Answer **all** questions.

1 Given that $y = 16x + x^{-1}$, find the two values of x for which $\dfrac{dy}{dx} = 0$. *(5 marks)*

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{1}{x^2 + 1}\,dx$$

giving your answer to four significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

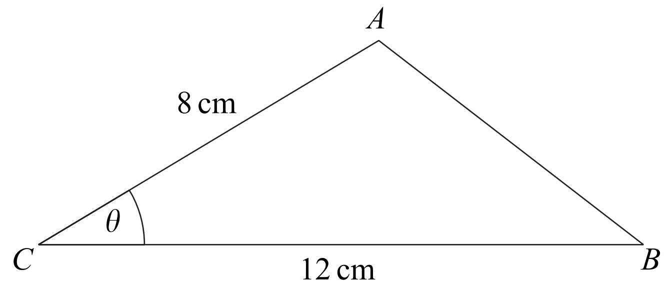
3 (a) Use logarithms to solve the equation $0.8^x = 0.05$, giving your answer to three decimal places. *(3 marks)*

(b) An infinite geometric series has common ratio r. The sum to infinity of the series is five times the first term of the series.

(i) Show that $r = 0.8$. *(3 marks)*

(ii) Given that the first term of the series is 20, find the least value of n such that the nth term of the series is less than 1. *(3 marks)*

4 The triangle ABC, shown in the diagram, is such that $AC = 8$ cm, $CB = 12$ cm and angle $ACB = \theta$ radians.

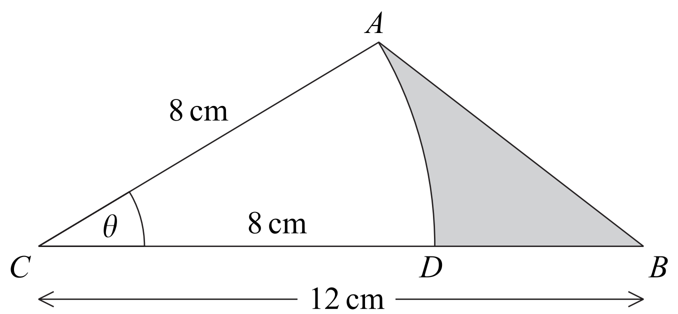


The area of triangle $ABC = 20\,\text{cm}^2$.

(a) Show that $\theta = 0.430$ correct to three significant figures. *(3 marks)*

(b) Use the cosine rule to calculate the length of AB, giving your answer to two significant figures. *(3 marks)*

(c) The point D lies on CB such that AD is an arc of a circle centre C and radius 8 cm. The region bounded by the arc AD and the straight lines DB and AB is shaded in the diagram.



Calculate, to two significant figures:

(i) the length of the arc AD; *(2 marks)*

(ii) the area of the shaded region. *(3 marks)*

5 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \qquad u_2 = 150 \qquad u_3 = 120$$

(a) Show that $p = 0.6$ and find the value of q. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L. Write down an equation for L and hence find the value of L. *(3 marks)*

6 (a) Describe the geometrical transformation that maps the curve with equation $y = \sin x$ onto the curve with equation:

(i) $y = 2\sin x$; *(2 marks)*

(ii) $y = -\sin x$; *(2 marks)*

(iii) $y = \sin(x - 30°)$. *(2 marks)*

(b) Solve the equation $\sin(\theta - 30°) = 0.7$, giving your answers to the nearest 0.1° in the interval $0° \leqslant \theta \leqslant 360°$. *(3 marks)*

(c) Prove that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$. *(4 marks)*

