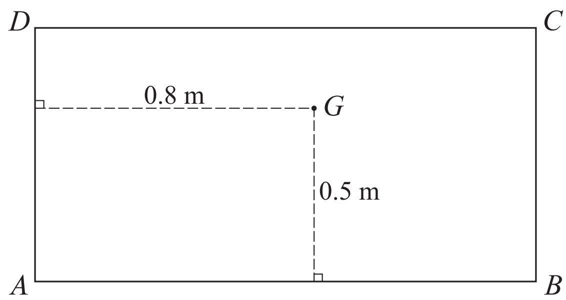
1 The driveshaft of an electric motor begins to rotate from rest and has constant angular acceleration. In the first 8 seconds it turns through 56 radians.

(i) Find the angular acceleration. [2]

(ii) Find the angle through which the driveshaft turns while its angular speed increases from $20\,\text{rad s}^{-1}$ to $36\,\text{rad s}^{-1}$. [2]

2 The region R is bounded by the curve $y = \sqrt{4a^2 - x^2}$ for $0 \leqslant x \leqslant a$, the x-axis, the y-axis and the line $x = a$, where a is a positive constant. The region R is rotated through 2π radians about the x-axis to form a uniform solid of revolution. Find the x-coordinate of the centre of mass of this solid. [7]

3



A non-uniform rectangular lamina $ABCD$ has mass 6 kg. The centre of mass G of the lamina is 0.8 m from the side AD and 0.5 m from the side AB (see diagram). The moment of inertia of the lamina about AD is $6.2\,\text{kg m}^2$ and the moment of inertia of the lamina about AB is $2.8\,\text{kg m}^2$.

The lamina rotates in a vertical plane about a fixed horizontal axis which passes through A and is perpendicular to the lamina.

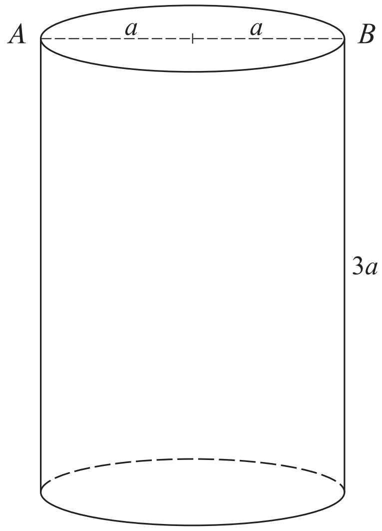
(i) Write down the moment of inertia of the lamina about this axis. [1]

The lamina is released from rest in the position where AB and DC are horizontal and DC is above AB. A frictional couple of constant moment opposes the motion. When AB is first vertical, the angular speed of the lamina is $2.4\,\text{rad s}^{-1}$.

(ii) Find the moment of the frictional couple. [5]

(iii) Find the angular acceleration of the lamina immediately after it is released. [3]

4



A uniform solid cylinder has radius a, height $3a$, and mass M. The line AB is a diameter of one of the end faces of the cylinder (see diagram).

(i) Show by integration that the moment of inertia of the cylinder about AB is $\frac{13}{4}Ma^2$. (You may assume that the moment of inertia of a uniform disc of mass m and radius a about a diameter is $\frac{1}{4}ma^2$.) [7]

The line AB is now fixed in a horizontal position and the cylinder rotates freely about AB, making small oscillations as a compound pendulum.

(ii) Find the approximate period of these small oscillations, in terms of a and g. [3]

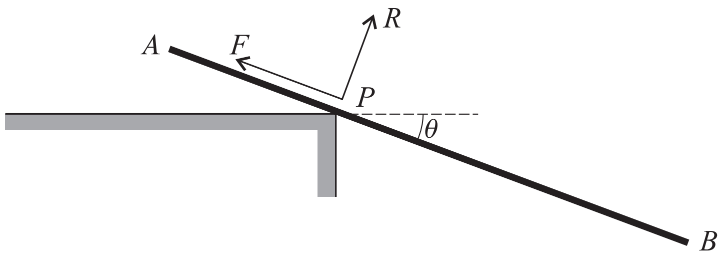
5 A ship S is travelling with constant speed $12\ \text{m s}^{-1}$ on a course with bearing 345°. A patrol boat B spots the ship S when S is 2400 m from B on a bearing of 050°. The boat B sets off in pursuit, travelling with constant speed $v\ \text{m s}^{-1}$ in a straight line.

