Answer **all** questions.

1 (a) Find $\dfrac{dy}{dx}$ when $y = \tan 3x$. *(2 marks)*

(b) Given that $y = \dfrac{3x+1}{2x+1}$, show that $\dfrac{dy}{dx} = \dfrac{1}{(2x+1)^2}$. *(3 marks)*

2 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^3 \frac{1}{\sqrt{1+x^3}}\,dx$$

giving your answer to three significant figures. *(4 marks)*

3 (a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\displaystyle\int \frac{2x^3+1}{x^4+2x}\,dx$. *(2 marks)*

(b) (i) Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x+1}\,dx = \frac{1}{4}\int\left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)du$$ *(3 marks)*

(ii) Hence show that $\displaystyle\int_0^4 x\sqrt{2x+1}\,dx = 19.9$ correct to three significant figures. *(4 marks)*

4 It is given that $2\operatorname{cosec}^2 x = 5 - 5\cot x$.

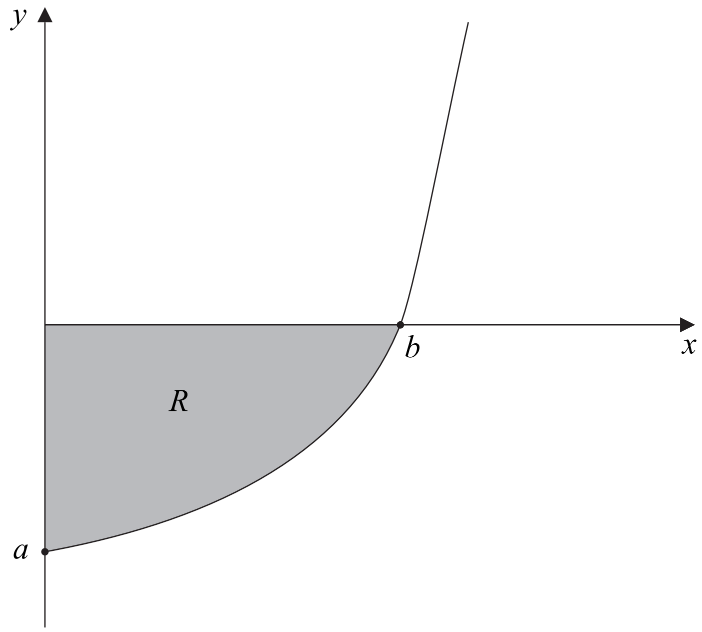
(a) Show that the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$ can be written in the form

$$2\cot^2 x + 5\cot x - 3 = 0$$ *(2 marks)*

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. *(2 marks)*

(c) Hence, or otherwise, solve the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leqslant \pi$. *(3 marks)*

5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



(a) State the value of a, and show that $b = \ln 3$. *(3 marks)*

(b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. *(1 mark)*

(c) The shaded region R is rotated through 360° about the x-axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. *(6 marks)*

(d) Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geqslant 0$. *(2 marks)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x-axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. *(2 marks)*

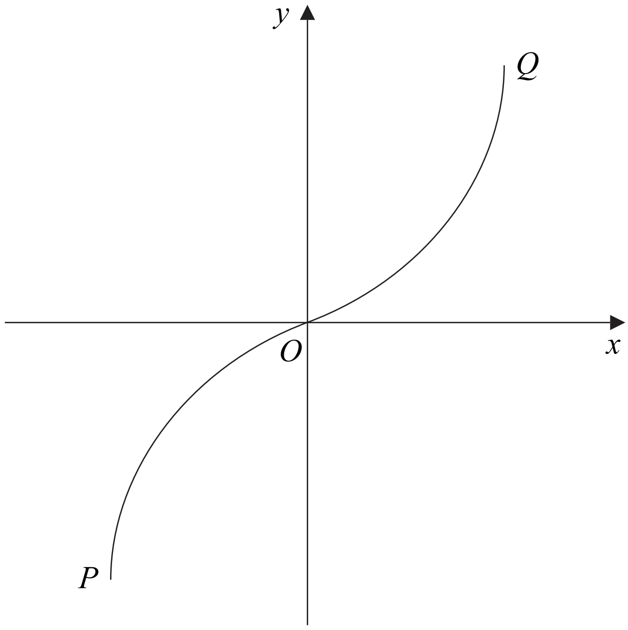
(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \dfrac{3 - x^3}{4}$. *(1 mark)*

