CIE-general-physics-and-Newtonian-mechanics

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Section I General Physics

Chapter 1 Physics and physical measurement

1.1 Measurement and uncertainties

1-1-1 SI units

In general, a physical quantity is made up of two parts: numerical magnitude + unit.

For example, the distance from school to your home is 1000 m. then 1000 is the numerical magnitude and m (meter) is its unit.

(i) SI Units

There are seven SI Units shown in **table 1.1**:

Table 1.1 SI Units

Base quantity	SI Units	
	Name	Symbol
Mass	Kilogram	kg
Length	Meter	m
Time	Second	s
Thermodynamic temperature	Kelvin	K
Electric current	Ampere	A
Amount of substance	Mole	mol
Luminous intensity	candela	cd

Other units are derived from these: (**table 1.2**)

Table 1.2 Examples of SI derived Units

Physical quantity	Defining equation	Derived unit	Special symbol
Speed	Distance × time	$m \cdot s^{-1}$	--
Acceleration	Speed/time	$m \cdot s^{-2}$	--
Force	mass×acceleration	$kg \cdot m \cdot s^{-2}$	N(Newton)
Work	force×distance	$N \cdot m$	J(joule)
Density	Mass/volume	$kg \cdot m^{-3}$	--
Charge	current×time	$A \cdot s$	C(coulomb)
Pressure	Force/area	$N \cdot m^{-2}$	Pa(Pascal)
Resistance	Voltage/current	$V \cdot A^{-1}$	Ω (ohm)
voltage	Energy/charge	$J \cdot C^{-1}$	V(volt)

1-1-2 Prefixes

Prefixes can be added to SI and derived units to make larger or smaller units as shown in **table 1.3**:

Table 1.3 Prefixes

Value	prefix	symbol	Value	prefix	symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zeta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	**m**
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	**M**	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

For example:

1 kilometer = 1 km = 10^3 m

1 microgram = 1 μ g = 10^{-6} g

1 mega meter = 1 **M** m = 10^6 m

1 millimeter = 1 **m** m = 10^{-3} m

1.2 measurement

1-2-1 uncertainties

There is an uncertainty associated with every measurement. Uncertainty arises from different sources.

(i) Systematic uncertainties:

That arises from the measuring system.

(ii) Random uncertainties:

That arises from the sensitivity of the measuring instrument or the readings obtained.

For example, measure the length of a desk using a tape, the readings:

1.5 m 1.6 m 1.7 m 1.4 m 1.3 m

You can find the average value of above readings: 2.5 m, and then the length of the desk can be written as

$L = 2.5 \pm 0.2$

Where 0.2 m is the uncertainty

(iii) Percentage uncertainty

Percentage uncertainty is the ratio of the uncertainty to the measured value, multiplied by 100. For example, in the measurement above, the percentage uncertainty is given by

$$\frac{0.2}{2.5}\times 100 = 8\%$$

(iv) Scientific notation

Numbers written in "powers of 10" are in scientific notation. For example, 4850000 can be written as 4.85×10^{6}. 0.00023 can be written as 2.3×10^{-4}.

The advantage of scientific notation is that it can clearly express the significant figures of the numbers.

For example, if we know that 4850000 has three significant numbers, it can be written as 4.85×10^{6}. If it has 5 significant numbers, it can be written as 4.8500×10^{6}.

(v) Combining uncertainties

Say that there are some measurements A, B, C…

★And if A = B + C or A = B – C

Then:

Uncertainty in A = Uncertainty in B + Uncertainty in C

Note: of course, if A = B + C – F – E, then

Uncertainty in A = Uncertainty in B + Uncertainty in C + Uncertainty in F + Uncertainty in E

★if $A = B\times C$ or $A = B/C$, then

Percentage Uncertainty in A = **Percentage** Uncertainty in B + **Percentage** Uncertainty in C

Note: if $A = B\times C\times E$ or $A = \frac{B}{C}\times E$, then

Percentage Uncertainty in A = **Percentage** Uncertainty in B + **Percentage** Uncertainty in C + **Percentage** Uncertainty in E

1-2-2 Uncertainties in graphs

A car accelerates from stationary, and here are some readings of speed at different time shown in the graph below:

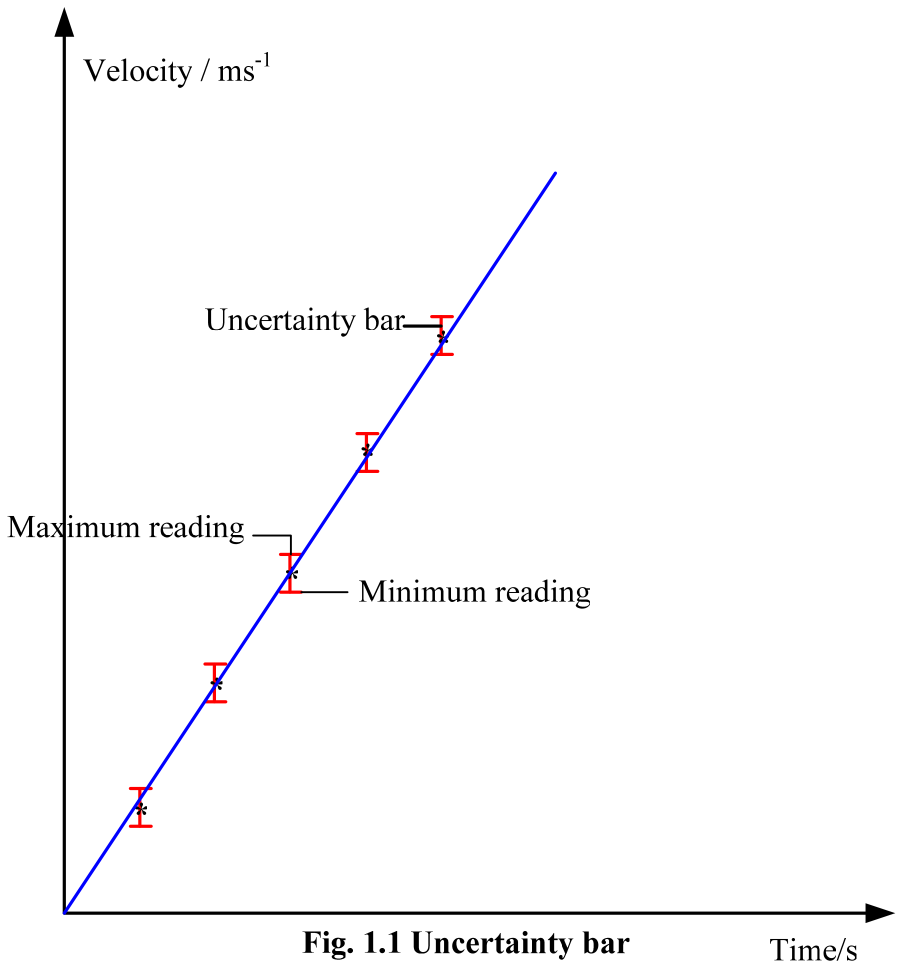


Fig. 1.1 Uncertainty bar

Say that the readings arise from random uncertainty; the uncertainty is estimated in the readings. And then calculate the maximum and minimum values for that reading, shown in the graph (**Fig. 1.1**).

In the graph, the short、 vertical lines are called uncertainty bars.

1.3 6 Worked examples

1. The length of each side of a sugar cube is measured as 10 mm with an uncertainty of ± 2 mm.

What is the absolute uncertainty in the volume of the sugar cube?

Solution:

Present uncertainty of each side is given by

$$\frac{2}{10}\times 100 = 20\%$$

The volume of the sugar cube is $10\times 10\times 10 = 1000\ mm^3$

Thus, the combining uncertainty is 20% + 20% + 20% = 60%
Therefore, the absolute uncertainty is given by
$\Delta e = 1000\,mm^3 \times 60\% = 600mm^3$

2. The current in a resistor is measured as 2.00 A ± 0.02 A. what is the (absolute) uncertainty and the percentage uncertainty in the current?
Solution:
From 2.00 A ± 0.02 A, the uncertainty is ± 0.02 A.
The percentage uncertainty is given by
$$\frac{\pm 0.02}{2.00} \times 100 = \pm 1\%$$

3. A volume is measured to be 25 mm^3, express the volume in m^3.
Solution:
$1mm = 10^{-3}\,m$, thus
$$(1mm)^3 = \left(10^{-3}\,m\right)^3 = 10^{-9}\,m^3$$

4. When measuring the acceleration of free fall at the surface of the Earth the following results (**table 1.4**) were obtained. The results are accurate? Or precise?