(i) Given that $v = 16$, find the bearing of the course which B should take in order to intercept S, and the time taken to make the interception. [8]

(ii) Given instead that $v = 10$, find the bearing of the course which B should take in order to get as close as possible to S. [4]

[Turn over

6



A uniform rod AB has mass m and length $2a$. The point P on the rod is such that $AP = \frac{2}{3}a$. The rod is placed in a horizontal position perpendicular to the edge of a rough horizontal table, with AP in contact with the table and PB overhanging the edge. The rod is released from rest in this position. When it has rotated through an angle θ, and no slipping has occurred at P, the normal reaction acting on the rod at P is R and the frictional force is F (see diagram).

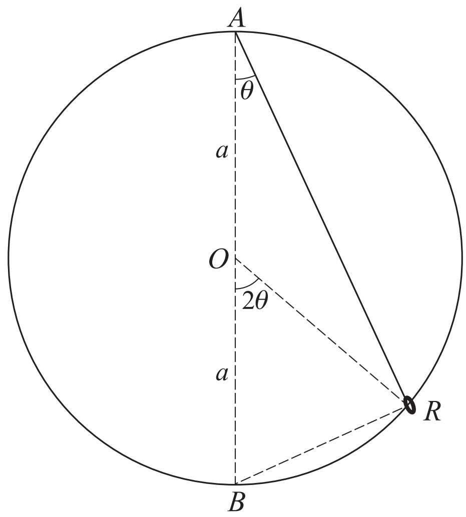
(i) Show that the angular acceleration of the rod is $\dfrac{3g\cos\theta}{4a}$. [4]

(ii) Find the angular speed of the rod, in terms of a, g and θ. [3]

(iii) Find F and R in terms of m, g and θ. [6]

(iv) Given that the coefficient of friction between the rod and the edge of the table is μ, show that the rod is on the point of slipping at P when $\tan\theta = \frac{1}{2}\mu$. [2]

7



A smooth circular wire, with centre O and radius a, is fixed in a vertical plane. The highest point on the wire is A and the lowest point on the wire is B. A small ring R of mass m moves freely along the wire. A light elastic string, with natural length a and modulus of elasticity $\frac{1}{2}mg$, has one end attached to A and the other end attached to R. The string AR makes an angle θ (measured anticlockwise) with the downward vertical, so that OR makes an angle 2θ with the downward vertical (see diagram). You may assume that the string does not become slack.

(i) Taking A as the level for zero gravitational potential energy, show that the total potential energy V of the system is given by

$$V = mga\left(\tfrac{1}{4} - \cos\theta - \cos^2\theta\right).$$ [4]

(ii) Show that $\theta = 0$ is the only position of equilibrium. [3]

(iii) By differentiating the energy equation with respect to time t, show that

$$\frac{d^2\theta}{dt^2} = -\frac{g}{4a}\sin\theta(1 + 2\cos\theta).$$ [5]

(iv) Deduce the approximate period of small oscillations about the equilibrium position $\theta = 0$. [3]

1 Two flywheels F and G are rotating freely, about the same axis and in the same direction, with angular speeds $21\,\text{rad}\,\text{s}^{-1}$ and $36\,\text{rad}\,\text{s}^{-1}$ respectively. The flywheels come into contact briefly, and immediately afterwards the angular speeds of F and G are $28\,\text{rad}\,\text{s}^{-1}$ and $34\,\text{rad}\,\text{s}^{-1}$, respectively, in the same direction. Given that the moment of inertia of F about the axis is $1.5\,\text{kg}\,\text{m}^2$, find the moment of inertia of G about the axis. [4]

2 A rotating turntable is slowing down with constant angular deceleration. It makes 16 revolutions as its angular speed decreases from $8\,\text{rad}\,\text{s}^{-1}$ to rest.