(c) (i) Use the iteration $x_{n+1} = \dfrac{3 - x_n{}^3}{4}$ with $x_1 = 0.5$ to find x_3, giving your answer to two decimal places. *(3 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \dfrac{3 - x^3}{4}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(3 marks)*

7 (a) The sketch shows the graph of $y = \sin^{-1} x$.



Write down the coordinates of the points P and Q, the end-points of the graph. *(2 marks)*

(b) Sketch the graph of $y = -\sin^{-1}(x - 1)$. *(3 marks)*

8 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{x+2} \qquad \text{for real values of } x, \quad x \neq -2$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 4$. *(4 marks)*

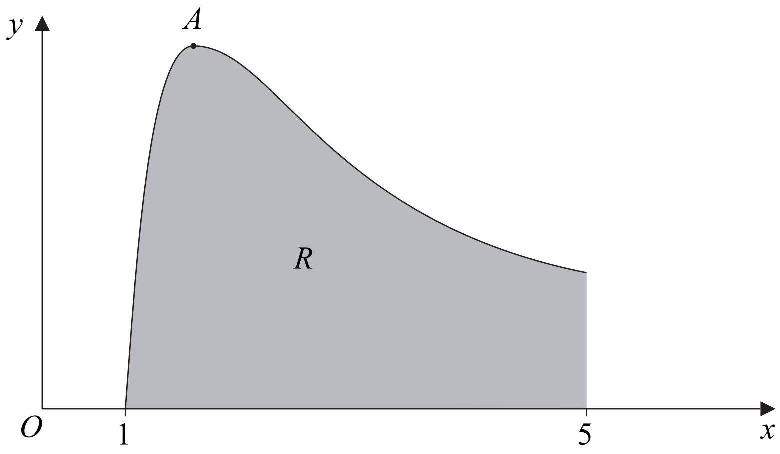
(c) (i) Explain why the function f does **not** have an inverse. *(1 mark)*

(ii) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

9 (a) Given that $y = x^{-2} \ln x$, show that $\dfrac{dy}{dx} = \dfrac{1 - 2\ln x}{x^3}$. *(4 marks)*

(b) Using integration by parts, find $\int x^{-2} \ln x \, dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2} \ln x$.



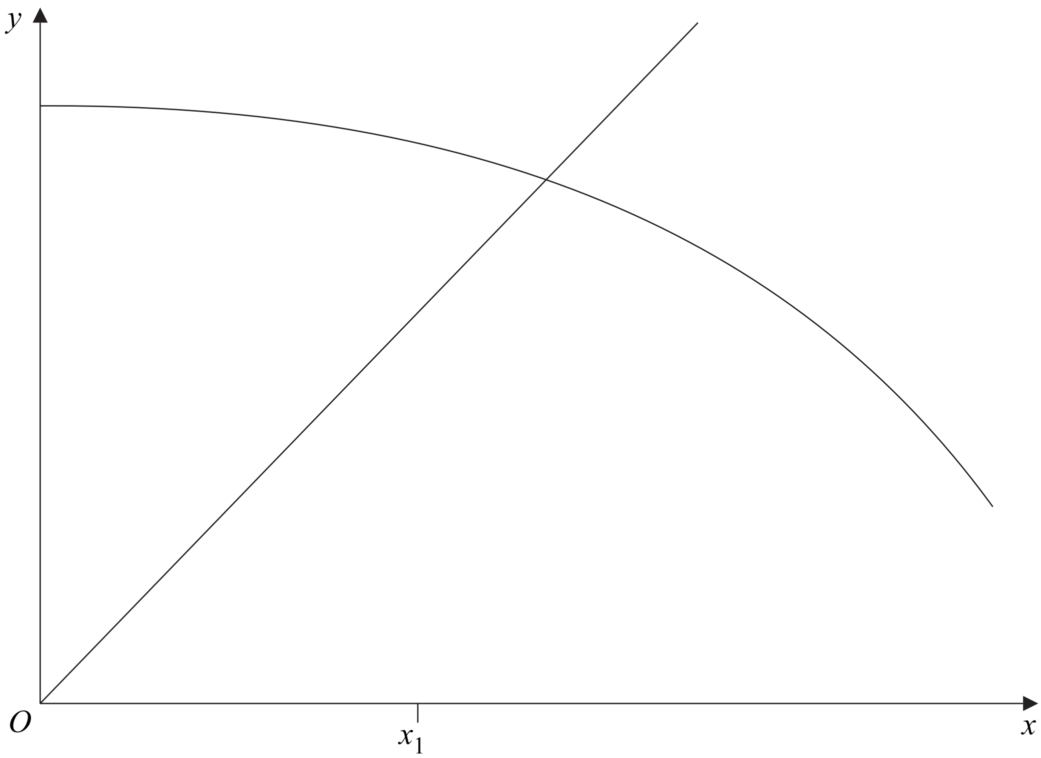
(i) Using the answer to part (a), find, in terms of e, the x-coordinate of the stationary point A. *(2 marks)*

