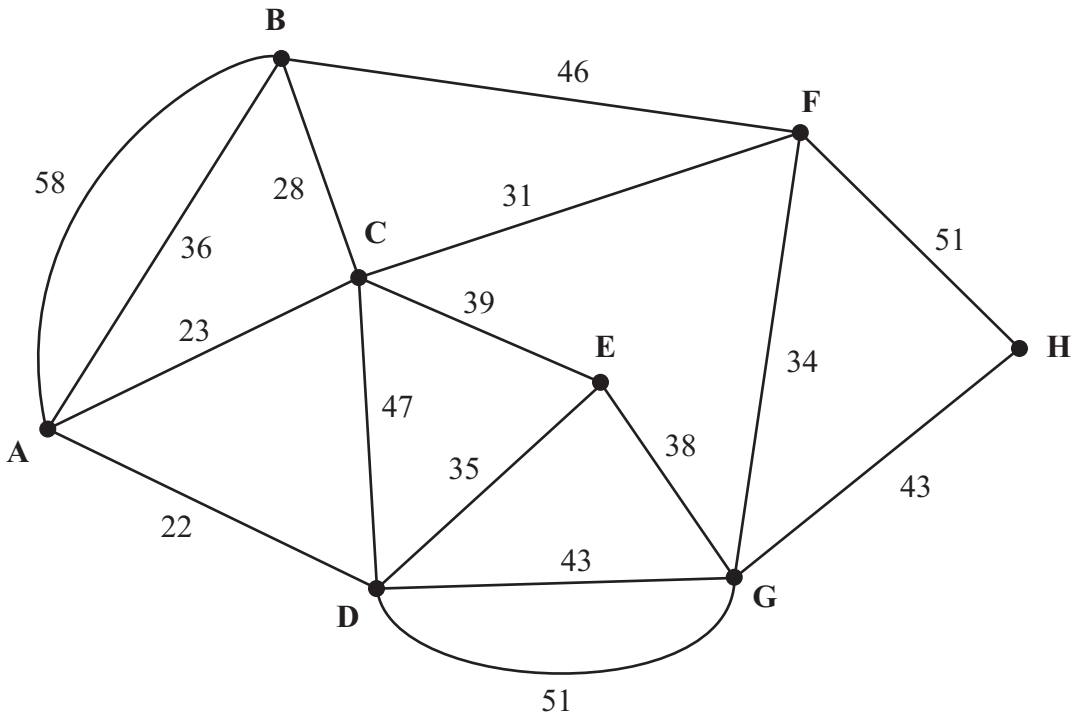


Figure 3

[The total weight of the network is 625 m]

Figure 3 models a network of paths in a park. The number on each arc represents the length, in m, of that path.
Rob needs to travel along each path to inspect the surface. He wants to minimise the length of his route.

(a) Use the route inspection algorithm to find the length of his route. State the arcs that should be repeated. You should make your method and working clear.

(6)

The surface on each path is to be renewed. A machine will be hired to do this task and driven along each path.
The machine will be delivered to point G and will start from there, but it may be collected from any point once the task is complete.

(b) Given that each path must be traversed at least once, determine the finishing point so that the length of the route is minimised. Give a reason for your answer and state the length of your route.

(3)

(Total 9 marks)

6.