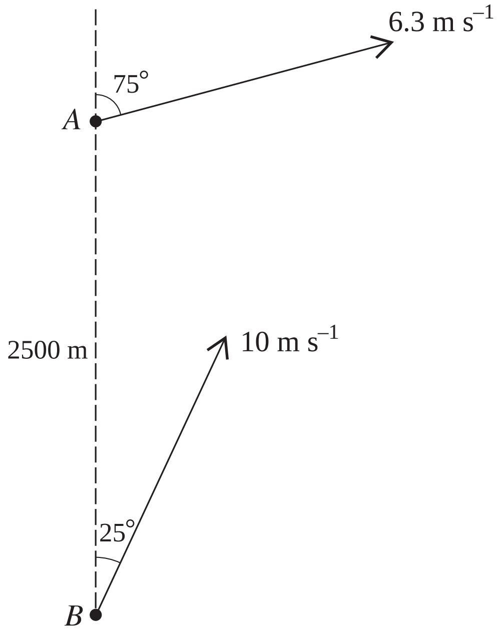
(i) Find the angular deceleration of the turntable. [2]

(ii) Find the angular speed of the turntable at the start of its last complete revolution before coming to rest. [2]

(iii) Find the time taken for the turntable to make its last complete revolution before coming to rest. [2]

3 The region bounded by the curve $y = 2x + x^2$ for $0 \leqslant x \leqslant 3$, the x-axis, and the line $x = 3$, is occupied by a uniform lamina. Find the coordinates of the centre of mass of this lamina. [9]

4



A boat A is travelling with constant speed $6.3\,\text{m}\,\text{s}^{-1}$ on a course with bearing 075°. Boat B is travelling with constant speed $10\,\text{m}\,\text{s}^{-1}$ on a course with bearing 025°. At one instant, A is 2500 m due north of B (see diagram).

(i) Find the magnitude and bearing of the velocity of A relative to B. [5]

(ii) Find the shortest distance between A and B in the subsequent motion. [3]

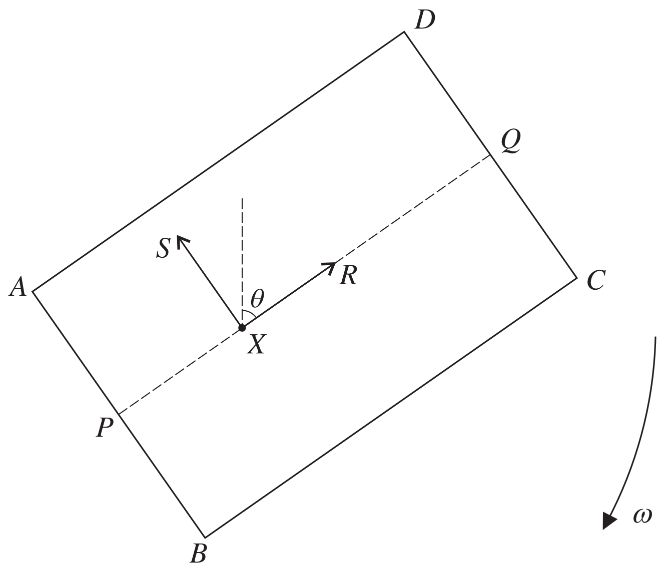
5 The region bounded by the curve $y = \sqrt{ax}$ for $a \leqslant x \leqslant 4a$ (where a is a positive constant), the x-axis, and the lines $x = a$ and $x = 4a$, is rotated through 2π radians about the x-axis to form a uniform solid of revolution of mass m.

(i) Show that the moment of inertia of this solid about the x-axis is $\frac{7}{5}ma^2$. [8]

The solid is free to rotate about a fixed horizontal axis along the line $y = a$, and makes small oscillations as a compound pendulum.