(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

END OF QUESTIONS

Figure 1 (for Question 6)



Answer **all** questions.

1 (a) Find the roots of the equation $m^2 + 2m + 2 = 0$ in the form $a + \mathrm{i}b$. *(2 marks)*

(b) (i) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 4x$$

(6 marks)

(ii) Hence express y in terms of x, given that $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ when $x = 0$. *(4 marks)*

2 (a) Find $\int_0^a x\mathrm{e}^{-2x}\,\mathrm{d}x$, where $a > 0$. *(5 marks)*

(b) Write down the value of $\lim_{a \to \infty} a^k \mathrm{e}^{-2a}$, where k is a positive constant. *(1 mark)*

(c) Hence find $\int_0^\infty x\mathrm{e}^{-2x}\,\mathrm{d}x$. *(2 marks)*

3 (a) Show that $y = x^3 - x$ is a particular integral of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$

(3 marks)

(b) By differentiating $(x^2 - 1)y = c$ implicitly, where y is a function of x and c is a constant, show that $y = \frac{c}{x^2 - 1}$ is a solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 0$$

(3 marks)

(c) Hence find the general solution of

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$

(2 marks)

4 (a) Use the series expansion

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$$

to write down the first four terms in the expansion, in ascending powers of x, of $\ln(1-x)$. *(1 mark)*

(b) The function f is defined by

$$\mathrm{f}(x) = \mathrm{e}^{\sin x}$$

Use Maclaurin's theorem to show that when $\mathrm{f}(x)$ is expanded in ascending powers of x:

(i) the first three terms are

$$1 + x + \frac{1}{2}x^2$$ *(6 marks)*

(ii) the coefficient of x^3 is zero. *(3 marks)*

(c) Find

$$\lim_{x \to 0} \frac{\mathrm{e}^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x}$$ *(4 marks)*

Turn over for the next question

5 (a) The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x,y) = x \ln x + \frac{y}{x}$$

and $$y(1) = 1$$

(i) Use the Euler formula

$$y_{r+1} = y_r + h\,f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(ii) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,f(x_r, y_r)$$

with your answer to part (a)(i) to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(4 marks)*

(b) (i) Show that $\frac{1}{x}$ is an integrating factor for the first-order differential equation