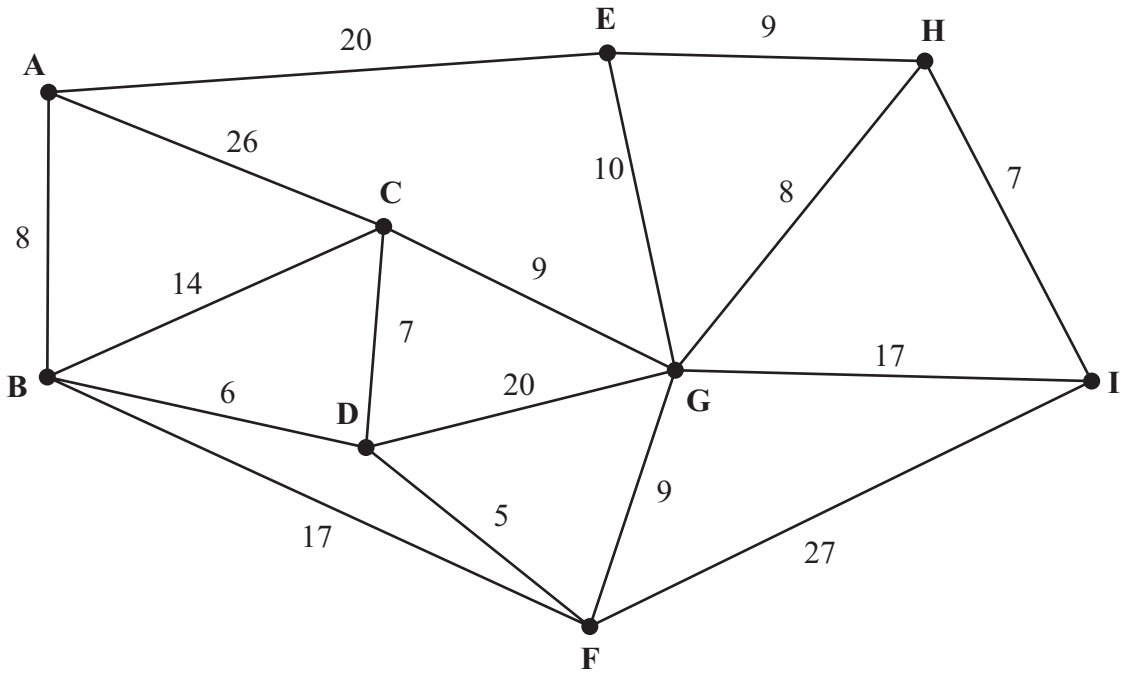


Figure 4

Figure 4 represents a network of roads. The number on each arc gives the length, in km, of that road.

(a) Use Dijkstra's algorithm to find the shortest distance from A to I. State your shortest route.

(6)

(b) State the shortest distance from A to G.

(1)

(Total 7 marks)

7. Rose makes hanging baskets which she sells at her local market. She makes two types, large and small. Rose makes x large baskets and y small baskets.

Each large basket costs £7 to make and each small basket costs £5 to make. Rose has £350 she can spend on making the baskets.

(a) Write down an inequality, in terms of x and y, to model this constraint.

(2)

Two further constraints are

$$y \leqslant 20 \text{ and}$$
$$y \leqslant 4x.$$

(b) Use these two constraints to write down statements that describe the numbers of large and small baskets that Rose can make.

(2)

(c) On the grid provided, show these three constraints and $x \geqslant 0$, $y \geqslant 0$. Hence label the feasible region, R.

(4)

Rose makes a profit of £2 on each large basket and £3 on each small basket. Rose wishes to maximise her profit, £P.

(d) Write down the objective function.

(1)

(e) Use your graph to determine the optimal numbers of large and small baskets Rose should make, and state the optimal profit.

(5)

(Total 14 marks)

8.