(ii) Find, in terms of a and g, the approximate period of these small oscillations. [4]

6



A uniform rectangular lamina $ABCD$ has mass m and sides $AB = 2a$ and $BC = 3a$. The mid-point of AB is P and the mid-point of CD is Q. The lamina is rotating freely in a vertical plane about a fixed horizontal axis which is perpendicular to the lamina and passes through the point X on PQ where $PX = a$. Air resistance may be neglected. When Q is vertically above X, the angular speed is $\sqrt{\frac{9g}{10a}}$. When XQ makes an angle θ with the upward vertical, the angular speed is ω, and the force acting on the lamina at X has components R parallel to PQ and S parallel to BA (see diagram).

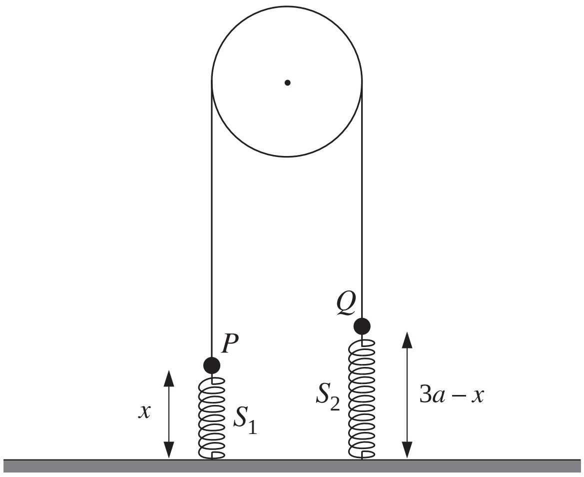
(i) Show that the moment of inertia of the lamina about the axis through X is $\frac{4}{3}ma^2$. [3]

(ii) At an instant when $\cos\theta = \frac{3}{5}$, show that $\omega^2 = \frac{6g}{5a}$. [3]

(iii) At an instant when $\cos\theta = \frac{3}{5}$, show that $R = 0$, and given also that $\sin\theta = \frac{4}{5}$ find S in terms of m and g. [9]

[Turn over

7



Particles P and Q, with masses $3m$ and $2m$ respectively, are connected by a light inextensible string passing over a smooth light pulley. The particle P is connected to the floor by a light spring S_1 with natural length a and modulus of elasticity mg. The particle Q is connected to the floor by a light spring S_2 with natural length a and modulus of elasticity $2mg$. The sections of the string not in contact with the pulley, and the two springs, are vertical. Air resistance may be neglected. The particles P and Q move vertically and the string remains taut; when the length of S_1 is x, the length of S_2 is $(3a - x)$ (see diagram).

(i) Find the total potential energy of the system (taking the floor as the reference level for gravitational potential energy). Hence show that $x = \frac{4}{3}a$ is a position of stable equilibrium. [9]

(ii) By differentiating the energy equation, and substituting $x = \frac{4}{3}a + y$, show that the motion is simple harmonic, and find the period. [9]

1 When the power is turned off, a fan disk inside a jet engine slows down with constant angular deceleration $0.8\,\text{rad s}^{-2}$.

(i) Find the time taken for the angular speed to decrease from $950\,\text{rad s}^{-1}$ to $750\,\text{rad s}^{-1}$. **[2]**

(ii) Find the angle through which the disk turns as the angular speed decreases from $220\,\text{rad s}^{-1}$ to $200\,\text{rad s}^{-1}$. **[2]**

(iii) Find the time taken for the disk to make the final 10 revolutions before coming to rest. **[3]**

2 A straight rod AB has length a. The rod has variable density, and at a distance x from A its mass per unit length is $ke^{-\frac{x}{a}}$, where k is a constant. Find, in an exact form, the distance of the centre of mass of the rod from A. **[7]**

3 A uniform rod XY, of mass 5 kg and length 1.8 m, is free to rotate in a vertical plane about a fixed horizontal axis through X. The rod is at rest with Y vertically below X when a couple of constant moment is applied to the rod. It then rotates, and comes instantaneously to rest when XY is horizontal.