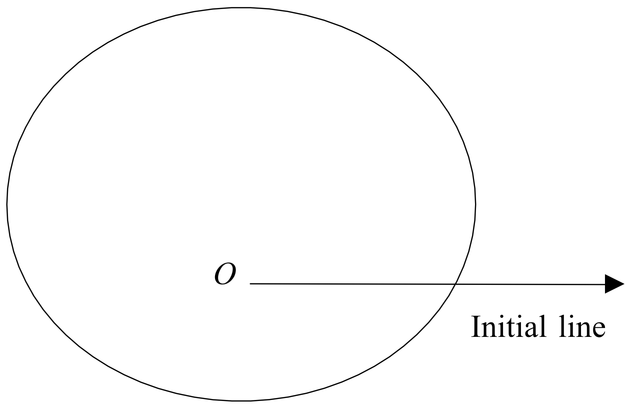
$$\frac{dy}{dx} - \frac{1}{x}y = x \ln x$$ *(3 marks)*

(ii) Solve this differential equation, given that $y = 1$ when $x = 1$. *(6 marks)*

(iii) Calculate the value of y when $x = 1.2$, giving your answer to three decimal places. *(1 mark)*

6 (a) A circle C_1 has cartesian equation $x^2 + (y-6)^2 = 36$. Show that the polar equation of C_1 is $r = 12\sin\theta$. *(4 marks)*

(b) A curve C_2 with polar equation $r = 2\sin\theta + 5,\ 0 \leqslant \theta \leqslant 2\pi$ is shown in the diagram.



Calculate the area bounded by C_2. *(6 marks)*

(c) The circle C_1 intersects the curve C_2 at the points P and Q. Find, in surd form, the area of the quadrilateral $OPMQ$, where M is the centre of the circle and O is the pole. *(6 marks)*

END OF QUESTIONS

Answer **all** questions.

1 (a) The polynomial $f(x)$ is defined by $f(x) = 3x^3 + 2x^2 - 7x + 2$.

(i) Find $f(1)$. *(1 mark)*

(ii) Show that $f(-2) = 0$. *(1 mark)*

(iii) Hence, or otherwise, show that

$$\frac{(x-1)(x+2)}{3x^3 + 2x^2 - 7x + 2} = \frac{1}{ax+b}$$

where a and b are integers. *(3 marks)*

(b) The polynomial $g(x)$ is defined by $g(x) = 3x^3 + 2x^2 - 7x + d$.

When $g(x)$ is divided by $(3x - 1)$, the remainder is 2. Find the value of d. *(3 marks)*

2 A curve is defined by the parametric equations

$$x = 3 - 4t \qquad y = 1 + \frac{2}{t}$$

(a) Find $\frac{dy}{dx}$ in terms of t. *(4 marks)*

(b) Find the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. *(4 marks)*

(c) Verify that the cartesian equation of the curve can be written as

$$(x-3)(y-1) + 8 = 0$$ *(3 marks)*

3 It is given that $3\cos\theta - 2\sin\theta = R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

(a) Find the value of R. *(1 mark)*

(b) Show that $\alpha \approx 33.7°$. *(2 marks)*

(c) Hence write down the maximum value of $3\cos\theta - 2\sin\theta$ and find a **positive** value of θ at which this maximum value occurs. *(3 marks)*

4 On 1 January 1900, a sculpture was valued at £80.

When the sculpture was sold on 1 January 1956, its value was £5000.

The value, £V, of the sculpture is modelled by the formula $V = A k^t$, where t is the time in years since 1 January 1900 and A and k are constants.

(a) Write down the value of A. *(1 mark)*

(b) Show that $k \approx 1.07664$. *(3 marks)*

(c) Use this model to:

(i) show that the value of the sculpture on 1 January 2006 will be greater than £200 000; *(2 marks)*

(ii) find the year in which the value of the sculpture will first exceed £800 000. *(3 marks)*

5 (a) (i) Obtain the binomial expansion of $(1-x)^{-1}$ up to and including the term in x^2. *(2 marks)*

(ii) Hence, or otherwise, show that

$$\frac{1}{3-2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of x. *(3 marks)*

(b) Obtain the binomial expansion of $\dfrac{1}{(1-x)^2}$ up to and including the term in x^2. *(2 marks)*

(c) Given that $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ can be written in the form $\dfrac{A}{(3-2x)} + \dfrac{B}{(1-x)} + \dfrac{C}{(1-x)^2}$,

find the values of A, B and C. *(5 marks)*

(d) Hence find the binomial expansion of $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$ up to and including the term in x^2. *(3 marks)*