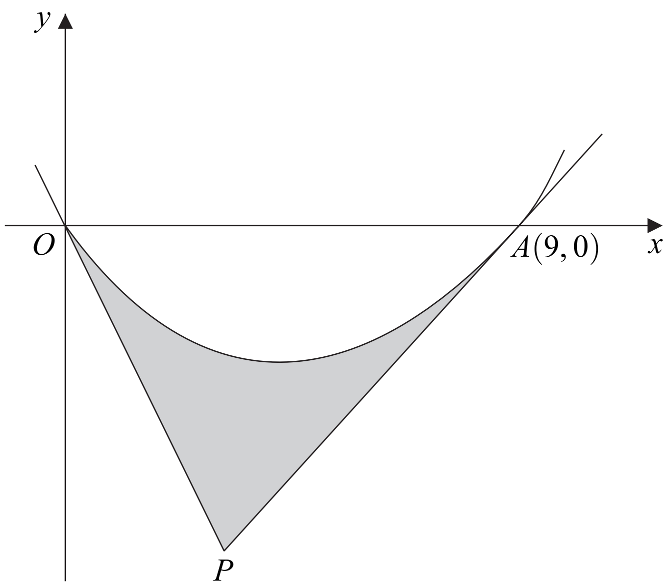
7 It is given that n satisfies the equation

$$2\log_a n - \log_a(5n - 24) = \log_a 4$$

(a) Show that $n^2 - 20n + 96 = 0$. *(3 marks)*

(b) Hence find the possible values of n. *(2 marks)*

8 A curve, drawn from the origin O, crosses the x-axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find $\frac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$. *(3 marks)*

(iii) Hence find the coordinates of the point P where the two tangents meet. *(3 marks)*

(c) Find $\int \left(x^{\frac{3}{2}} - 3x\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP. *(5 marks)*

END OF QUESTIONS

Answer **all** questions.

1 (a) Show that

$$\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{2r+1}{r^2(r+1)^2}$$ *(2 marks)*

(b) Hence find the sum of the first n terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$$ *(4 marks)*

2 The cubic equation

$$x^3 + px^2 + qx + r = 0$$

where p, q and r are real, has roots α, β and γ.

(a) Given that

$$\alpha + \beta + \gamma = 4 \qquad \text{and} \qquad \alpha^2 + \beta^2 + \gamma^2 = 20$$

find the values of p and q. *(5 marks)*

(b) Given further that one root is $3 + \mathrm{i}$, find the value of r. *(5 marks)*

3 The complex numbers z_1 and z_2 are given by

$$z_1 = \frac{1+\mathrm{i}}{1-\mathrm{i}} \qquad \text{and} \qquad z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$$

(a) Show that $z_1 = \mathrm{i}$. *(2 marks)*

(b) Show that $|z_1| = |z_2|$. *(2 marks)*

(c) Express both z_1 and z_2 in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(3 marks)*

(d) Draw an Argand diagram to show the points representing z_1, z_2 and $z_1 + z_2$. *(2 marks)*

(e) Use your Argand diagram to show that

$$\tan\frac{5}{12}\pi = 2 + \sqrt{3}$$ *(3 marks)*

4 (a) Prove by induction that

$$2 + (3 \times 2) + (4 \times 2^2) + \ldots + (n+1)\,2^{n-1} = n\,2^n$$

for all integers $n \geqslant 1$. *(6 marks)*

(b) Show that

$$\sum_{r=n+1}^{2n} (r+1)\,2^{r-1} = n\,2^n(2^{n+1} - 1)$$ *(3 marks)*

5 The complex number z satisfies the relation

$$|z + 4 - 4\mathrm{i}| = 4$$

(a) Sketch, on an Argand diagram, the locus of z. *(3 marks)*

(b) Show that the greatest value of $|z|$ is $4(\sqrt{2} + 1)$. *(3 marks)*

(c) Find the value of z for which

$$\arg(z + 4 - 4\mathrm{i}) = \tfrac{1}{6}\pi$$

Give your answer in the form $a + \mathrm{i}b$. *(3 marks)*

Turn over for the next question

6 It is given that $z = e^{i\theta}$.

(a) (i) Show that

$$z + \frac{1}{z} = 2\cos\theta$$ *(2 marks)*

(ii) Find a similar expression for

$$z^2 + \frac{1}{z^2}$$ *(2 marks)*

(iii) Hence show that

$$z^2 - z + 2 - \frac{1}{z} + \frac{1}{z^2} = 4\cos^2\theta - 2\cos\theta$$ *(3 marks)*