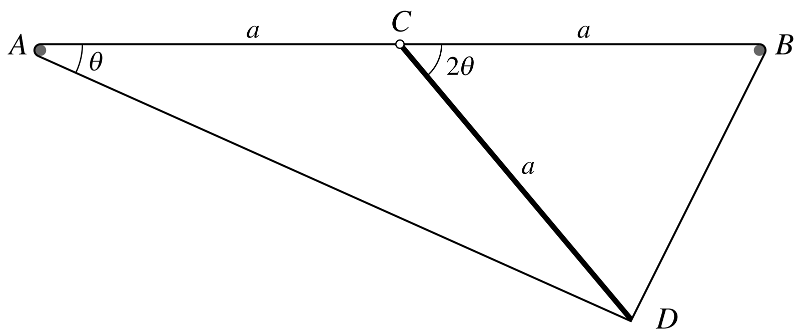
(i) Find the moment of the couple. **[4]**

(ii) Find the angular acceleration of the rod

(a) immediately after the couple is first applied, **[3]**

(b) when XY is horizontal. **[2]**

4



Two small smooth pegs A and B are fixed at a distance $2a$ apart on the same horizontal level, and C is the mid-point of AB. A uniform rod CD, of mass m and length a, is freely pivoted at C and can rotate in the vertical plane containing AB, with D below the level of AB. A light elastic string, of natural length a and modulus of elasticity $3mg$, passes round the peg A and its ends are attached to C and D. Another light elastic string, of natural length a and modulus of elasticity $4mg$, passes round the peg B and its ends are also attached to C and D. The angle CAD is θ, where $0 < \theta < \frac{1}{2}\pi$, so that the angle BCD is 2θ (see diagram).

(i) Taking AB as the reference level for gravitational potential energy, show that the total potential energy of the system is

$$\tfrac{1}{2}mga(14 - 2\cos 2\theta - \sin 2\theta).$$ **[5]**

(ii) Find the value of θ for which the system is in equilibrium. **[3]**

(iii) Determine whether this position of equilibrium is stable or unstable. **[2]**

5 The region inside the circle $x^2 + y^2 = a^2$ is rotated about the x-axis to form a uniform solid sphere of radius a and volume $\frac{4}{3}\pi a^3$. The mass of the sphere is $10M$.

(i) Show by integration that the moment of inertia of the sphere about the x-axis is $4Ma^2$. (You may assume the standard formula $\frac{1}{2}mr^2$ for the moment of inertia of a uniform disc about its axis.) **[6]**

The sphere is free to rotate about a fixed horizontal axis which is a diameter of the sphere. A particle of mass M is attached to the lowest point of the sphere. The sphere with the particle attached then makes small oscillations as a compound pendulum.

(ii) Find, in terms of a and g, the approximate period of these oscillations. **[5]**

6 Two ships P and Q are moving on straight courses with constant speeds. At one instant Q is 80 km from P on a bearing of 220°. Three hours later, Q is 36 km due south of P.

(i) Show that the velocity of Q relative to P is 19.1 km h^{-1} in the direction with bearing 063.8° (both correct to 3 significant figures). **[5]**

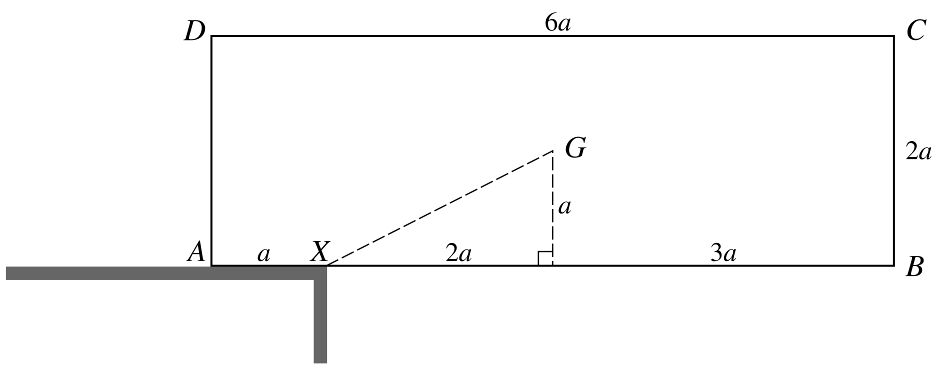
(ii) Find the shortest distance between the two ships in the subsequent motion. **[2]**

Given that the speed of P is 28 km h^{-1} and Q is travelling in the direction with bearing 105°, find

(iii) the bearing of the direction in which P is travelling, **[3]**

(iv) the speed of Q. **[2]**

7



A uniform rectangular block of mass m and cross-section $ABCD$ has $AB = CD = 6a$ and $AD = BC = 2a$. The point X is on AB such that $AX = a$ and G is the centre of $ABCD$. The block is placed with AB perpendicular to the straight edge of a rough horizontal table. AX is in contact with the table and XB overhangs the edge (see diagram). The block is released from rest in this position, and it rotates without slipping about a horizontal axis through X.