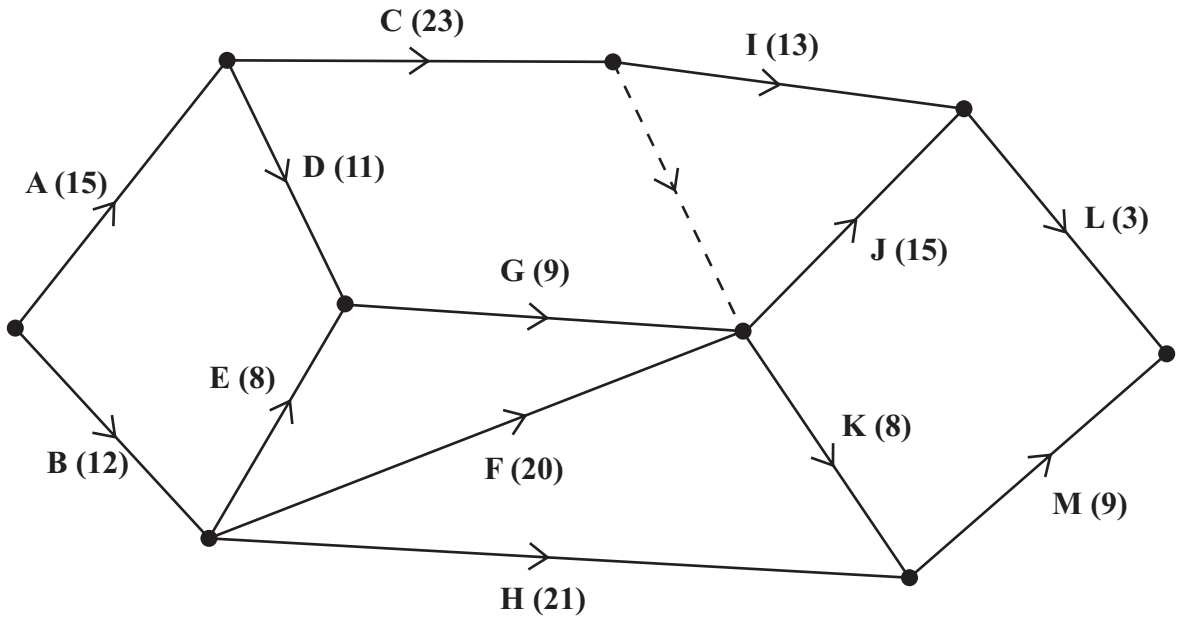


Figure 5

A construction project is modelled by the activity network shown in Figure 5. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 2 in the answer book, showing the early and late event times.

(4)

(b) State the critical activities.

(1)

(c) Find the total float for activities M and H. You **must** make the numbers you use in your calculations clear.

(3)

(d) On the grid provided, draw a cascade (Gantt) chart for this project.

(4)

An inspector visits the project at 1pm on days 16 and 31 to check the progress of the work.

(e) Given that the project is on schedule, which activities **must** be happening on each of these days?

(3)

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

1. (a)

	A	B	C	D	E	F
A	-	135	180	70	95	225
B	135	-	215	125	205	240
C	180	215	-	150	165	155
D	70	125	150	-	100	195
E	95	205	165	100	-	215
F	225	240	155	195	215	-

(b)

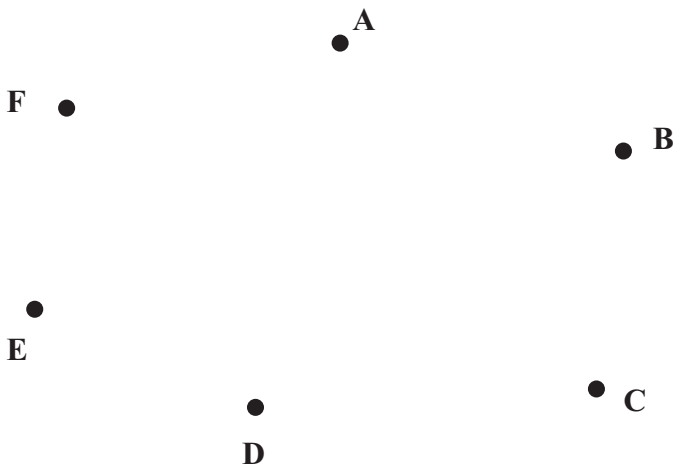


Diagram 1

(c) Total weight of tree _____

Q1

(Total 5 marks)

2.

Q2

(Total 9 marks)

3.