(b) Hence solve the quartic equation

$$z^4 - z^3 + 2z^2 - z + 1 = 0$$

giving the roots in the form $a + ib$. *(5 marks)*

7 (a) Use the definitions

$$\sinh\theta = \frac{1}{2}(e^{\theta} - e^{-\theta}) \qquad \text{and} \qquad \cosh\theta = \frac{1}{2}(e^{\theta} + e^{-\theta})$$

to show that:

(i) $2\sinh\theta\cosh\theta = \sinh 2\theta$; *(2 marks)*

(ii) $\cosh^2\theta + \sinh^2\theta = \cosh 2\theta$. *(3 marks)*

(b) A curve is given parametrically by

$$x = \cosh^3\theta, \quad y = \sinh^3\theta$$

(i) Show that

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \frac{9}{4}\sinh^2 2\theta\cosh 2\theta$$ *(6 marks)*

(ii) Show that the length of the arc of the curve from the point where $\theta = 0$ to the point where $\theta = 1$ is

$$\frac{1}{2}\left[(\cosh 2)^{\frac{3}{2}} - 1\right]$$ *(6 marks)*

END OF QUESTIONS

Mock papers 3

Leave blank

1. (a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$,

(ii) $x + 2$.

(3)

(b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0.$$

(4)

Leave blank

Question 1 continued

Q1

(Total 7 marks)

Leave blank

2. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum of the first 20 terms, giving your answer to the nearest whole number.

(2)

Leave blank

Question 2 continued

(Total 6 marks)

Q2

Leave blank

3. (a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

Leave blank

Question 3 continued

Q3

(Total 7 marks)

Turn over

Leave blank

4. (a) Show that the equation

$$3\sin^2\theta - 2\cos^2\theta = 1$$

can be written as

$$5\sin^2\theta = 3.$$

(2)

(b) Hence solve, for $0° \leqslant \theta < 360°$, the equation

$$3\sin^2\theta - 2\cos^2\theta = 1,$$

giving your answers to 1 decimal place.

(7)

Leave blank

Question 4 continued

(Total 9 marks)

Q4

Turn over

Leave blank

5. Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(6)

Leave blank

Question 5 continued

(Total 6 marks)

Q5

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6. **Figure 1**

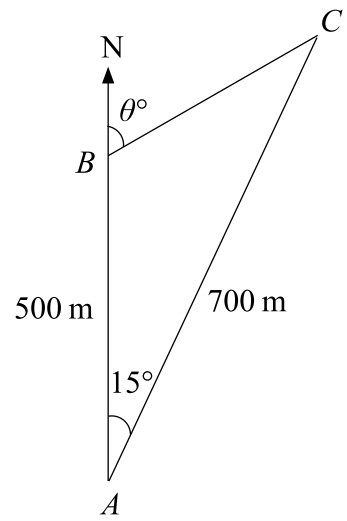

Figure 1 shows 3 yachts A, B and C which are assumed to be in the same horizontal plane. Yacht B is 500 m due north of yacht A and yacht C is 700 m from A. The bearing of C from A is 015°.

(a) Calculate the distance between yacht B and yacht C, in metres to 3 significant figures.
(3)

The bearing of yacht C from yacht B is $\theta°$, as shown in Figure 1.

(b) Calculate the value of θ.
(4)

Leave blank

Question 6 continued

(Total 7 marks)

Q6

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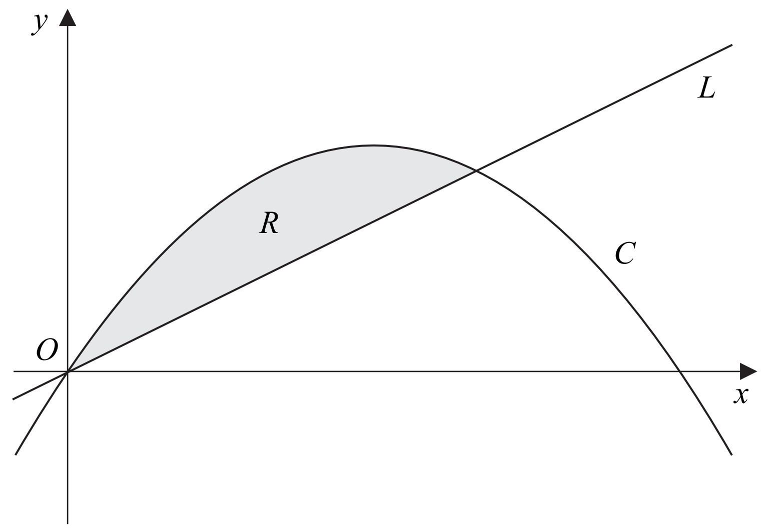
7. **Figure 2**