Table 1.4

Acceleration of free fall / ms $^{-2}$
7.69
7.70
7.69
7.71
7.66

Solution:
We know that the acceleration of free fall is 9.801 ms $^{-2}$, thus the results are inaccurate. But the results are to be in conformity with one another, the results are precise.

5. The frequency *f* of the fundamental vibration of a standing wave of fixed length is measured for different values of the tension *T* in the string, using the apparatus shown (**Fig. 1.2**).

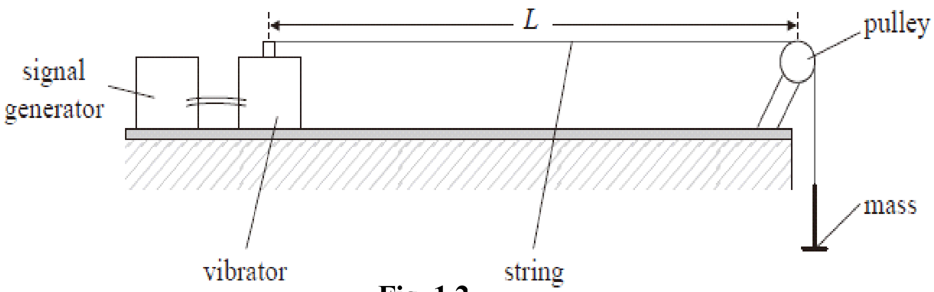


Fig. 1.2

In order to find the relationship between the speed v of the wave and the tension T in the string, the speed v is calculated from the relation

$$v = 2fL$$

Where L is the length of the string.

The data points are shown plotted on the axes below (**Fig. 1.3**). The uncertainty in v is ±5 ms^{-1} and the uncertainty in T is negligible.

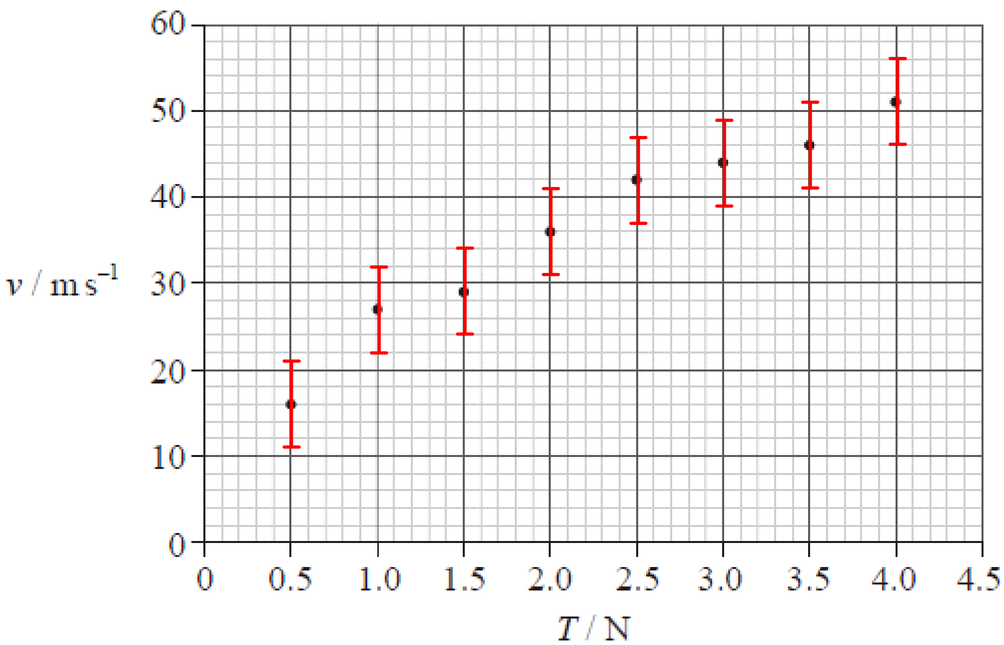


Fig. 1.3 Data points

(a) Draw error bars on the first and last data points to show the uncertainty in speed v.

(b) The original hypothesis is that the speed is directly proportional to the tension T.

Explain why the data do not support this hypothesis.

Solution:

If the speed is directly proportional to the tension T, it is a straight line goes through the origin and the error. But on the graph, it can not be drawn.

(c) It is suggested that the relationship between speed and tension is of the form

$$v = k\sqrt{T}$$

where k is a constant.

To test whether the data support this relationship, a graph of v^2 against T is plotted as shown below (**Fig. 1.4**).

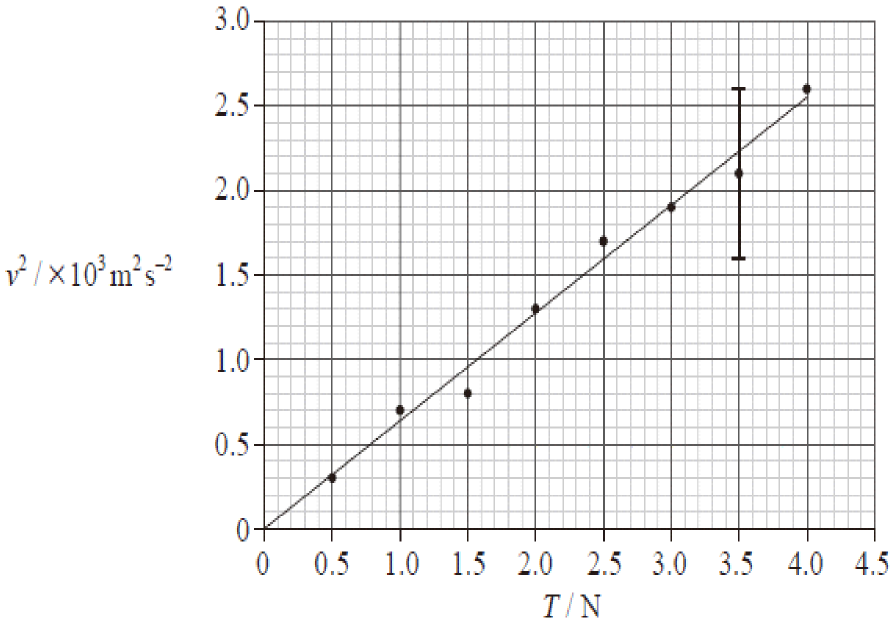


Fig. 1.4 graph of v^2 against T

The best-fit line shown takes into account the uncertainties for each data point. The uncertainty in v^2 for $T = 3.5$ N is shown as an error bar on the graph.

(i) State the value of the uncertainty in v^2 for $T = 3.5$ N.

Solution:

From the graph, the uncertainty in v^2 for $T = 3.5$ N is $500 m^2 s^{-2}$

(ii) At $T = 1.0$ N the speed $v = 27 \pm 5$ ms^{-1}. Calculate the uncertainty in v^2.