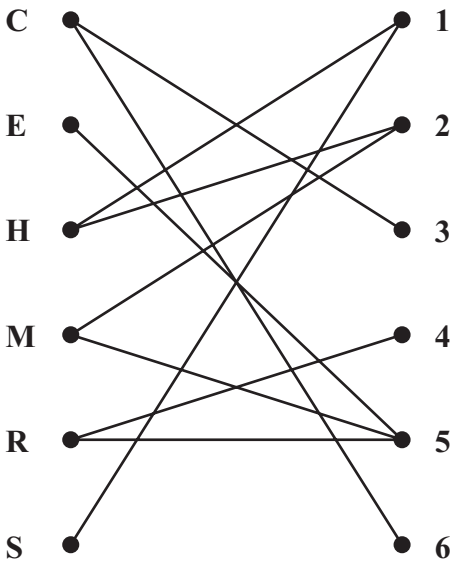


Figure 1

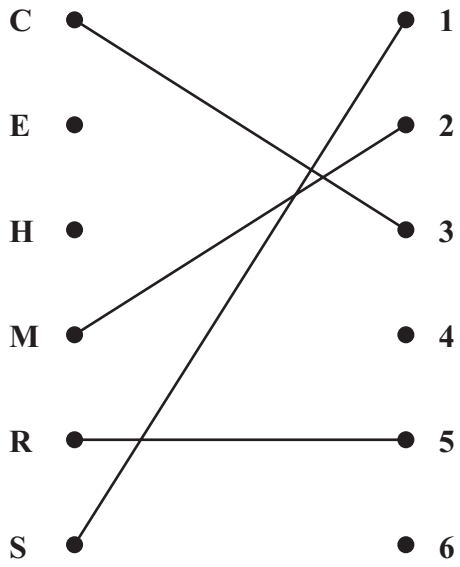


Figure 2

(Question 3 continued)

C ●	● 1
E ●	● 2
H ●	● 3
M ●	● 4
R ●	● 5
S ●	● 6

Q3

(Total 7 marks)

4.

(Question 4 continued)

Q4

(Total 9 marks)

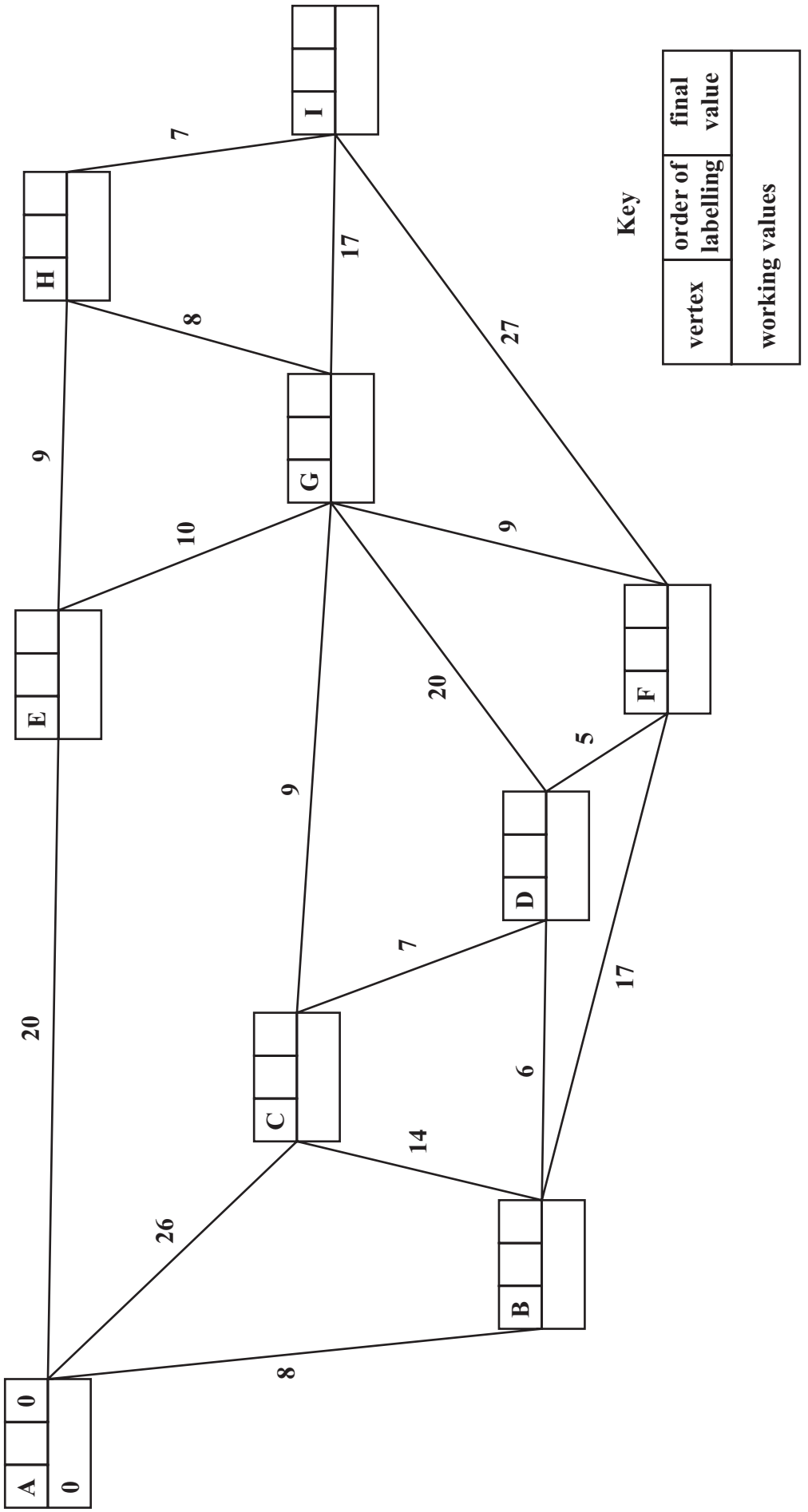
5.

