

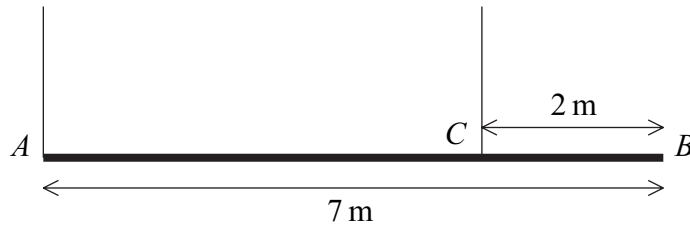
NOTICE TO CUSTOMER:

The sale of this product is intended for use of the original purchaser only and for use only on a single computer system. Duplicating, selling, or otherwise distributing this product is a violation of the law ; **your license of the product will be terminated at any moment if you are selling or distributing the products.**

No parts of this book may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher.

Answer **all** questions.

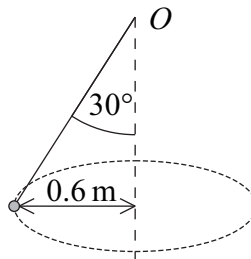
- 1 A uniform beam, AB , has mass 20 kg and length 7 metres. A rope is attached to the beam at A . A second rope is attached to the beam at the point C , which is 2 metres from B . Both of the ropes are vertical. The beam is in equilibrium in a horizontal position, as shown in the diagram.



Find the tensions in the two ropes.

(6 marks)

- 2 A particle, of mass 2 kg, is attached to one end of a light inextensible string. The other end is fixed to the point O . The particle is set into motion, so that it describes a horizontal circle of radius 0.6 metres, with the string at an angle of 30° to the vertical. The centre of the circle is vertically below O .



- (a) Show that the tension in the string is 22.6 N, correct to three significant figures.

(3 marks)

- (b) Find the speed of the particle.

(4 marks)

3 A particle moves in a straight line and at time t has velocity v , where

$$v = 2t - 12e^{-t}, \quad t \geq 0$$

- (a) (i) Find an expression for the acceleration of the particle at time t . (2 marks)
- (ii) State the range of values of the acceleration of the particle. (3 marks)
- (b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t . (4 marks)

4 A car has a maximum speed of 42 m s^{-1} when it is moving on a horizontal road. When the speed of the car is $v \text{ m s}^{-1}$, it experiences a resistance force of magnitude $30v$ newtons.

- (a) Show that the maximum power of the car is 52920 W . (2 marks)
- (b) The car has mass 1200 kg . It travels, from rest, up a slope inclined at 5° to the horizontal.

- (i) Show that, when the car is travelling at its maximum speed $V \text{ m s}^{-1}$ up the slope,

$$V^2 + 392 \sin 5^\circ V - 1764 = 0 \quad (4 \text{ marks})$$

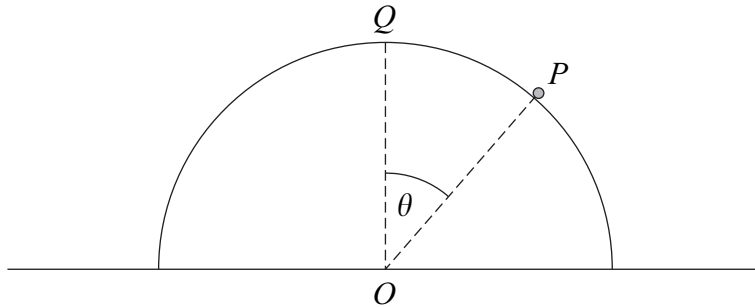
- (ii) Hence find V . (2 marks)

5 A car, of mass 1600 kg , is travelling along a straight horizontal road at a speed of 20 m s^{-1} when the driving force is removed. The car then freewheels and experiences a resistance force. The resistance force has magnitude $40v$ newtons, where $v \text{ m s}^{-1}$ is the speed of the car after it has been freewheeling for t seconds.