Strategy:

★if $A = B \times C$ or $A = B/C$, then

Percentage Uncertainty in A = **Percentage** Uncertainty in B + **Percentage** Uncertainty in C

Solution:

Percentage Uncertainty in v^2 = **Percentage** Uncertainty in v + **Percentage** Uncertainty in v

Thus, **Percentage** Uncertainty in v^2 = $2 \times \frac{5}{27} \times 100 = 37\%$

Therefore, the uncertainty in v^2 is $\Delta v^2 = 27 \times 27 \times 37\% = 270 m^2 s^{-2}$

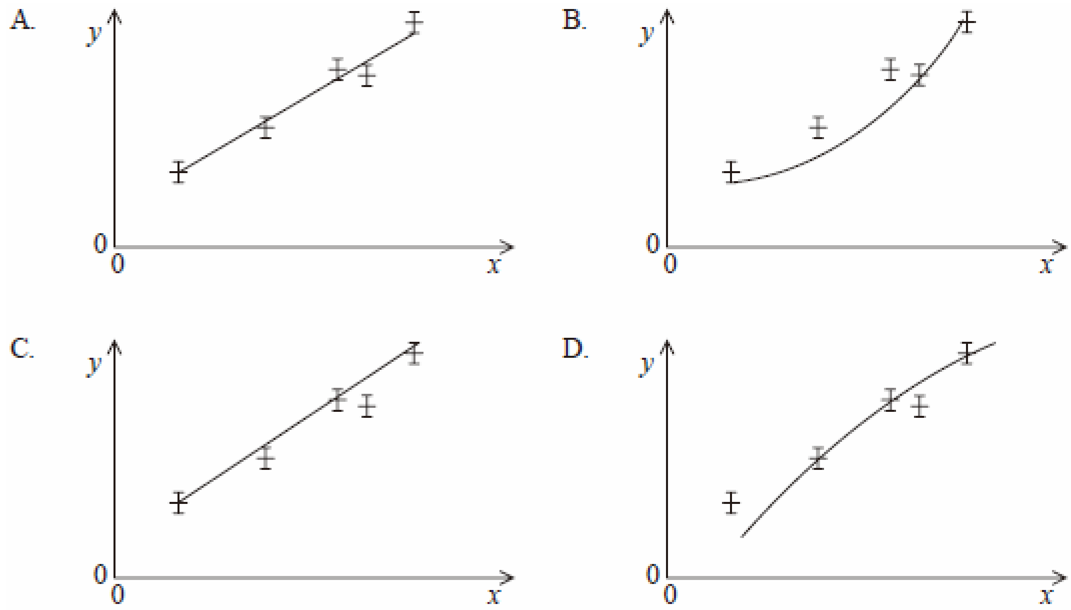
(d) Use the graph in (c) to determine k without its uncertainty.

Solution:

The gradient of the straight line represents k^2. Thus

$k^2 = \frac{2.55 \times 10^3}{4}$, gives $k = 25$

6. Which of the following graphs shows the best-fit line for the plotted points?



Solution:

The best-fit line should be goes through the error bar. Thus choose **(A)**

1.4 Vectors and scalars

1.4.1 Addition of vectors

1.1 Definition of scalars and vectors

Scalar: quantity has direction only.

Examples of scalar: mass, temperatures, volume, work…

Vector: quantity both has magnitude and direction

Examples of vectors: force, acceleration, displacement, velocity, momentum…

Representation of vectors: any vectors can be represented by a straight line with an arrow whose length represents the magnitude of the vectors, and the direction of the arrow gives the direction of the vectors.

Vector Notation: use an arrow $\vec{A}$, $\vec{S}$, $\vec{B}$…

Or use the bold letter **A, B, S**…

When considering the magnitude of a vector only, we can use the italic letter *A, B, S*…

1.2 Addition of vectors:

When adding vectors, the units of the vectors must be the same, the direction must be taken into account.

Addition Principles:

i : if two vectors are in the same direction: the magnitude of the resultant vector is equal to the sum of their magnitudes, in the same direction.

ii : if two vectors are in the opposite direction: the magnitude of the resultant vector is equal to the difference of the magnitude of the two vectors and is in the direction of the greater vector.

iii: if two vectors are placed tail-to-tail at an angle θ, it can also be represented as a closed triangle (**Fig. 2.1**).

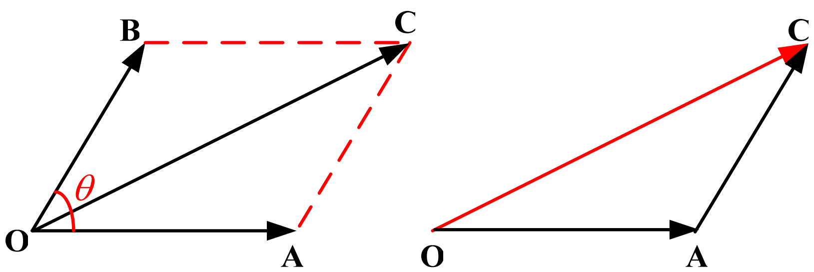


Fig. 2.1 addition principles of vectors

$\overrightarrow{OA}+\overrightarrow{AC}=\overrightarrow{OC}$ Because $\overrightarrow{OB}=\overrightarrow{AC}$

$\overrightarrow{OA}$ *and* $\overrightarrow{OB}$ are placed tail to tail to form two adjacent sides of a parallelogram and the diagonal $\overrightarrow{OC}$ gives the sum of the vectors $\overrightarrow{OA}$ *and* $\overrightarrow{OB}$. This is also called as 'parallelogram rule of vector addition.

Addition Methods:

(i): Graphical Methods----using scale drawings

For example:

$\mathbf{F_1}$ and $\mathbf{F_2}$ are at right angle, and $\mathbf{F_1}$ = 3 N, $\mathbf{F_2}$ = 4 N, determine the resultant force **F** (**Fig. 2.2**).

Let 1cm=1N

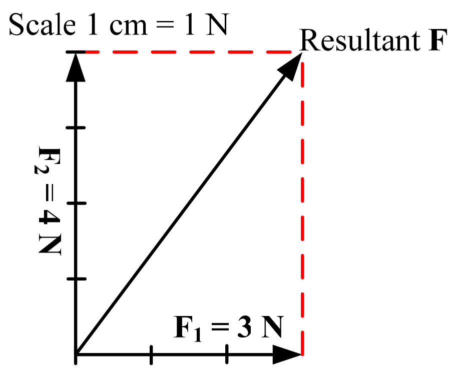


Fig. 2.2 addition methods—scale drawings

Measure the length of the resultant vector, we get length = 5cm, then resultant force, F = 5 N.

(ii) Algebraic Methods

For example:

$\mathbf{F_1}$ and $\mathbf{F_2}$ are at right angle, and $\mathbf{F_1}$ = 3 N, $\mathbf{F_2}$ = 4 N, determine the resultant force **F (Fig. 2.3)**.

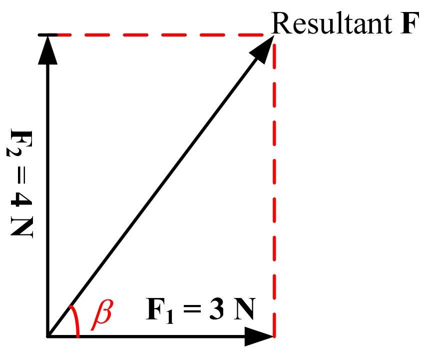


Fig. 2.3 Algebraic methods

Using the Pythagorean Theorem:

Magnitude of the resultant force, $F = \sqrt{F_1^2 + F_2^2} = \sqrt{3^2 + 4^2} = 5N$

The angle β between F and F_1 is given by:

$$\tan\beta = \frac{F_2}{F_1} = \frac{4}{3}$$

Or

$$\sin\beta = \frac{F_2}{F} = \frac{4}{5}$$

Or

$$\cos\beta = \frac{F_1}{F} = \frac{3}{5}$$

1.4.2 Resolving a vector into two perpendicular components

For example, for a vector $\overrightarrow{OC}$, θ is known, resolving it horizontally and vertically (**Fig. 2.4**).

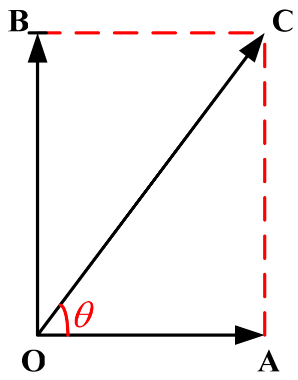


Fig. 2.4 Resolving a vector into two perpendicular components

Magnitude of Horizontally component $OA = OC\cos\theta$

Magnitude of vertically component $OB = OC\sin\theta$

Thus, a force can be resolved into two perpendicular components (**Fig. 2.5**): F and θ are known.

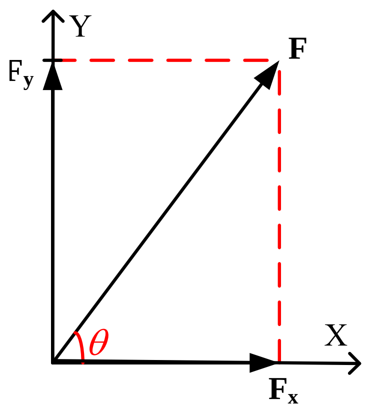


Fig. 2.5 Resolving a force into two perpendicular components

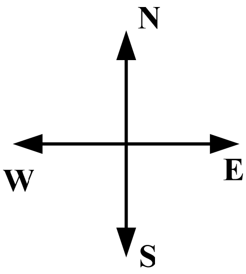
$F_x = F\cos\theta$ $\qquad$ $F_y = F\sin\theta$

1.4.3 10 Worked examples

1. Representation of vectors:

(i) A displacement of 500 m due east

Represent the displacement:



Let scale: $1cm = 100m$

Then

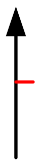
Note: of course you can also let scale: $1cm = 250m$

Then:

(ii) A force of $\vec{F} = 100N$ (or **F**=100N) due north.