(Question 5 continued)

Q5

(Total 9 marks)

6. (a)



(b) Shortest distance A to G _____

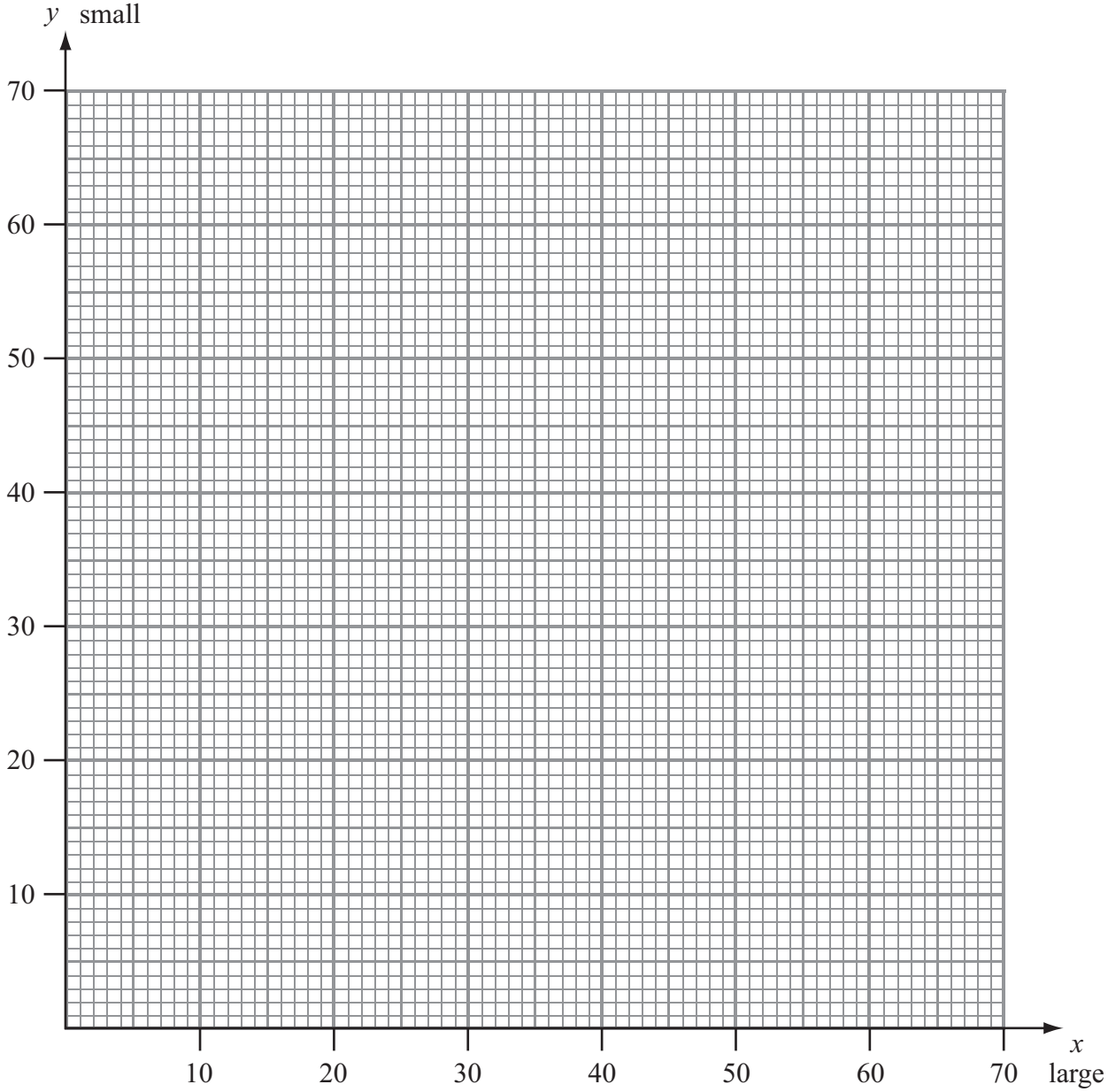
Shortest route _____

(Total 7 marks)

Q6

7.