Find an expression for v in terms of t . (7 marks)

Turn over for the next question

- 6 A particle P , of mass m kg, is placed at the point Q on the top of a smooth upturned hemisphere of radius 3 metres and centre O . The plane face of the hemisphere is fixed to a horizontal table. The particle is set into motion with an initial horizontal velocity of 2 m s^{-1} . When the particle is on the surface of the hemisphere, the angle between OP and OQ is θ and the particle has speed $v \text{ m s}^{-1}$.



- (a) Show that $v^2 = 4 + 6g(1 - \cos \theta)$. (4 marks)
- (b) Find the value of θ when the particle leaves the hemisphere. (5 marks)

- 7 A particle, of mass 10 kg, is attached to one end of a light elastic string of natural length 0.4 metres and modulus of elasticity 100 N. The other end of the string is fixed to the point O .

- (a) Find the length of the elastic string when the particle hangs in equilibrium directly below O . (2 marks)
- (b) The particle is pulled down and held at a point P , which is 1 metre vertically below O .

Show that the elastic potential energy of the string when the particle is in this position is 45 J. (2 marks)

- (c) The particle is released from rest at the point P . In the subsequent motion, the particle has speed $v \text{ m s}^{-1}$ when it is x metres **below** O .
- (i) Show that, while the string is taut,

$$v^2 = 39.6x - 25x^2 - 14.6 \quad (7 \text{ marks})$$

- (ii) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. (3 marks)

END OF QUESTIONS

Answer **all** questions.

1 A study undertaken by Goodhealth Hospital found that the number of patients each month, X , contracting a particular superbug can be modelled by a Poisson distribution with a mean of 1.5 .

- (a) (i) Calculate $P(X = 2)$. *(2 marks)*
- (ii) Hence determine the probability that exactly 2 patients will contract this superbug in each of three consecutive months. *(2 marks)*
- (b) (i) Write down the distribution of Y , the number of patients contracting this superbug in a given 6-month period. *(1 mark)*
- (ii) Find the probability that at least 12 patients will contract this superbug during a given 6-month period. *(2 marks)*
- (c) State **two** assumptions implied by the use of a Poisson model for the number of patients contracting this superbug. *(2 marks)*

2 Year 12 students at Newstatus School choose to participate in one of four sports during the Spring term.

The students' choices are summarised in the table.

	Squash	Badminton	Archery	Hockey	Total
Male	5	16	30	19	70
Female	4	20	33	53	110
Total	9	36	63	72	180

- (a) Use a χ^2 test, at the 5% level of significance, to determine whether the choice of sport is independent of gender. *(10 marks)*
- (b) Interpret your result in part (a) as it relates to students choosing hockey. *(2 marks)*

- 3 The time, T minutes, that parents have to wait before seeing a mathematics teacher at a school parents' evening can be modelled by a normal distribution with mean μ and standard deviation σ .

At a recent parents' evening, a random sample of 9 parents was asked to record the times that they waited before seeing a mathematics teacher.

The times, in minutes, are

5 12 10 8 7 6 9 7 8

- (a) Construct a 90% confidence interval for μ . (7 marks)
- (b) Comment on the headteacher's claim that the mean time that parents have to wait before seeing a mathematics teacher is 5 minutes. (2 marks)

- 4 (a) A random variable X has probability density function defined by

$$f(x) = \begin{cases} k & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

- (i) Show that $k = \frac{1}{b-a}$. (1 mark)
- (ii) Prove, using integration, that $E(X) = \frac{1}{2}(a+b)$. (4 marks)
- (b) The error, X grams, made when a shopkeeper weighs out loose sweets can be modelled by a rectangular distribution with the following probability density function:

$$f(x) = \begin{cases} k & -2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Write down the value of the mean, μ , of X . (1 mark)
- (ii) Evaluate the standard deviation, σ , of X . (2 marks)
- (iii) Hence find $P\left(X < \frac{2-\mu}{\sigma}\right)$. (3 marks)