Let scale: $1cm = 50N$

Then



2. Addition of the vectors $\vec{F}_1 = 3.5N$, $\vec{F}_2 = 7.5N$

(i) $\vec{F}_1$ *and* $\vec{F}_2$ are in the same direction.

Magnitude of the resultant $\mathrm{F} = F_1 + F_2 = 11N$

Direction: the same direction of $\vec{F}_1$ *and* $\vec{F}_2$

(ii) $\vec{F}_1$ *and* $\vec{F}_2$ are in the opposite direction.

Magnitude of the resultant $\mathrm{F} = F_2 - F_1 = 4N$

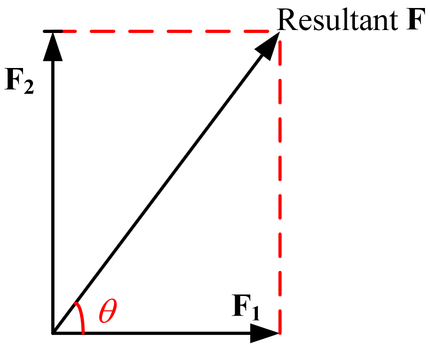
Direction: the same direction of $\vec{F}_2$

(iii) $\vec{F}_1$ *and* $\vec{F}_2$ are at right angles to each other.

Using the algebraic methods:

Magnitude of the resultant:

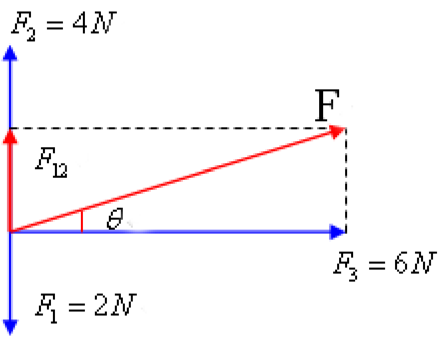
$\mathrm{F} = \sqrt{F_1^2 + F_2^2} = \sqrt{12.25 + 68.5} = 9N$



Direction:

$$\tan\theta = \frac{F_2}{F_1} = \frac{7.5}{3.5} = 2.14 \quad Then \quad \theta = \arctan 2.14$$

3. Calculate the resultant force of F_1, F_2, F_3



Strategy: ① calculate the resultant of F_1 *and* F_2

$$F_{12} = F_2 - F_1 = 2N$$

② calculate the resultant force of F_{12} *and* F_3, that is the resultant force of F_1, F_2 *and* F_3

Magnitude of resultant force:

$$F = \sqrt{F_{12}^2 + F_3^2} = \sqrt{2^2 + 6^2} = 6.32N$$

Direction:

$$\tan\theta = \frac{F_{12}}{F_3} = \frac{2}{6} = \frac{1}{3}$$

$$\theta = \arctan\frac{1}{3}$$

4. A crane is used to raise one end of a steel girder off the ground, as shown in **fig. 2.6**. When the cable attached to the end of the girder is at 20°to the vertical, the force of the cable on the girder is 6.5kN. Calculate the horizontal and vertical components of this force.

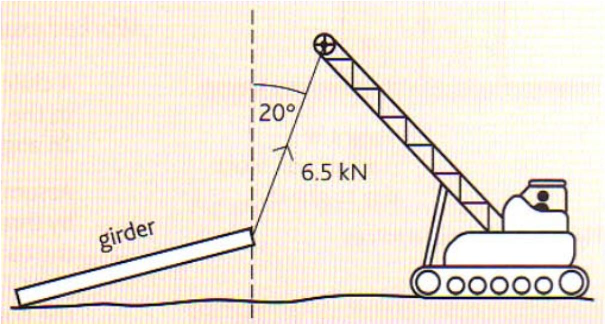
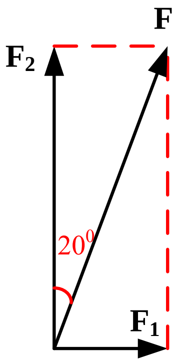


Fig. 2.6

Strategy:

Resolving the force F = 6.5 kN



$F_1 = F\sin 20^o = 6.5\sin 20^o = 2.2kN$ (Horizontal components of the force)

$F_2 = F\cos 20^o = 6.5\cos 20^o = 6.1kN$ (Vertical components of the force)

5. (a) (i) State what is meant by a scalar quantity.

Scalar quantity: quantity has direction only.

(ii) State two examples of scalar quantities.

Example 1: mass

Example 2: temperatures

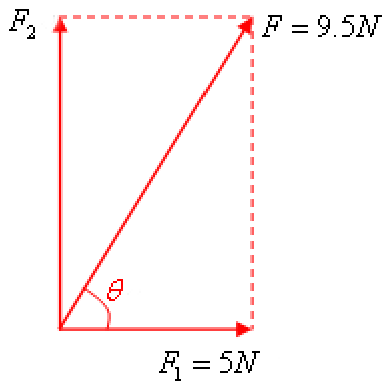
(b) An object is acted upon by two forces at right angles to each other. One of the forces has a magnitude of 5.0 N and the resultant force produced on the object is 9.5 N.

Determine

(i) The magnitude of the other force,

Strategy: adding of vectors, using the Algebraic Methods

Draw the forces below:



And $F_1^2 + F_2^2 = F^2$

$5^2 + F_2^2 = 9.5^2$

So, $F_2 = 8.1N$

(ii) The angle between the resultant force and the 5.0 N force.

$$\cos\theta = \frac{F_1}{F} = \frac{5}{9.5} = 0.53$$

So $\theta = \arccos 0.53 = 58^0$

6. (a) State the difference between vector and scalar quantities.

Answers: Vector quantities have direction and scalar quantities do not.

(b) State one example of a vector quantity (other than force) and one example of a scalar quantity.

Vector quantity: velocity, acceleration.

Scalar quantity: mass, temperature.

(c) A 12.0 N force and a 8.0 N force act on a body of mass 6.5 kg at the same time. For this body, calculate

(i) The maximum resultant acceleration that it could experience,

Strategy: by the Newton's second law, $F = ma$, the maximum resultant acceleration when the body has the maximum resultant force. And when the two forces are at the same direction, the body has the maximum resultant force.

So, resultant force, $F = F_1 + F_2 = 12 + 8 = 20N$

So the maximum resultant acceleration, $a = \frac{F}{m} = \frac{20}{6.5} = 3.1ms^{-2}$

(ii) The minimum resultant acceleration that it could experience.

Strategy: by the Newton's second law, $F = ma$, the minimum resultant acceleration when the body has the minimum resultant force. And when the two forces are at the opposite direction, the body has the minimum resultant force.

That is, resultant force, $F = F_1 - F_2 = 12 - 8 = 4N$

So the minimum resultant acceleration, $a = \frac{F}{m} = \frac{4}{6.5} = 0.62ms^{-2}$

7. **Figure 2.7** shows a uniform steel girder being held horizontally by a crane. Two cables are attached to the ends of the girder and the tension in each of these cables is T.

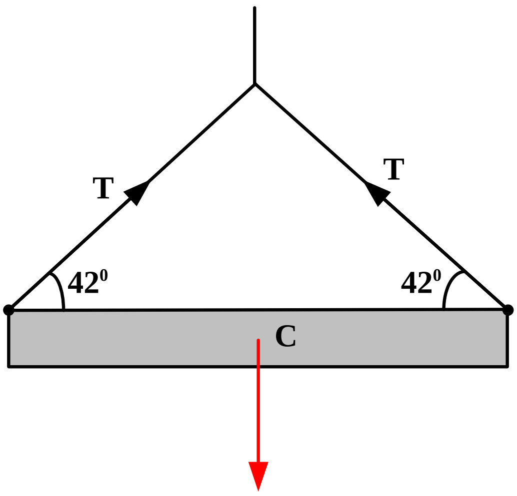


Fig. 2.7

(a) If the tension, T, in each cable is 850 N, calculate

(i) The horizontal component of the tension in each cable,

Answers: $T_h = T\cos 42 = 850 \times \cos 42 = 632\ N$

(ii) The vertical component of the tension in each cable,

$T_v = T\sin 42 + T\sin 42 = 1138N$

(iii) The weight of the girder.