(Question 7 continued)



Q7

(Total 14 marks)

8. (a)

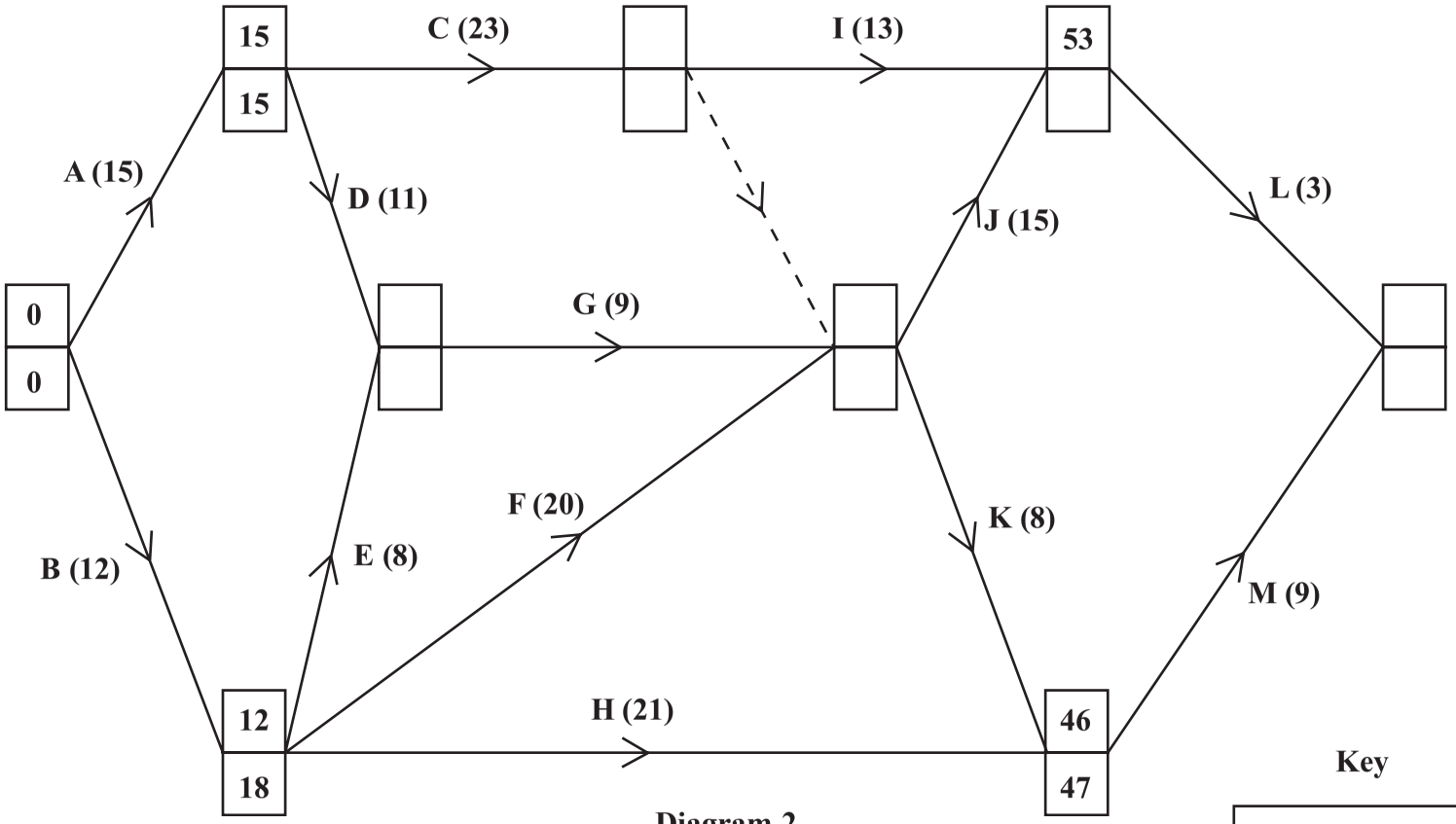


Diagram 2

(b) Critical activities _____

(c) _____