- 5 The Globe Express agency organises trips to the theatre. The cost, £ X , of these trips can be modelled by the following probability distribution:

x	40	45	55	74
$P(X=x)$	0.30	0.24	0.36	0.10

- (a) Calculate the mean and standard deviation of X . *(4 marks)*
- (b) For special celebrity charity performances, Globe Express increases the cost of the trips to £ Y , where

$$Y = 10X + 250$$

Determine the mean and standard deviation of Y . *(2 marks)*

- 6 In previous years, the marks obtained in a French test by students attending Topnotch College have been modelled satisfactorily by a normal distribution with a mean of 65 and a standard deviation of 9.

Teachers in the French department at Topnotch College suspect that this year their students are, on average, underachieving.

In order to investigate this suspicion, the teachers selected a random sample of 35 students to take the French test and found that their mean score was 61.5.

- (a) Investigate, at the 5% level of significance, the teachers' suspicion. *(6 marks)*
- (b) Explain, in the context of this question, the meaning of a Type I error. *(2 marks)*

- 7 Engineering work on the railway network causes an increase in the journey time of commuters travelling into work each morning.

The increase in journey time, T hours, is modelled by a continuous random variable with probability density function

$$f(t) = \begin{cases} 4t(1 - t^2) & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $E(T) = \frac{8}{15}$. (3 marks)

(b) (i) Find the cumulative distribution function, $F(t)$, for $0 \leq t \leq 1$. (2 marks)

(ii) Hence, or otherwise, for a commuter selected at random, find

$$P(\text{mean} < T < \text{median}) \quad (5 \text{ marks})$$

- 8 Bottles of sherry nominally contain 1000 millilitres. After the introduction of a new method of filling the bottles, there is a suspicion that the mean volume of sherry in a bottle has changed.

In order to investigate this suspicion, a random sample of 12 bottles of sherry is taken and the volume of sherry in each bottle is measured.

The volumes, in millilitres, of sherry in these bottles are found to be

996	1006	1009	999	1007	1003
998	1010	997	996	1008	1007

Assuming that the volume of sherry in a bottle is normally distributed, investigate, at the 5% level of significance, whether the mean volume of sherry in a bottle differs from 1000 millilitres. (10 marks)

END OF QUESTIONS

8. The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} 2(x-2) & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f(x)$ for all values of x . (3)

(b) Write down the mode of X . (1)

Find

(c) $E(X)$, (3)

(d) the median of X . (4)

(e) Comment on the skewness of this distribution. Give a reason for your answer. (2)

Leave
blank

- 1 The random variable T is normally distributed with mean μ and standard deviation σ . It is given that $P(T > 80) = 0.05$ and $P(T > 50) = 0.75$. Find the values of μ and σ . [6]
- 2 A village has a population of 600 people. A sample of 12 people is obtained as follows. A list of all 600 people is obtained and a three-digit number, between 001 and 600 inclusive, is allocated to each name in alphabetical order. Twelve three-digit random numbers, between 001 and 600 inclusive, are obtained and the people whose names correspond to those numbers are chosen.
- (i) Find the probability that all 12 of the numbers chosen are 500 or less. [3]
- (ii) When the selection has been made, it is found that all of the numbers chosen are 500 or less. One of the people in the village says, "The sampling method must have been biased." Comment on this statement. [2]
- 3 The random variable G has the distribution $Po(\lambda)$. A test is carried out of the null hypothesis $H_0 : \lambda = 4.5$ against the alternative hypothesis $H_1 : \lambda \neq 4.5$, based on a single observation of G . The critical region for the test is $G \leq 1$ and $G \geq 9$.
- (i) Find the significance level of the test. [5]
- (ii) Given that $\lambda = 5.5$, calculate the probability that the test results in a Type II error. [3]
- 4 The random variable Y has the distribution $N(\mu, \sigma^2)$. The results of 40 independent observations of Y are summarised by
- $$\Sigma y = 3296.0, \quad \Sigma y^2 = 286\,800.40.$$
- (i) Calculate unbiased estimates of μ and σ^2 . [4]
- (ii) Use your answers to part (i) to estimate the probability that a single random observation of Y will be less than 60.0. [2]
- (iii) Explain whether it is necessary to know that Y is normally distributed in answering part (i) of this question. [1]
- 5 Over a long period the number of visitors per week to a stately home was known to have the distribution $N(500, 100^2)$. After higher car parking charges were introduced, a sample of four randomly chosen weeks gave a mean number of visitors per week of 435. You should assume that the number of visitors per week is still normally distributed with variance 100^2 .
- (i) Test, at the 10% significance level, whether there is evidence that the mean number of visitors per week has fallen. [7]
- (ii) Explain why it is necessary to assume that the distribution of the number of visitors per week (after the introduction of higher charges) is normal in order to carry out the test. [2]