Strategy: the girder is at a uniform state, so the weight of the girder is equal to the vertical component of the tension.

So weight, $W = T_v = 1138N$

(b) On **Figure 2.7** draw an arrow to show the line of action of the weight of the girder.

8. Which of the following contains three scalar quantities?

A	Mass	Charge	Speed
B	Density	Weight	Mass

C	Speed	Weight	Charge
D	Charge	Weight	Density

Solution:

Scalar: quantity has direction only.

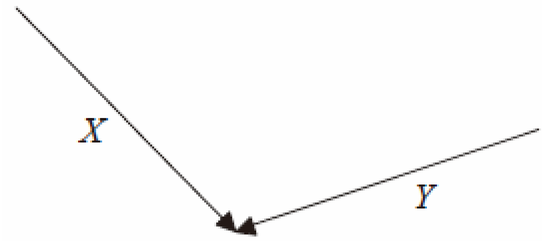
Examples of scalar: mass, temperatures, volume, work…

Vector: quantity both has magnitude and direction

Examples of vectors: force, acceleration, displacement, velocity, momentum…

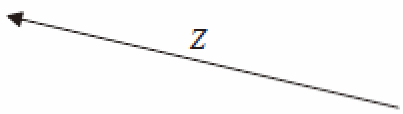
And $weight = m \cdot \vec{g}$ is a vector. Thus choose (A)

9. The diagram below shows two vectors **X** and **Y**.

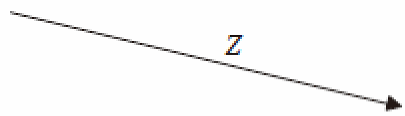


Which of the following best represents the vector **Z** = **X** – **Y**.

A.



B.

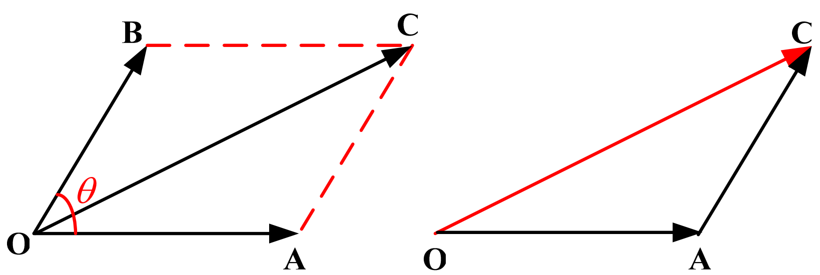


C.

D.

Strategy:

If two vectors are placed tail-to-tail at an angle θ, it can also be represented as a closed triangle.

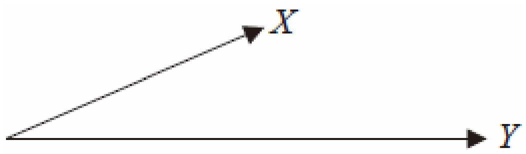


$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$ Because $\overrightarrow{OB} = \overrightarrow{AC}$

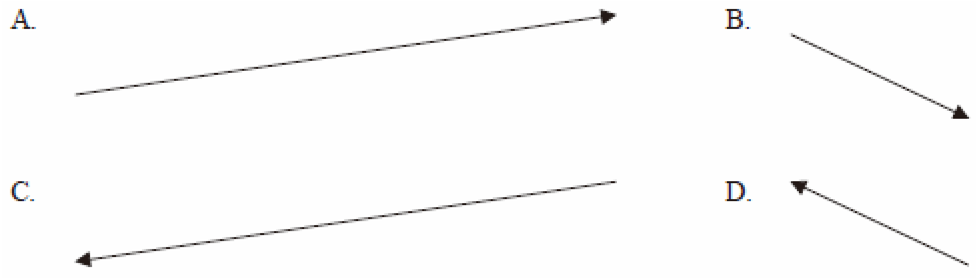
Solution:

And **X** = **Z** + **Y, thus choose (B)**

10. The magnitude and direction of two vectors **X** and **Y** are represented by the vector diagram below.

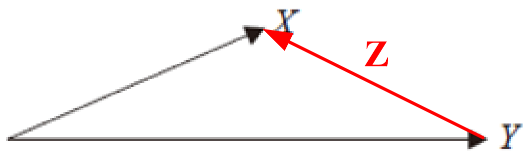


Which of the following best represents the vector (**X**–**Y**)?



Solution:

Let **X** minus **Y** to be **Z**: **X**–**Y = Z,** thus **X** = **Z** + **Y**



Choose (**D**):

Section II Newtonian mechanics

Chapter 1 Kinematics

1.1 Linear motion

1.1.1 Displacement and velocity

Distance: is the magnitude of the path covered, is a scalar.

SI unit: metre (m)

Displacement: the change in position between the starting point and the end point.

SI unit: metre (m)

Displacement is a vector; its direction is from the starting point to end point.

For example:

(i) An ant crawl along the arc that start from O to A (**Fig. 1.1**),

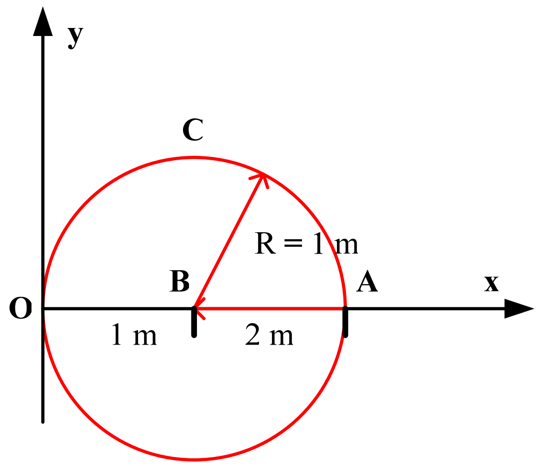


Fig. 1.1

Then:

$$\text{Distance} = \pi R = \pi = 3.14m$$

$$\text{Displacement} = \overrightarrow{OA} = 2m$$

(ii) The ant now goes on crawling from A to B,

$$\text{Distance} = \overset{\frown}{OCA} + AB = \pi R + 1 = 4.14m$$

$$\text{Displacement} = \overrightarrow{OB} = 1m$$

(iii) The ant now goes back from B to O,

Note: the ant start from O then go back to O. that is starting point is O, the end point is O.

$$\text{Distance} = \overset{\frown}{OCA} + AO = \pi R + 2 = 5.14m$$

$$\text{Displacement} = \overrightarrow{OO} = 0m$$

Speed: the distance traveled by a moving object over a period of time.
Constant speed: the moving object doesn't change its speed.

$$(average)speed = \frac{distance}{time\ taken}$$

$$v = \frac{s}{t}$$

Unit: m/s or ms^{-1}

Velocity: the speed in a given direction.
Average velocity: the change in position (displacement) over a period of time.

$$\vec{v}_{average} = \frac{change\ in\ position}{time\ taken} = \frac{displacement}{time\ taken} = \frac{\Delta\vec{x}}{t} = \frac{\vec{s}}{t}$$

Where $\vec{s}$ is displacement

Unit: m/s or ms^{-1}

Velocity is a vector; the direction is the same as the direction of the displacement.

Instantaneous velocity: the velocity that the moving object has at any one instance

1.1.2 Acceleration

Changing velocity (non-uniform) means an acceleration is present.
We can define acceleration as the change of velocity per unit time.
Uniform acceleration: the acceleration is constant, means the velocity of the moving object changes the same rate.
Average acceleration: change in velocity over a period of time.

$$\text{Average acceleration} = \frac{change\ in\ velocity}{time\ taken}$$

In symbol:

$$\vec{a}_{average} = \frac{\Delta\vec{v}}{t} = \frac{v-u}{t}$$

Where, v is the final velocity, u is the initial velocity.
SI unit: Meters per second squared (m/s^2)
Acceleration is a vector; the direction is the same as the direction of the change of velocity.