(i) Find the moment of inertia of the block about the axis of rotation. **[3]**

For the instant when XG is horizontal,

(ii) show that the angular acceleration of the block is $\dfrac{3\sqrt{5}g}{25a}$, **[2]**

(iii) find the angular speed of the block, **[3]**

(iv) show that the force exerted by the table on the block has magnitude $\dfrac{2\sqrt{70}}{25}mg$. **[8]**

1 A wheel is rotating and is slowing down with constant angular deceleration. The initial angular speed is $80\,\text{rad}\,\text{s}^{-1}$, and after 15 s the wheel has turned through 1020 radians.

(i) Find the angular deceleration of the wheel. **[2]**

(ii) Find the angle through which the wheel turns in the last 5 s before it comes to rest. **[2]**

(iii) Find the total number of revolutions made by the wheel from the start until it comes to rest. **[3]**

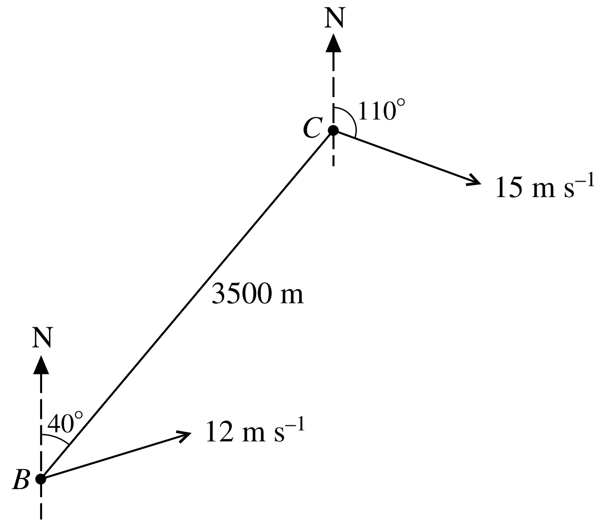
2 The region bounded by the x-axis, the y-axis, the line $x = \ln 3$, and the curve $y = e^{-x}$ for $0 \leqslant x \leqslant \ln 3$, is occupied by a uniform lamina. Find, in an exact form, the coordinates of the centre of mass of this lamina. **[9]**

3 A circular disc is rotating in a horizontal plane with angular speed $16\,\text{rad}\,\text{s}^{-1}$ about a fixed vertical axis passing through its centre O. The moment of inertia of the disc about the axis is $0.9\,\text{kg}\,\text{m}^2$. A particle, initially at rest just above the surface of the disc, drops onto the disc and sticks to it at a point 0.4 m from O. Afterwards, the angular speed of the disc with the particle attached is $15\,\text{rad}\,\text{s}^{-1}$.

(i) Find the mass of the particle. **[4]**

(ii) Find the loss of kinetic energy. **[3]**

4



From a boat B, a cruiser C is observed 3500 m away on a bearing of 040°. The cruiser C is travelling with constant speed $15\,\text{m}\,\text{s}^{-1}$ along a straight line course with bearing 110° (see diagram). The boat B travels with constant speed $12\,\text{m}\,\text{s}^{-1}$ on a straight line course which takes it as close as possible to the cruiser C.