- 6 The number of house sales per week handled by an estate agent is modelled by the distribution $Po(3)$.
- (i) Find the probability that, in one randomly chosen week, the number of sales handled is
 - (a) greater than 4, [2]
 - (b) exactly 4. [2]
 - (ii) Use a suitable approximation to the Poisson distribution to find the probability that, in a year consisting of 50 working weeks, the estate agent handles more than 165 house sales. [5]
 - (iii) One of the conditions needed for the use of a Poisson model to be valid is that house sales are independent of one another.
 - (a) Explain, in non-technical language, what you understand by this condition. [1]
 - (b) State another condition that is needed. [1]

7 A continuous random variable X_1 has probability density function given by

$$f(x) = \begin{cases} kx & 0 \leq x \leq 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (i) Show that $k = \frac{1}{2}$. [2]
 - (ii) Sketch the graph of $y = f(x)$. [2]
 - (iii) Find $E(X_1)$ and $\text{Var}(X_1)$. [5]
 - (iv) Sketch the graph of $y = f(x - 1)$. [2]
 - (v) The continuous random variable X_2 has probability density function $f(x - 1)$ for all x . Write down the values of $E(X_2)$ and $\text{Var}(X_2)$. [2]
- 8 Consultations are taking place as to whether a site currently in use as a car park should be developed as a shopping mall. An agency acting on behalf of a firm of developers claims that at least 65% of the local population are in favour of the development. In a survey of a random sample of 12 members of the local population, 6 are in favour of the development.
- (i) Carry out a test, at the 10% significance level, to determine whether the result of the survey is consistent with the claim of the agency. [7]
 - (ii) A local residents' group claims that no more than 35% of the local population are in favour of the development. Without further calculations, state with a reason what can be said about the claim of the local residents' group. [2]
 - (iii) A test is carried out, at the 15% significance level, of the agency's claim. The test is based on a random sample of size $2n$, and exactly n of the sample are in favour of the development. Find the smallest possible value of n for which the outcome of the test is to reject the agency's claim. [4]

Answer **all** questions.

- 1 The number of accidents, X , occurring during one week at Joanne's place of work can be modelled by a Poisson distribution with a mean of 0.7.

The number of accidents, Y , occurring during one week at Pete's place of work can be modelled by a Poisson distribution with a mean of 1.3.

- (a) (i) Determine $P(X < 3)$. *(1 mark)*
- (ii) Calculate $P(Y = 2)$. *(2 marks)*
- (b) Find the probability that, during a particular week, there are at least 4 accidents in total at these two places of work. *(3 marks)*

- 2 The marks achieved by Pat in her homework assignments may be assumed to be normally distributed with mean μ .

The marks achieved by Pat in a random sample of 8 assignments were recorded as follows:

60 65 62 67 69 71 63 66

Construct a 99% confidence interval for μ . *(5 marks)*

- 3 The handicap committee of a golf club has indicated that the mean score achieved by the club's members in the past was 85.9.

A group of members believes that recent changes to the golf course have led to a change in the mean score achieved by the club's members and decides to investigate this belief.