1.1.3 Equations for uniform acceleration

Consider a body is moving along a straight line with uniform acceleration, and its velocity increases from *u* (initial velocity) to *v* (final velocity) in time *t*.

First equation:

$$\text{acceleration} = \frac{change\ in\ velocity}{time\ taken}$$

$$a = \frac{v-u}{t}$$

So

$$at = v - u \quad \text{or} \quad v = u + at \ \ldots\ldots \ ①$$

Second equation:

$$average\ velocity = \frac{change\ in\ position}{time\ taken} = \frac{displacement}{time\ taken}$$

$$\bar{v} = \frac{\Delta\vec{x}}{t} = \frac{\vec{s}}{t}$$

Because the body is moving along a straight line in one direction, the magnitude of the displacement is equal to the distance.

And for the acceleration is uniform,

$$the\ average\ velocity, \bar{v} = \frac{v+u}{2}$$

So

$$\bar{v} = \frac{v+u}{2} = \frac{s}{t} \quad \text{or} \quad s = \frac{(v+u)}{2}t \ \ldots\ldots \ ②$$

Third equation:

From equation ①, $v = u + at$ and equation ②, $s = \frac{(v+u)}{2}t$

$$s = \frac{(u+at+u)}{2}t = ut + \frac{1}{2}at^2 \ \ldots\ldots \ ③$$

Fourth equation:

From equation, $v = u + at$

We get:

$v^2 = (u+at)^2$

$v^2 = u^2 + 2uat + a^2t^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$

But $s = ut + \frac{1}{2}at^2$

So

$$v^2 = u^2 + 2as \quad \ldots\ldots \text{④}$$

1.1.4 Displacement—time graphs

Note: for a body moving along a straight line, we can only draw the displacement—time graphs (**Fig. 3.2**)

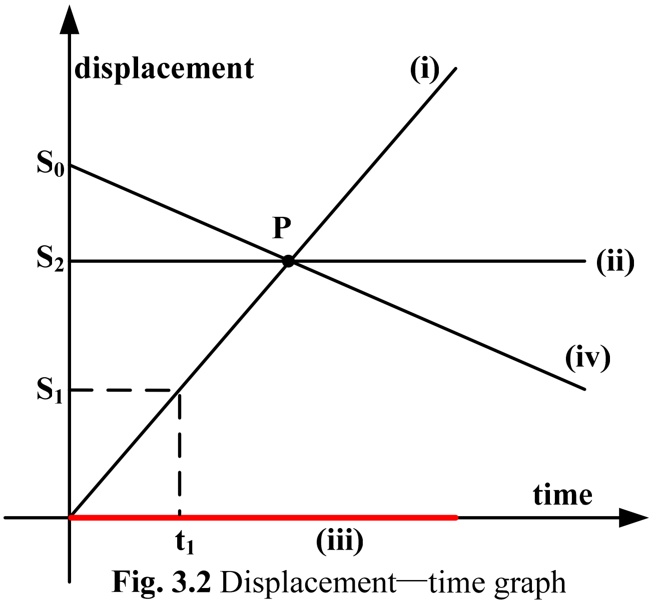


Fig. 3.2 Displacement—time graph

(i) Represents the body moving along a straight line with constant velocity; And the slope or gradient of the displacement—time graph represents the velocity of the body.

(ii) The body keeps rest with displacement S_2.

(iii) The body keeps rest with zero displacement.

(iv) The body moving along the opposite direction with constant velocity and initial displacement S_0.

(v) The point P means the displacement when the objects meeting with each other.

(vi) Displacement of the body is S_1 at time t_1.

1.1.5 Velocity—time graphs

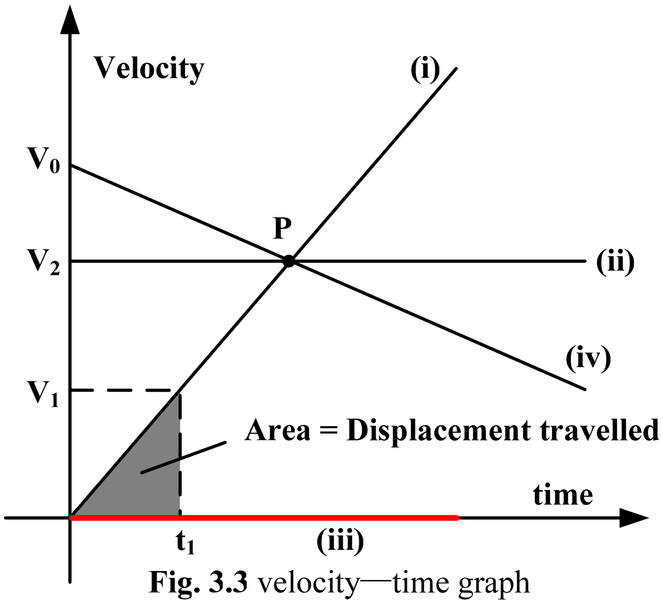


Fig. 3.3 velocity—time graph

(i) represents the body moving along a straight line with constant acceleration; And the slope or gradient of the velocity—time graph represents the acceleration of the body.

(ii) The body moving with constant velocity V_2.

(iii) The body keeps rest with zero velocity.

(iv)The body moving along a straight line with constant deceleration with initial velocity V_0; and the slope or gradient of the velocity—time graph represents the deceleration of the body

(v)The point P means the same velocity when the objects meeting with each other.

(vi)Velocity of the body is V_1 at time t_1 and the area under a velocity—time graph measures the displacement traveled.

1.2 Non-linear motion

1.2.1 Free-fall motion

The motion of a body that is only acted on by gravity and falls down from rest is called **free-fall motion.** This motion can occur only in a space without air. If air resistance is quite small and neglectable, the falling of a body in the air can also be referred to as a free-fall motion.

Galileo pointed out: free-fall motion is a uniformly accelerated rectilinear motion with zero initial velocity.

1.2.1.2 Acceleration of free-fall body

All bodies in free-fall motion have the same acceleration. This acceleration is called **free-fall acceleration** or **gravitational acceleration**. It is usually denoted by g.

The magnitude of gravitational acceleration g / ($m \cdot s^{-2}$)

Standard value: $g = 9.80665 m/s^2$

The direction of gravitational acceleration g is always vertically downward. Its magnitude can be measured through experiments.

Precise experiments show that the magnitude of g varies in different places on the earth. For example, at the equator $g = 9.780 m/s^2$. We take $9.81 m/s^2$ for g in general calculations. In rough calculations, $10 m/s^2$ is used.

As free-fall motion is uniformly accelerated rectilinear motion with zero initial velocity, the fundamental equations and the deductions for uniformly accelerated rectilinear motion are applicable for free-fall motion. What is only needed is to take zero for the initial velocity (u) and replace acceleration a with g.

1.2.2 Drag force and terminal speed

Any object moving through a fluid experiences a force that drags on it due to the fluid. The drag force depends on:

(i) The shape of the object

(ii) Its speed

(iii) The viscosity of the fluid which is a measure of how easily the fluid flows past a surface.

Note: the faster an object travels in a fluid, the greater the drag force on it.

1.2.2.1 Drag force in air

Considering an object released from rest in air, and then the speed of the object increases as it falls, so the drag force on it due to the fluid increases. The resultant force on the object is the difference between the force of gravity on it (its weight) and the drag force. As the drag force increases, the resultant force decreases, so the acceleration becomes less as it falls. If it continues falling, it attains terminal speed, when the drag force on it is equal and opposite to its weight. Its acceleration is then zero and its speed remains constant as it falls.

And at any instant, the resultant force F = mg—D, where m is the mass of the object and D is the drag force.

Therefore, the acceleration of the object, $a = \frac{mg - D}{m} = g - \frac{D}{m}$

Note:

(i) The initial acceleration = g because the speed is zero, and therefore the drag force is zero; at the instant the object is released.

(ii) At the terminal speed, the potential energy lost by the object is converted, as it falls, to internal energy of the fluid by the drag force.

1.2.2.2 Drag force in liquid

An object moving through a fluid experiences a ***resistive force, or drag***, that is proportional to the **viscosity** of the fluid. If the object is moving *slowly enough*, ***the drag force is proportional to its speed v***. if the object is a sphere of radius *r*, the force is

$$\boxed{F = 6\pi\eta rv}$$

Where η is again the coefficient of viscosity. This equation is known as **Stokes's law**. **Stokes's law** can be used to relate the speed of a sphere falling in a liquid to the viscosity of that liquid.