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.
(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).
(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.
(6)

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Question 7 continued

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Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 10 marks)

Leave blank

8. A circle C has centre M (6, 4) and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

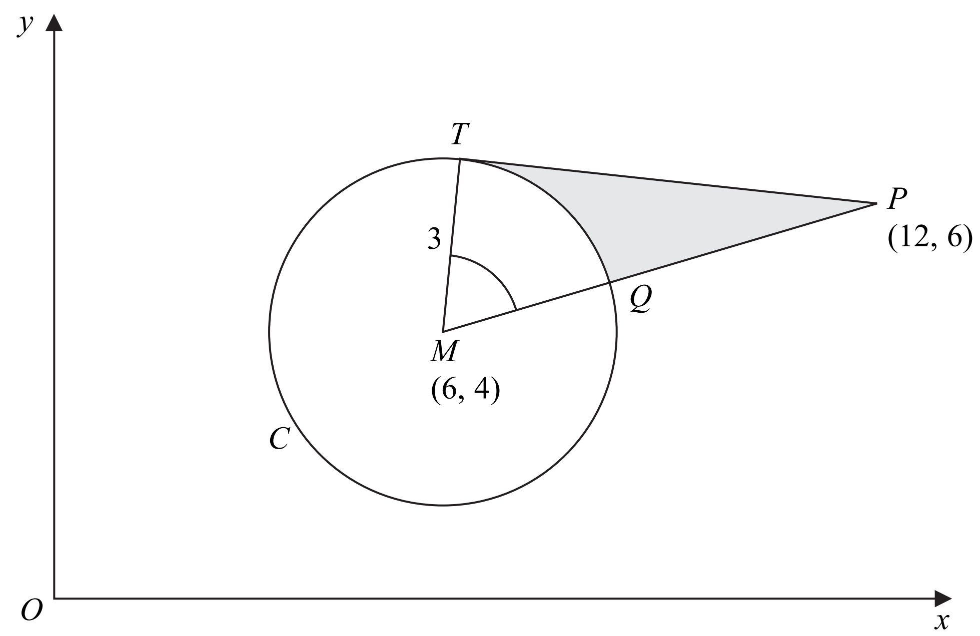

Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point P (12, 6). The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)

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Question 8 continued

Turn over

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Question 8 continued

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Question 8 continued

(Total 11 marks)

Q8

Turn over

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9. **Figure 4**

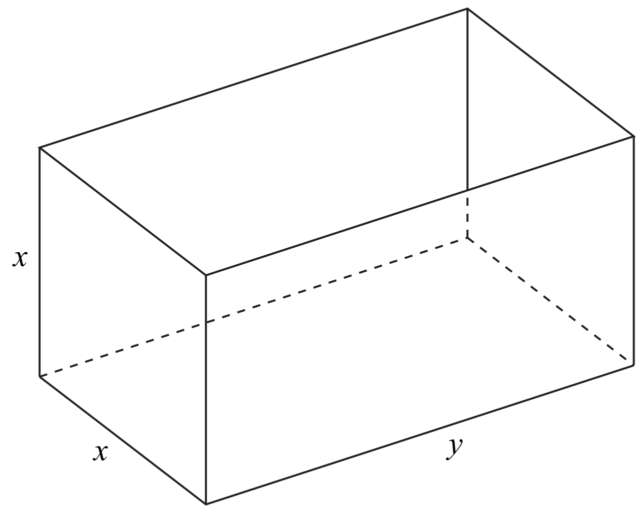

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is $100\,\text{m}^3$.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2 .$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