A random sample of the scores, x , of 100 club members was taken and is summarised by

$$\sum x = 8350 \quad \text{and} \quad \sum (x - \bar{x})^2 = 15\,321$$

where \bar{x} denotes the sample mean.

Test, at the 5% level of significance, the group's belief that the mean score of 85.9 has changed. *(8 marks)*

- 4 The number of mistakes, X , that Holli makes as a learner driver when she drives from Ampthill to Bedford can be modelled by the following discrete probability distribution:

x	≤ 1	2	3	4	5	6	≥ 7
$P(X = x)$	0	0.40	0.25	0.18	0.12	k	0

- (a) Find the value of k . *(1 mark)*
- (b) Find:
- (i) $E(X)$; *(1 mark)*
- (ii) $\text{Var}(X)$. *(3 marks)*
- (c) When Holli makes the return journey by the same route, the number of mistakes, Y , that she makes can be approximated by

$$Y = 2X - 3$$

Find:

- (i) $E(Y)$; *(1 mark)*
- (ii) the standard deviation of Y . *(3 marks)*

- 5 Jasmine's French teacher states that a homework assignment should take, on average, 30 minutes to complete.

Jasmine believes that he is understating the mean time that the assignment takes to complete and so decides to investigate. She records the times, in minutes, that it takes for a random sample of 10 students to complete the French assignment, with the following results:

29 33 36 42 30 28 31 34 37 35

- (a) Test, at the 1% level of significance, Jasmine's belief that her French teacher has understated the mean time that it should take to complete the homework assignment. *(7 marks)*
- (b) State an assumption that you must make in order for the test used in part (a) to be valid. *(1 mark)*

Turn over for the next question

- 6 The waiting time, T minutes, before being served at a local newsagents can be modelled by a continuous random variable with probability density function

$$f(t) = \begin{cases} \frac{3}{8}t^2 & 0 \leq t < 1 \\ \frac{1}{16}(t+5) & 1 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the graph of f . (3 marks)
- (b) For a customer selected at random, calculate $P(T \geq 1)$. (2 marks)
- (c) (i) Show that the cumulative distribution function for $1 \leq t \leq 3$ is given by

$$F(t) = \frac{1}{32}(t^2 + 10t - 7) \quad (5 \text{ marks})$$

- (ii) Hence find the median waiting time. (4 marks)

- 7 A statistics unit is required to determine whether or not there is an association between students' performances in mathematics at Key Stage 3 and at GCE.

A survey of the results of 500 students showed the following information:

		GCE Grade				Total
		A	B	C	Below C	
Key Stage 3 Level	8	60	55	47	43	205
	7	55	32	31	26	144
	6	40	38	35	38	151
Total		155	125	113	107	500

- (a) Use a χ^2 test at the 10% level of significance to determine whether there is an association between students' performances in mathematics at Key Stage 3 and at GCE. (9 marks)
- (b) Comment on the number of students who gained a grade A at GCE having gained a level 7 at Key Stage 3. (1 mark)

END OF QUESTIONS

Answer **all** questions.

- 1 Alan's journey time, in minutes, to travel home from work each day is known to be normally distributed with mean μ .

Alan records his journey time, in minutes, on a random sample of 8 days as being

36 38 39 40 50 35 36 42

Construct a 95% confidence interval for μ . (5 marks)

- 2 The number of computers, A , bought during one day from the Amplebuy computer store can be modelled by a Poisson distribution with a mean of 3.5.

The number of computers, B , bought during one day from the Bestbuy computer store can be modelled by a Poisson distribution with a mean of 5.0.