(i) Show that the bearing of the course of B is 073°, correct to the nearest degree. **[4]**

(ii) Find the magnitude and the bearing of the velocity of C relative to B. **[3]**

(iii) Find the shortest distance between B and C in the subsequent motion. **[3]**

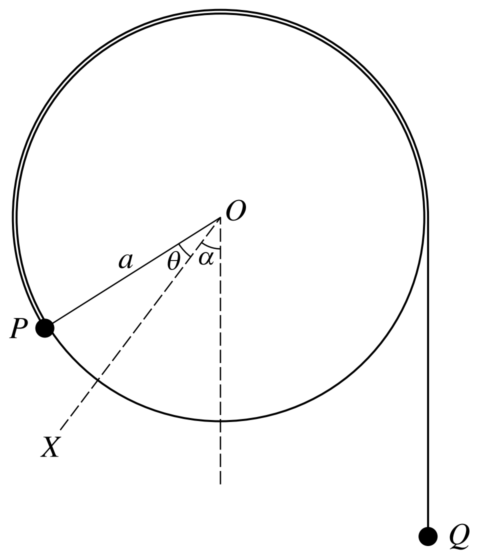
5 A uniform rod AB has mass m and length $6a$. The point C on the rod is such that $AC = a$. The rod can rotate freely in a vertical plane about a fixed horizontal axis passing through C and perpendicular to the rod.

(i) Show by integration that the moment of inertia of the rod about this axis is $7ma^2$. **[5]**

The rod starts at rest with B vertically below C. A couple of constant moment $\dfrac{6mga}{\pi}$ is then applied to the rod.

(ii) Find, in terms of a and g, the angular speed of the rod when it has turned through one and a half revolutions. **[6]**

6



A light pulley of radius a is free to rotate in a vertical plane about a fixed horizontal axis passing through its centre O. Two particles, P of mass $5m$ and Q of mass $3m$, are connected by a light inextensible string. The particle P is attached to the circumference of the pulley, the string passes over the top of the pulley, and Q hangs below the pulley on the opposite side to P. The section of string not in contact with the pulley is vertical. The fixed line OX makes an angle α with the downward vertical, where $\cos\alpha = \frac{4}{5}$, and OP makes an angle θ with OX (see diagram).

You are given that the total potential energy of the system (using a suitable reference level) is V, where

$$V = mga(3\sin\theta - 4\cos\theta - 3\theta).$$

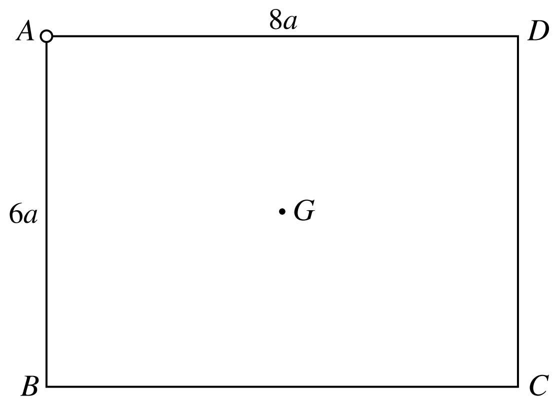
(i) Show that $\theta = 0$ is a position of stable equilibrium. **[5]**

(ii) Show that the kinetic energy of the system is $4ma^2\dot{\theta}^2$. **[2]**

(iii) By differentiating the energy equation, then making suitable approximations for $\sin\theta$ and $\cos\theta$, find the approximate period of small oscillations about the equilibrium position $\theta = 0$. **[5]**

[Question 7 is printed overleaf.]

7



The diagram shows a uniform rectangular lamina $ABCD$ with $AB = 6a$, $AD = 8a$ and centre G. The mass of the lamina is m. The lamina rotates freely in a vertical plane about a fixed horizontal axis passing through A and perpendicular to the lamina.

(i) Find the moment of inertia of the lamina about this axis. **[3]**

The lamina is released from rest with AD horizontal and BC below AD.

(ii) For an instant during the subsequent motion when AD is vertical, show that the angular speed of the lamina is $\sqrt{\frac{3g}{50a}}$ and find its angular acceleration. **[5]**

At an instant when AD is vertical, the force acting on the lamina at A has magnitude F.

(iii) By finding components parallel and perpendicular to GA, or otherwise, show that $F = \frac{\sqrt{493}}{20}mg$. **[8]**