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Question 9 continued

Turn over

Leave blank

Question 9 continued

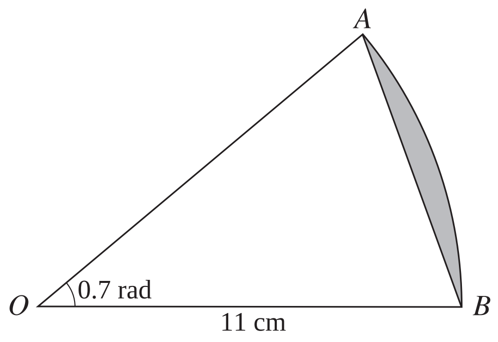
Q9

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

1



The diagram shows a sector AOB of a circle with centre O and radius 11 cm. The angle AOB is 0.7 radians. Find the area of the segment shaded in the diagram. [4]

2 Use the trapezium rule, with 3 strips each of width 2, to estimate the value of

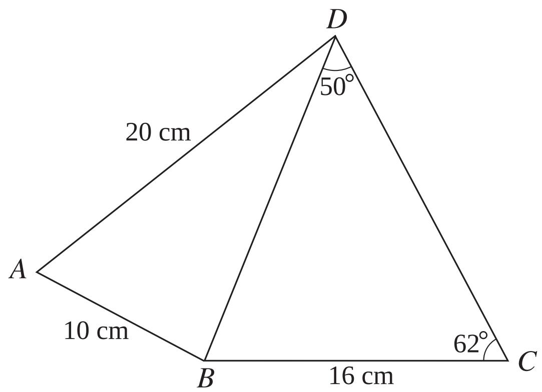
$$\int_1^7 \sqrt{x^2+3}\,dx.$$ [4]

3 Express each of the following as a single logarithm:

(i) $\log_a 2 + \log_a 3$, [1]

(ii) $2\log_{10} x - 3\log_{10} y$. [3]

4



In the diagram, angle $BDC = 50°$ and angle $BCD = 62°$. It is given that $AB = 10$ cm, $AD = 20$ cm and $BC = 16$ cm.

(i) Find the length of BD. [2]

(ii) Find angle BAD. [3]

5 The gradient of a curve is given by $\frac{dy}{dx} = 12\sqrt{x}$. The curve passes through the point (4, 50). Find the equation of the curve. [6]

6 A sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

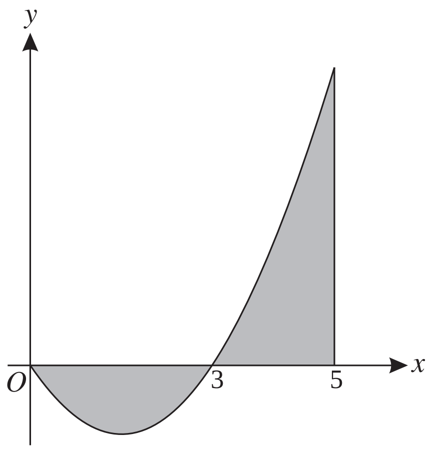
$$u_n = 2n + 5, \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_1, u_2 and u_3. [2]

(ii) State what type of sequence it is. [1]

(iii) Given that $\sum_{n=1}^{N} u_n = 2200$, find the value of N. [5]

7



The diagram shows part of the curve $y = x^2 - 3x$ and the line $x = 5$.

(i) Explain why $\int_0^5 (x^2 - 3x)\,dx$ does not give the total area of the regions shaded in the diagram. [1]

(ii) Use integration to find the exact total area of the shaded regions. [7]

8 The first term of a geometric progression is 10 and the common ratio is 0.8.

(i) Find the fourth term. [2]

(ii) Find the sum of the first 20 terms, giving your answer correct to 3 significant figures. [2]

(iii) The sum of the first N terms is denoted by S_N, and the sum to infinity is denoted by S_∞.