Consider a solid sphere of radius r dropped into the top of a column of liquid (**Fig. 1.1**). At the top of the column, the sphere accelerates downward under the influence of gravity. However there are two additional forces, both acting upward: the constant buoyant force and a speed-dependent retarding force given by Stokes's law. When the sum of the upward forces is equal to the gravitational force, the sphere travels with a constant speed v_t, called the ***terminal speed***. To determine this speed, we write the equation for the equilibrium of forces:

$$F_{grav} = F_{buoyant} + F_{drag}$$

We can express the gravitational force in terms of the density ρ of the sphere, its volume $\frac{4}{3}\pi r^3$, and *g*:

$$F_{grav} = \frac{4}{3}\pi r^3 \rho g$$

The buoyant force is equal to the weight of the displaced liquid, which has a density $'\rho$:

$$F_{buoyant} = \frac{4}{3}\pi r^3 \rho' g$$

The retarding force is expressed by Stokes's law with the speed v_t:

$$F_{drag} = 6\pi\eta r v_t$$

Combining these equations, we get an expression for the terminal speed:

$$v_t = \frac{2r^2 g}{9\eta}(\rho - \rho')$$

The terminal speed is also called the sedimentation speed by biologists and geologists.

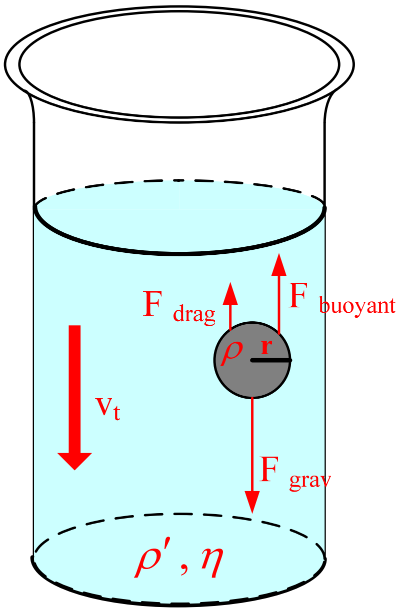


Fig. 1.1 A sphere falling in a viscous liquid reaches a terminal speed v_t that depends upon the radius and density of the sphere and the density and viscosity of the liquid.

Note: *Stokes's law* applies for situations in which the fluid flow is laminar, but not when the flow becomes turbulent.

But whenever an object moves rapidly enough, the retarding force F depends not on the speed (*Stokes's law*), but on the square of the speed:

$$F = bv^2$$

Where b is a constant determined for each different case.

An object falling from rest through the air falls with increasing speed until, at the terminal speed v_t, the retarding force of the air is equal in magnitude to the gravitational force:

$$mg = bv_t^2$$

Thus, the terminal speed can be written as

$$v_t = \sqrt{\frac{mg}{b}}$$

Where the constant b depends on the density ρ of the air and the area A of the body presented to the air flow. Then the equation for the terminal speed is

$$v_t = \sqrt{\frac{mg}{C_D \frac{\rho}{2} A}}$$

Where **C_D** is called the **drag coefficient**. This equation also holds for objects moving horizontally through the air at any speed if mg is replaced by the retarding, or drag, force on the object. Thus, the aerodynamic drag on a moving object, such as a car, becomes approximately

$$\boxed{F_{drag} = 0.65 C_D A v^2}$$

1.3 17 worked examples

1. An aero plane taking off accelerates uniformly on a runway from a velocity of $3ms^{-1}$ to a velocity of $90ms^{-1}$ in 45s.

Calculate:

(i) Its acceleration.

(ii) The distance on the runway.

Solution: data: $u = 3ms^{-1}$ $v = 93ms^{-1}$ $t = 45s$

Strategy: $v = u + at \Rightarrow a = \frac{v-u}{t}$, $s = ut + \frac{1}{2}at^2$

Answers:

$$a = \frac{v-u}{t} = \frac{93-3}{45} = 2ms^{-1}$$

$$s = ut + \frac{1}{2}at^2 = 3 \times 45 + \frac{1}{2} \times 2 \times 45^2 = 2160m = 2.16km$$

2. A car accelerates uniformly from a velocity of $15ms^{-1}$ to a velocity of $25ms^{-1}$ with a distance of 125m.

Calculate:
(i) Its acceleration
(ii) The time taken

Solution:

Data: $u = 15ms^{-1}$ $v = 25ms^{-1}$ $s = 125m$

Strategy: $v^2 = u^2 + 2as \Rightarrow a = \frac{(v^2 - u^2)}{2s}$

$$v = u + at \Rightarrow t = \frac{v-u}{a}$$

Answers: $a = \frac{(v^2 - u^2)}{2s} = \frac{25^2 - 15^2}{2 \times 125} = 1.6ms^{-2}$

$$v = u + at \Rightarrow t = \frac{v-u}{a} = \frac{25-15}{1.6} = 6.25s$$

3. A racing car starts from rest and accelerates uniformly at $2ms^{-2}$ in 30seconds, it then travels at a constant speed for 2min and finally decelerates at $3ms^{-2}$ until it stops, determine the maximum speed in km/h and the total distance in km it covered.

Strategy:

First stage: $u = 0ms^{-1}$ $a = 2ms^{-2}$ $t = 30s$, $v = u + at = 60ms^{-1}$

Second stage: moving with a constant speed $60ms^{-1}$ for 2min.

Third stage: $u = 60ms^{-1}$ $v = 0ms^{-1}$ $a = -3ms^{-2}$ ($deceleration$)

Answers:

First stage: $v = u + at = 60ms^{-1}$

$$s_1 = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 30^2 = 900m$$

Second stage: the final speed of the first stage is the constant speed of the second stage.

$s_2 = vt = 60 \times 2\min = 60 \times 2 \times 60 = 7200m$
$(1\min = 60s)$

Third stage: $v^2 = u^2 + 2as \Rightarrow s_3 = \frac{v^2 - u^2}{2a} = \frac{0 - 60^2}{2 \times (-3)} = 600m$

So

Maximum speed = $60ms^{-1} = \frac{60}{1000} \times 60 \times 60 = 216km/h$

Total distance $= s_1 + s_2 + s_3 = 900 + 7200 + 600 = 8700m = 8.7km$

4. Figure 4.1 shows the shuttle spacecraft as it is launched into space.

Fig. 4.1 shuttle spacecraft launching into space

During the first 5 minutes of the launch the average acceleration of the shuttle is $14.5ms^{-2}$.

a. Calculate the speed of the shuttle after the first 5 minutes.

b. Calculate how far the shuttle travels in the first 5 minutes.

Data: $u = 0ms^{-1}$, $\bar{a} = 14.5ms^{-2}$, $t = 5\min = 300\sec$

Strategy: $v = u + at$, $s = ut + \frac{1}{2}at^2$

Answers: a. $v = u + at = 0 + 14.5 \times 300 = 4350m = 4.35km$

b. $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 14.5 \times 300^2 = 652500m = 652.5km$

5. Figure 5.1 shows an incomplete velocity—time graph for a boy running a distance of 100m.

a. What is his acceleration during the first 4 seconds?

b. How far does the boy travel during (i) the first 4 seconds, (ii) the next 9 seconds?

c. Copy and complete the graph showing clearly at what time he has covered the distance of 100m. Assume his speed remains constant at the value shown by the horizontal portion of the graph.

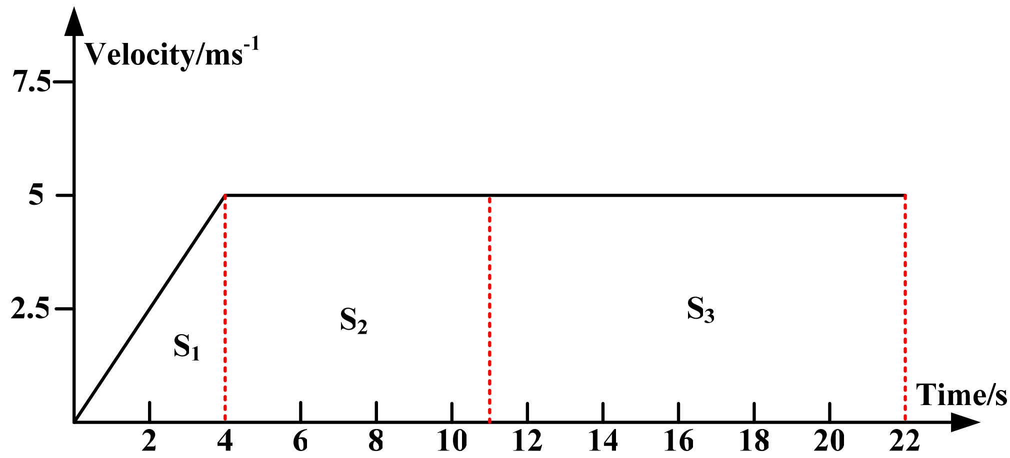


Fig. 5.1 velocity—time graph

Solution:

a. the gradient of the velocity—time graph represents the acceleration of the body.