Turn over for the next question

6 (a) Express $\cos 2x$ in the form $a\cos^2 x + b$, where a and b are constants. *(2 marks)*

(b) Hence show that $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{\pi}{a}$, where a is an integer. *(5 marks)*

7 The quadrilateral $ABCD$ has vertices $A(2, 1, 3)$, $B(6, 5, 3)$, $C(6, 1, -1)$ and $D(2, -3, -1)$.

The line l_1 has vector equation $\mathbf{r} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(a) (i) Find the vector $\overline{A}B$. *(2 marks)*

(ii) Show that the line AB is parallel to l_1. *(1 mark)*

(iii) Verify that D lies on l_1. *(2 marks)*

(b) The line l_2 passes through $D(2, -3, -1)$ and $M(4, 1, 1)$.

(i) Find the vector equation of l_2. *(2 marks)*

(ii) Find the angle between l_2 and AC. *(3 marks)*

8 (a) Solve the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

to find t in terms of x, given that $x = 70$ when $t = 0$. *(6 marks)*

(b) Liquid fuel is stored in a tank. At time t minutes, the depth of fuel in the tank is x cm. Initially there is a depth of 70 cm of fuel in the tank. There is a tap 6 cm above the bottom of the tank. The flow of fuel out of the tank is modelled by the differential equation

$$\frac{dx}{dt} = -2(x-6)^{\frac{1}{2}}$$

(i) Explain what happens when $x = 6$. *(1 mark)*

(ii) Find how long it will take for the depth of fuel to fall from 70 cm to 22 cm. *(2 marks)*

END OF QUESTIONS

Answer **all** questions.

1 Describe the geometrical transformation defined by the matrix

$$\begin{bmatrix} 0.6 & 0 & 0.8 \\ 0 & 1 & 0 \\ -0.8 & 0 & 0.6 \end{bmatrix}$$ *(3 marks)*

2 The matrices **P** and **Q** are defined in terms of the constant k by

$$\mathbf{P} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -1 & k \\ 5 & 3 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} 5 & 4 & 1 \\ 3 & k & -1 \\ 7 & 3 & 2 \end{bmatrix}$$

(a) Express $\det \mathbf{P}$ and $\det \mathbf{Q}$ in terms of k. *(3 marks)*

(b) Given that $\det(\mathbf{PQ}) = 16$, find the two possible values of k. *(4 marks)*

3 (a) The plane Π has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$.

(i) Find a vector which is perpendicular to both $\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$. *(2 marks)*

(ii) Hence find an equation for Π in the form $\mathbf{r} \cdot \mathbf{n} = d$. *(2 marks)*

(b) The line L has equation $\left(\mathbf{r} - \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix} \right) \times \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} = \mathbf{0}$.

Verify that $\mathbf{r} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} + t \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ is also an equation for L. *(2 marks)*

(c) Determine the position vector of the point of intersection of Π and L. *(4 marks)*

4 The vectors **a**, **b** and **c** are given by

$$\mathbf{a} = \mathbf{i} - \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \quad \text{and} \quad \mathbf{c} = 4\mathbf{i} - \mathbf{j} + 5\mathbf{k}$$

(a) (i) Evaluate $\begin{vmatrix} 1 & -1 & -1 \\ 2 & 3 & -1 \\ 4 & -1 & 5 \end{vmatrix}$. *(2 marks)*

(ii) Hence determine whether **a**, **b** and **c** are linearly dependent or independent. *(1 mark)*

(b) (i) Evaluate **b** . **c**. *(2 marks)*

(ii) Show that $\mathbf{b} \times \mathbf{c}$ can be expressed in the form $m\mathbf{a}$, where m is a scalar. *(2 marks)*

(iii) Use these results to describe the geometrical relationship between **a**, **b** and **c**. *(1 mark)*