Show that the inequality $S_\infty - S_N < 0.01$ can be written as

$$0.8^N < 0.0002,$$

and use logarithms to find the smallest possible value of N. [7]

9 **(i)**

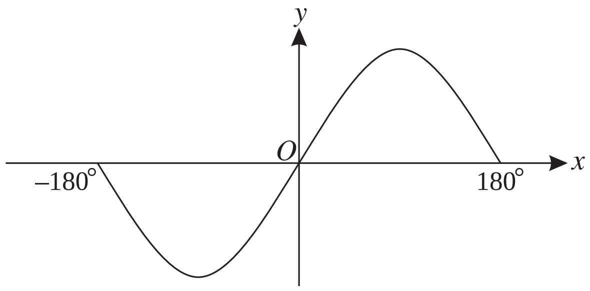


Fig. 1

Fig. 1 shows the curve $y = 2\sin x$ for values of x such that $-180^\circ \leqslant x \leqslant 180^\circ$. State the coordinates of the maximum and minimum points on this part of the curve. [2]

(ii)

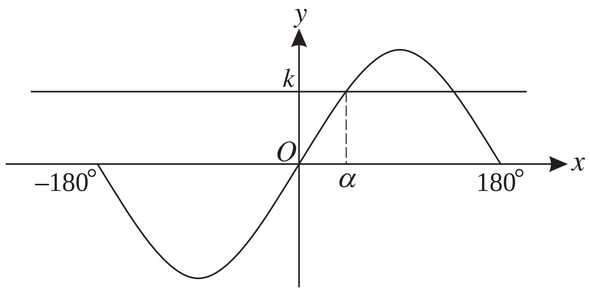


Fig. 2

Fig. 2 shows the curve $y = 2\sin x$ and the line $y = k$. The smallest positive solution of the equation $2\sin x = k$ is denoted by α. State, in terms of α, and in the range $-180^\circ \leqslant x \leqslant 180^\circ$,

(a) another solution of the equation $2\sin x = k$, [1]

(b) one solution of the equation $2\sin x = -k$. [1]

(iii) Find the x-coordinates of the points where the curve $y = 2\sin x$ intersects the curve $y = 2 - 3\cos^2 x$, for values of x such that $-180^\circ \leqslant x \leqslant 180^\circ$. [6]

10 **(i)** Find the binomial expansion of $(2x + 5)^4$, simplifying the terms. [4]

(ii) Hence show that $(2x + 5)^4 - (2x - 5)^4$ can be written as

$$320x^3 + kx,$$

where the value of the constant k is to be stated. [2]

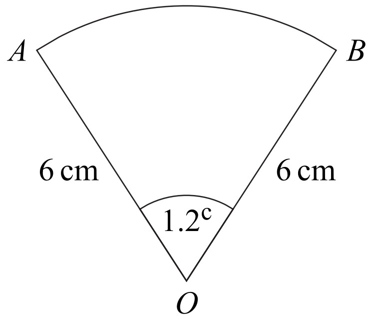
(iii) Verify that $x = 2$ is a root of the equation

$$(2x + 5)^4 - (2x - 5)^4 = 3680x - 800,$$

and find the other possible values of x. [6]

Answer **all** questions.

1 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle AOB is 1.2 radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) Find the perimeter of the sector OAB. *(3 marks)*

2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x}\,dx$$

giving your answer to three decimal places. *(4 marks)*

3 (a) Write down the values of p, q and r given that:

(i) $64 = 8^p$;

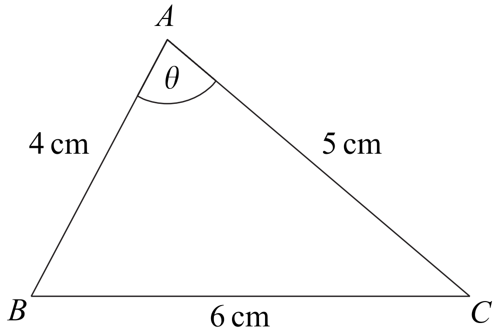
(ii) $\frac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. *(3 marks)*

(b) Find the value of x for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$ *(2 marks)*

4 The triangle ABC, shown in the diagram, is such that $BC = 6\,\text{cm}$, $AC = 5\,\text{cm}$ and $AB = 4\,\text{cm}$. The angle BAC is θ.