(a) (i) Calculate $P(A = 4)$. (2 marks)

(ii) Determine $P(B \leq 6)$. (1 mark)

(iii) Find the probability that a total of fewer than 10 computers is bought from these two stores on one particular day. (3 marks)

(b) Calculate the probability that a total of fewer than 10 computers is bought from these two stores on at least 4 out of 5 consecutive days. (3 marks)

(c) The numbers of computers bought from the Choicebuy computer store over a 10-day period are recorded as

8 12 6 6 9 15 10 8 6 12

(i) Calculate the mean and variance of these data. (2 marks)

(ii) State, giving a reason based on your results in part (c)(i), whether or not a Poisson distribution provides a suitable model for these data. (2 marks)

- 3 The handicap committee of a golf club has indicated that the mean score achieved by the club's members in the past was 85.9.

A group of members believes that recent changes to the golf course have led to a change in the mean score achieved by the club's members and decides to investigate this belief.

A random sample of the scores, x , of 100 club members was taken and is summarised by

$$\sum x = 8350 \quad \text{and} \quad \sum (x - \bar{x})^2 = 15\,321$$

where \bar{x} denotes the sample mean.

Test, at the 5% level of significance, the group's belief that the mean score of 85.9 has changed. (8 marks)

- 4 The number of fish, X , caught by Pearl when she goes fishing can be modelled by the following discrete probability distribution:

x	1	2	3	4	5	6	≥ 7
$\mathbf{P}(X = x)$	0.01	0.05	0.14	0.30	k	0.12	0

- (a) Find the value of k . (1 mark)
- (b) Find:
- (i) $E(X)$; (1 mark)
- (ii) $\text{Var}(X)$. (3 marks)
- (c) When Pearl sells her fish, she earns a profit, in pounds, given by

$$Y = 5X + 2$$

Find:

- (i) $E(Y)$; (1 mark)
- (ii) the standard deviation of Y . (3 marks)

- 5 Jasmine's French teacher states that a homework assignment should take, on average, 30 minutes to complete.

Jasmine believes that he is understating the mean time that the assignment takes to complete and so decides to investigate. She records the times, in minutes, that it takes for a random sample of 10 students to complete the French assignment, with the following results:

29 33 36 42 30 28 31 34 37 35

- (a) Test, at the 1% level of significance, Jasmine's belief that her French teacher has understated the mean time that it should take to complete the homework assignment. *(7 marks)*
- (b) State an assumption that you must make in order for the test used in part (a) to be valid. *(1 mark)*
- 6 The waiting time, T minutes, before being served at a local newsagents can be modelled by a continuous random variable with probability density function

$$f(t) = \begin{cases} \frac{3}{8}t^2 & 0 \leq t < 1 \\ \frac{1}{16}(t+5) & 1 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Sketch the graph of f . *(3 marks)*
- (b) For a customer selected at random, calculate $P(T \geq 1)$. *(2 marks)*
- (c) (i) Show that the cumulative distribution function for $1 \leq t \leq 3$ is given by

$$F(t) = \frac{1}{32}(t^2 + 10t - 7) \quad (5 \text{ marks})$$

- (ii) Hence find the median waiting time. *(4 marks)*

- 7 A statistics unit is required to determine whether or not there is an association between students' performances in mathematics at Key Stage 3 and at GCE.

A survey of the results of 500 students showed the following information:

		GCE Grade				Total
		A	B	C	Below C	
Key Stage 3 Level	8	60	55	47	43	205
	7	55	32	31	26	144
	6	40	38	35	38	151
Total		155	125	113	107	500

- (a) Use a χ^2 test at the 10% level of significance to determine whether there is an association between students' performances in mathematics at Key Stage 3 and at GCE. (9 marks)
- (b) Comment on the number of students who gained a grade A at GCE having gained a level 7 at Key Stage 3. (1 mark)

- 8 The continuous random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0 & x \leq -4 \\ \frac{x+4}{9} & -4 \leq x \leq 5 \\ 1 & x \geq 5 \end{cases}$$

- (a) Determine the probability density function, $f(x)$, of X . (2 marks)
- (b) Sketch the graph of f . (2 marks)
- (c) Determine $P(X > 2)$. (2 marks)
- (d) Evaluate the mean and variance of X . (2 marks)

END OF QUESTIONS