_____

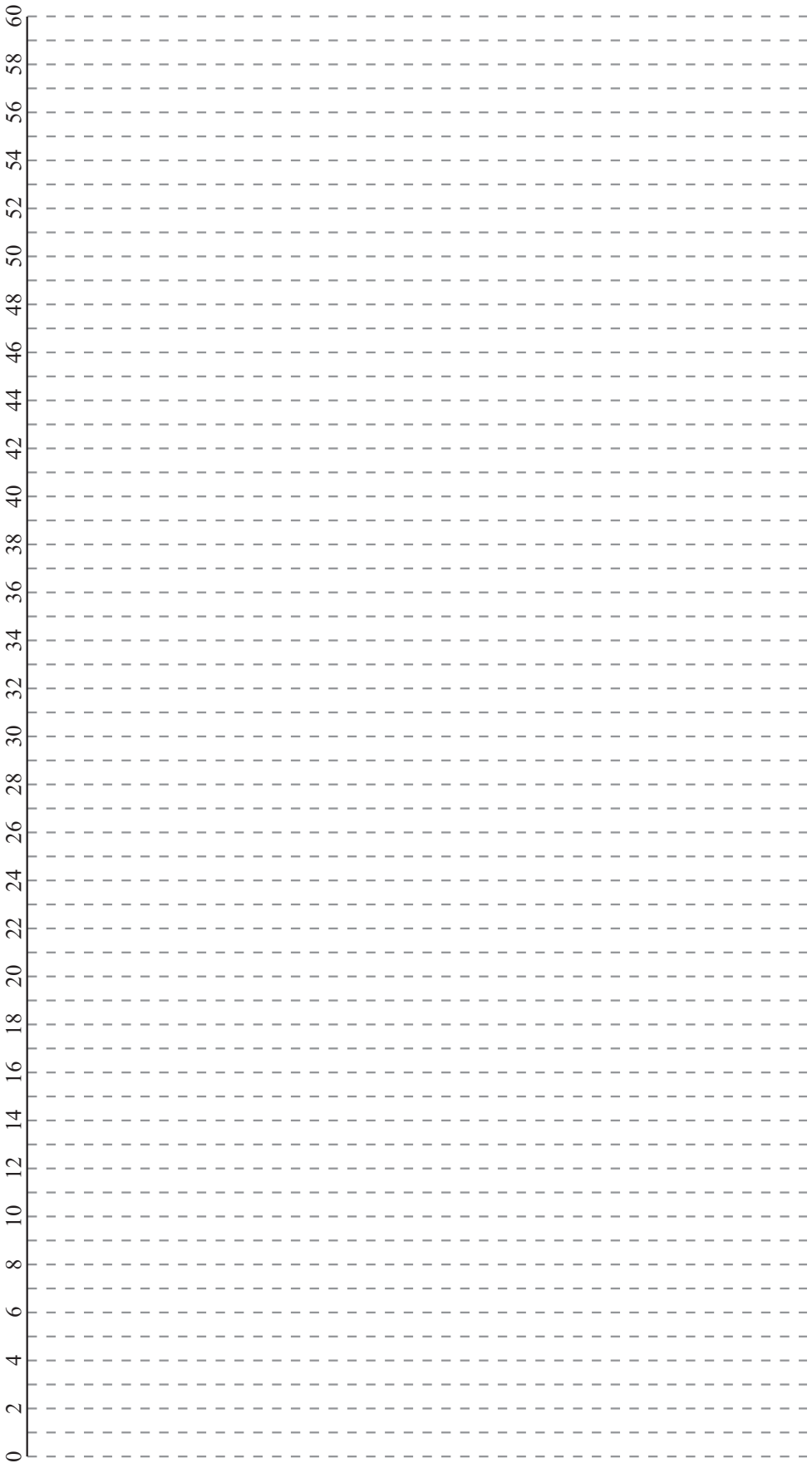
_____

_____

_____

(Question 8 continued)

(d)



(e) Day 16

Day 31

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END