(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

(b) Hence use a trigonometrical identity to show that $\sin\theta = \frac{3\sqrt{7}}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

5 The second term of a geometric series is 48 and the fourth term is 3.

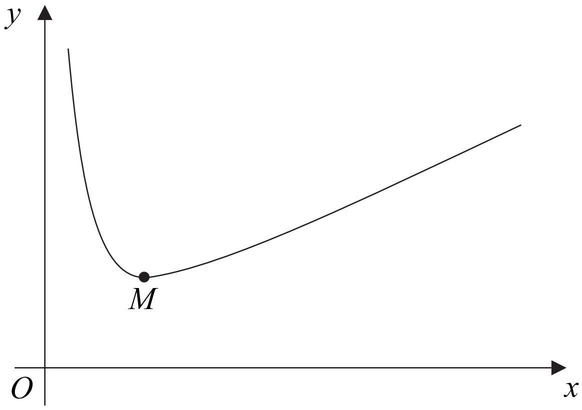
(a) Show that one possible value for the common ratio, r, of the series is $-\frac{1}{4}$ and state the other value. *(4 marks)*

(b) In the case when $r = -\frac{1}{4}$, find:

(i) the first term; *(1 mark)*

(ii) the sum to infinity of the series. *(2 marks)*

6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{dy}{dx}$. *(3 marks)*

(ii) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(iii) Find an equation of the normal to C at the point $(1, 6)$. *(4 marks)*

(b) (i) Find $\int \left(x + 1 + \frac{4}{x^2} \right) dx$. *(3 marks)*

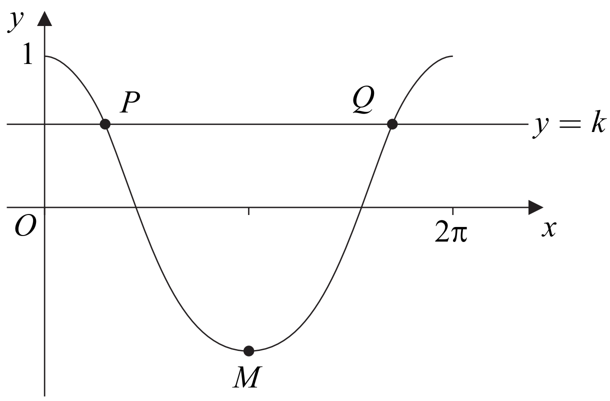
(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis. *(2 marks)*

7 (a) The first four terms of the binomial expansion of $(1 + 2x)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$. *(3 marks)*

8 (a) Solve the equation $\cos x = 0.3$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 2\pi$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 2\pi$, at the points P and Q. The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of Q in terms of π and α. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. *(2 marks)*

(d) Solve the equation $\cos 2x = \cos\dfrac{4\pi}{5}$ in the interval $0 \leqslant x \leqslant 2\pi$, giving the values of x in terms of π. *(4 marks)*

Turn over for the next question

9 (a) Solve the equation $3\log_a x = \log_a 8$. *(2 marks)*

(b) Show that

$$3\log_a 6 - \log_a 8 = \log_a 27$$ *(3 marks)*

(c) (i) The point $P(3, p)$ lies on the curve $y = 3\log_{10} x - \log_{10} 8$.

Show that $p = \log_{10}\left(\frac{27}{8}\right)$. *(2 marks)*

(ii) The point $Q(6, q)$ also lies on the curve $y = 3\log_{10} x - \log_{10} 8$.

Show that the gradient of the line PQ is $\log_{10} 2$. *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 (a) Given that

$$4\cosh^2 x = 7\sinh x + 1$$

find the two possible values of $\sinh x$. *(4 marks)*

(b) Hence obtain the two possible values of x, giving your answers in the form $\ln p$. *(3 marks)*

2 (a) Sketch on one diagram:

(i) the locus of points satisfying $|z - 4 + 2\mathrm{i}| = 2$; *(3 marks)*

(ii) the locus of points satisfying $|z| = |z - 3 - 2\mathrm{i}|$. *(3 marks)*

(b) Shade on your sketch the region in which

both $|z - 4 + 2\mathrm{i}| \leqslant 2$

and $|z| \leqslant |z - 3 - 2\mathrm{i}|$ *(2 marks)*

3 The cubic equation

$$z^3 + 2(1 - \mathrm{i})z^2 + 32(1 + \mathrm{i}) = 0$$

has roots α, β and γ.