(c) The points A, B and C have position vectors **a**, **b** and **c** respectively relative to an origin O. The points O, A, B and C are four of the eight vertices of a cuboid. Determine the volume of this cuboid. *(2 marks)*

5 The transformation T maps (x, y) to (x', y'), where

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) Describe the difference between *an invariant line* and *a line of invariant points* of T. *(1 mark)*

(b) Evaluate the determinant of the matrix $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and describe the geometrical significance of the result in relation to T. *(2 marks)*

(c) Show that T has a line of invariant points, and find a cartesian equation for this line. *(2 marks)*

(d) (i) Find the image of the point $(x, -x + c)$ under T. *(2 marks)*

(ii) Hence show that all lines of the form $y = -x + c$, where c is an arbitrary constant, are invariant lines of T. *(2 marks)*

(e) Describe the transformation T geometrically. *(3 marks)*

6 (a) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

(5 marks)

(b) (i) Hence, or otherwise, show that the system of equations

$$\begin{aligned} x + \quad y + \quad z &= p \\ 3x + \ 3y + 5z &= q \\ 15x + 15y + 9z &= r \end{aligned}$$

has no unique solution whatever the values of p, q and r. *(2 marks)*

(ii) Verify that this system is consistent when $24p - 3q - r = 0$. *(2 marks)*

(iii) Find the solution of the system in the case where $p = 1$, $q = 8$ and $r = 0$. *(5 marks)*

7 The matrix $\mathbf{M} = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -3 & 1 \\ 3 & -5 & 3 \end{bmatrix}$.

(a) Given that $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ are eigenvectors of **M**, find the eigenvalues corresponding to **u** and **v**. *(5 marks)*

(b) Given also that the third eigenvalue of **M** is 1, find a corresponding eigenvector. *(6 marks)*

(c) (i) Express the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in terms of **u** and **v**. *(1 mark)*

(ii) Deduce that $\mathbf{M}^n \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \lambda^n \mathbf{u} + \mu^n \mathbf{v}$, where λ and μ are scalar constants whose values should be stated. *(4 marks)*

(iii) Hence prove that, for all positive **odd** integers n,

$$\mathbf{M}^n \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2^n \\ 0 \\ 2^n \end{bmatrix}$$ *(3 marks)*

END OF QUESTIONS

mock papers 5

Leave blank

1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

Leave blank

2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2} .$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

Leave blank

3. $$f(x) = \ln(x+2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \; x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

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4.

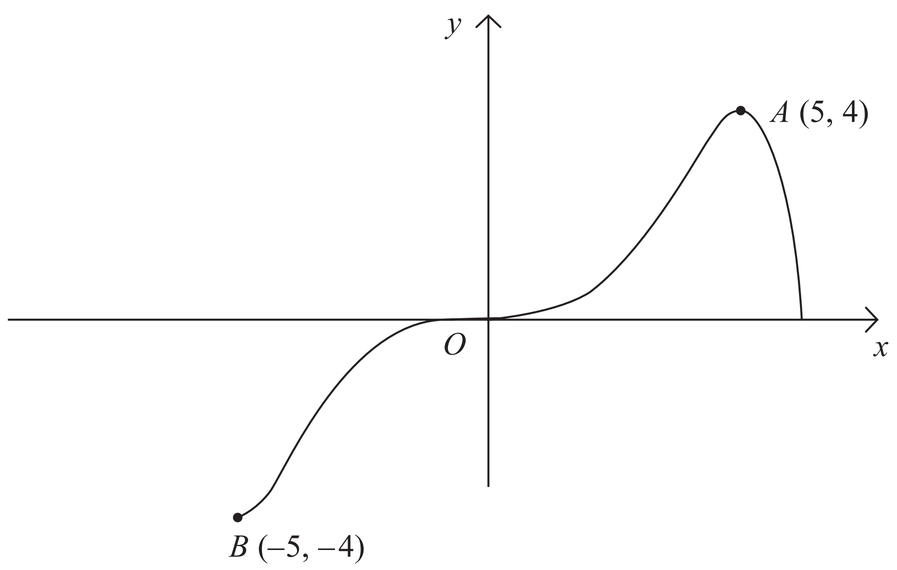

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$.
The curve passes through the origin *O* and the points *A*(5, 4) and *B*(–5, –4).