(a) It is given that α is of the form $k\mathrm{i}$, where k is real. By substituting $z = k\mathrm{i}$ into the equation, show that $k = 4$. *(5 marks)*

(b) Given that $\beta = -4$, find the value of γ. *(2 marks)*

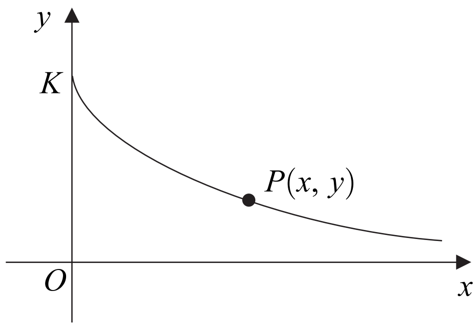
4 (a) Given that $y = \operatorname{sech} t$, show that:

(i) $\dfrac{dy}{dt} = -\operatorname{sech} t \tanh t$; *(3 marks)*

(ii) $\left(\dfrac{dy}{dt}\right)^2 = \operatorname{sech}^2 t - \operatorname{sech}^4 t$. *(2 marks)*

(b) The diagram shows a sketch of part of the curve given parametrically by

$$x = t - \tanh t \qquad y = \operatorname{sech} t$$



The curve meets the y-axis at the point K, and $P(x, y)$ is a general point on the curve. The arc length KP is denoted by s. Show that:

(i) $\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2 = \tanh^2 t$; *(4 marks)*

(ii) $s = \ln \cosh t$; *(3 marks)*

(iii) $y = e^{-s}$. *(2 marks)*

(c) The arc KP is rotated through 2π radians about the x-axis. Show that the surface area generated is

$$2\pi(1 - e^{-s})$$ *(4 marks)*

Turn over for the next question

5 (a) Prove by induction that, if n is a positive integer,

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$ *(5 marks)*

(b) Find the value of $\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^6$. *(2 marks)*

(c) Show that

$$(\cos\theta + \mathrm{i}\sin\theta)(1 + \cos\theta - \mathrm{i}\sin\theta) = 1 + \cos\theta + \mathrm{i}\sin\theta$$ *(3 marks)*

(d) Hence show that

$$\left(1 + \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^6 + \left(1 + \cos\frac{\pi}{6} - \mathrm{i}\sin\frac{\pi}{6}\right)^6 = 0$$ *(4 marks)*

6 (a) Find the three roots of $z^3 = 1$, giving the non-real roots in the form $\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta \leqslant \pi$. *(2 marks)*

(b) Given that ω is one of the non-real roots of $z^3 = 1$, show that

$$1 + \omega + \omega^2 = 0$$ *(2 marks)*

(c) By using the result in part (b), or otherwise, show that:

(i) $\dfrac{\omega}{\omega + 1} = -\dfrac{1}{\omega}$; *(2 marks)*

(ii) $\dfrac{\omega^2}{\omega^2 + 1} = -\omega$; *(1 mark)*

(iii) $\left(\dfrac{\omega}{\omega + 1}\right)^k + \left(\dfrac{\omega^2}{\omega^2 + 1}\right)^k = (-1)^k\, 2\cos\frac{2}{3}k\pi$, where k is an integer. *(5 marks)*

7 (a) Use the identity $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$ with $A = (r+1)x$ and $B = rx$ to show that

$$\tan rx \tan(r+1)x = \frac{\tan(r+1)x}{\tan x} - \frac{\tan rx}{\tan x} - 1$$ *(4 marks)*

(b) Use the method of differences to show that

$$\tan\frac{\pi}{50}\tan\frac{2\pi}{50} + \tan\frac{2\pi}{50}\tan\frac{3\pi}{50} + \ldots + \tan\frac{19\pi}{50}\tan\frac{20\pi}{50} = \frac{\tan\dfrac{2\pi}{5}}{\tan\dfrac{\pi}{50}} - 20$$ *(5 marks)*

END OF QUESTIONS