In separate diagrams, sketch the graph with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}(|x|)$,

(3)

(c) $y = 2\mathrm{f}(x+1)$.

(4)

On each sketch, show the coordinates of the points corresponding to *A* and *B*.

Leave blank

5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \qquad t \geqslant 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.
(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.
(4)

(c) Calculate the number of atoms that will be left when $t = 22\,920$.
(2)

(d) In the space provided on page 13, sketch the graph of R against t .
(2)

Leave blank

6. (a) Use the double angle formulae and the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2} .$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4 .$$

(3)

Leave blank

7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x,\ -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.

(4)

Leave blank

8. The functions f and g are defined by

$$f : x \mapsto 1 - 2x^3, \ \ x \in \mathbb{R}$$

$$g : x \mapsto \frac{3}{x} - 4, \ \ x > 0, \ \ x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$gf : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $gf(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = gf(x)$.

(5)

mock papers 6

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1.

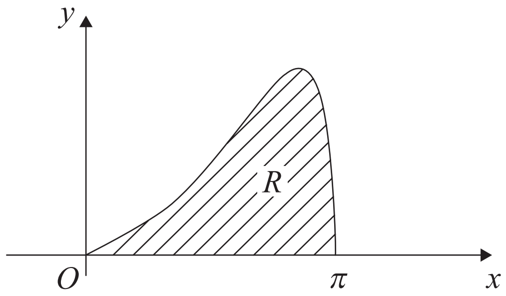

Figure 1

The curve shown in Figure 1 has equation $y = e^x \sqrt{(\sin x)}$, $0 \leqslant x \leqslant \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in Figure 1.

(a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(4)

Leave blank

2. (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction.

(5)

(b) Use your expansion, with a suitable value of x, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places.

(2)

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3.

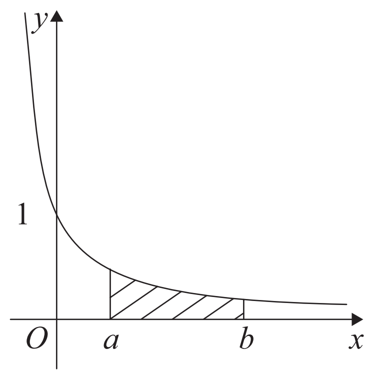

Figure 2

The curve shown in Figure 2 has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(5)

Leave blank

4. (i) Find $\int \ln(\frac{x}{2})\,dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx$.

(5)

Leave blank

5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

Leave blank

6. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l_1.

(2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C.

(c) Find the acute angle between l_1 and l_2.

(3)

(d) Find the position vector of the point C.

(4)

Leave blank

7.

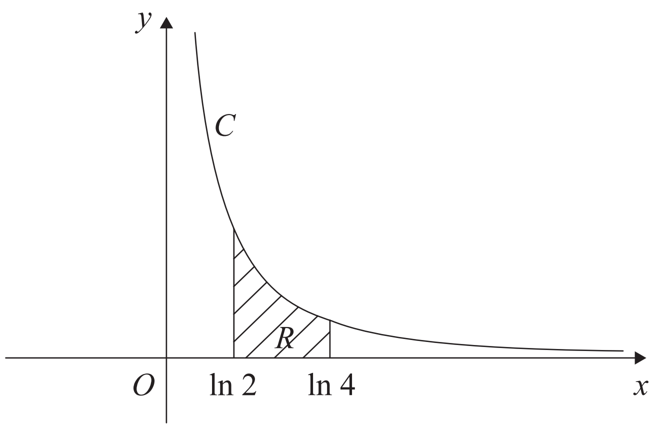

Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

Leave blank

8. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $cm^3\,s^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.}$$

(3)

When $h = 25$, water is leaking out of the hole at 400 $cm^3\,s^{-1}$.

(b) Show that $k = 0.02$

(1)

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$.

(6)

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)