During the first 4 seconds, $\text{gradient} = \frac{5}{4} = 1.25$

$\text{acceleration} = 1.25ms^{-2}$

b. (i) the area under a velocity—time graph measures the displacement traveled.

$area\ S_1 = \frac{1}{2} \times 4 \times 5 = 10$

$\text{Displacement} = 10m$

(ii) The next 9 seconds, $\text{area } S_2 = 9 \times 5 = 45$

$\text{Displacement} = 45m$

c. during the first 13 seconds, the distance covered is 10 + 45 = 55m,

The area needed $S_3 = 100 - 55 = 45$

So from 13s to 22 s, he covers $S_3 = 45$ m.

6. A constant resultant horizontal force of 1.8×10^3 N acts on a car of mass 900 kg, initially at rest on a level road.

(a) Calculate

(i) The acceleration of the car,

Strategy: by the Newton's second law, $F = ma$, $a = \frac{F}{m}$

So $a = \frac{F}{m} = \frac{1.8 \times 10^3}{900Kg} = 2ms^{-2}$

(ii) The speed of the car after 8.0 s,

Strategy: initial velocity, $u=0$, $t=8.0s$, $a=2ms^{-2}$. And from the equation: $v=u+at$, gives

$v=0+2\times 8=16ms^{-1}$

(iii) The momentum of the car after 8.0 s,

Strategy: The product of an object's mass m and velocity v is called its momentum:

$momentum=mv=900\times 16=1.44\times 10^{4}kgms^{-1}$

(iv) The distance traveled by the car in the first 8.0 s of its motion,

Strategy: $s=ut+\frac{1}{2}at^{2}$

$S=0+\frac{1}{2}\times 2\times 8^{2}=64m$

(v) The work done by the resultant horizontal force during the first 8.0 s.

Strategy: Work done=force × distance moved in direction of force.

$W=FS=1.8\times 10^{3}\times 64=115.2kJ$

(b) On the axes below (**Fig. 6.1**) sketch the graphs for speed, *v*, and distance traveled, *s*, against time, *t*, for the first 8.0 s of the car's motion.

Strategy: for the first 8.0 s, the car is moving with constant acceleration, $a=2ms^{-2}$, so the gradient of the v—t graph is equal to $2ms^{-2}$

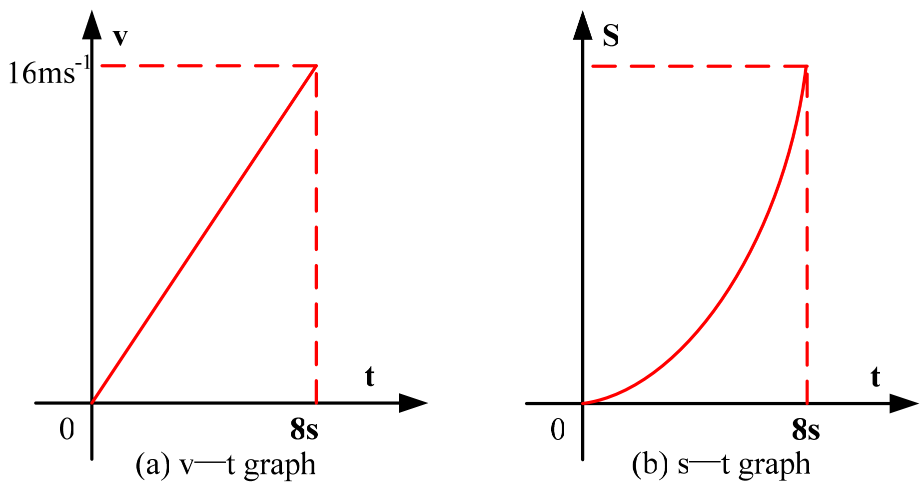


Fig. 6.1 (a) v—t graph (b) s—t graph

(c) In practice the resultant force on the car changes with time. Air resistance is one factor that affects the resultant force acting on the vehicle.

You may be awarded marks for the quality of written communication in your answer.

(i) Suggest, with a reason, how the resultant force on the car changes as its speed increases.

Answers: the resultant force decreases as its speed increases, because the air resistance increases as its speed increases, and the engine force of the car is constant, so the constant force decreases.

(ii) Explain, using Newton's laws of motion, why the vehicle has a maximum speed.

As the velocity increases, the air resistance increases, so the resultant force decreases, which means the acceleration of the car decreases, but the velocity is still increasing till the resultant force is zero (acceleration of the car is zero), according to the Newton's first law, then the vehicle has a maximum speed.

7. Fig. 7.1 represents the motion of two cars, A and B, as they move along a straight, horizontal road.

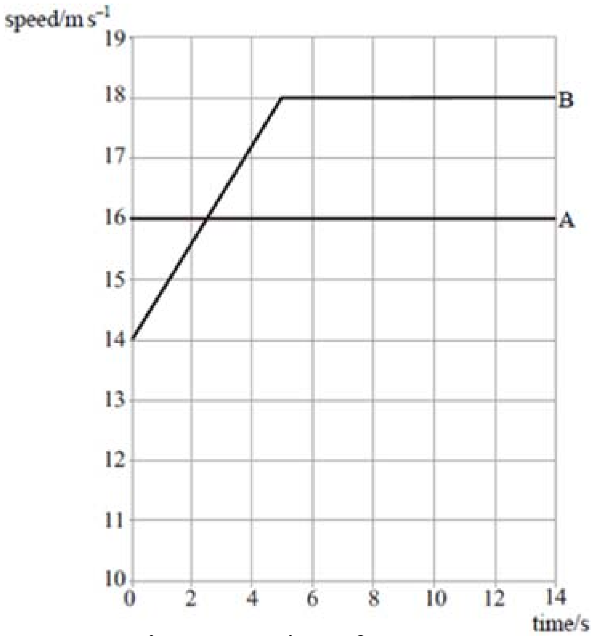


Fig. 7.1 motion of two cars

(a) Describe the motion of each car as shown on the graph.

(i) Car A: is moving with constant speed $16ms^{-1}$

(ii) Car B: accelerates in the first 5 seconds, and then moving with constant speed $18ms^{-1}$.

(b) Calculate the distance traveled by each car during the first 5.0 s.

(i) Car A:

Strategy: car A moving with constant speed, so distance of car A, So $S_A = ut = 16 \times 5 = 80m$

(ii) Car B:

Strategy: in the first 5 seconds, car B accelerates, and from the graph, the gradient of the v—t graph for B is $\frac{18-14}{5} = 0.8$, that is the acceleration of B is $a = 0.8ms^{-2}$

So $S_B = ut + \frac{1}{2}at^2 = 14 \times 5 + \frac{1}{2} \times 0.8 \times 5^2 = 80m$

(c) At time t = 0, the two cars are level. Explain why car A is at its maximum distance ahead of B at t = 2.5 s

Because car A is faster than car B at the first 2.5s, so for the first 2.5s, the distance between them increases till they have the same speed at 2.5s. After 2.5s, car B is faster than car A, so the distance then decreases. So at the time 2.5s, car A is at its maximum distance ahead of B.

8. A car accelerates from rest to a speed of $26ms^{-1}$. **Table 8.1** shows how the speed of the car varies over the first 30 seconds of motion.

Table 8.1

Time/s	0	5.0	10.0	15.0	20.0	25.0	30.0
Speed/ms^{-1}	0	16.5	22.5	24.5	25.5	26.0	26.0

(a) Draw a graph of speed against time on the grid provided (**Fig. 8.1**).

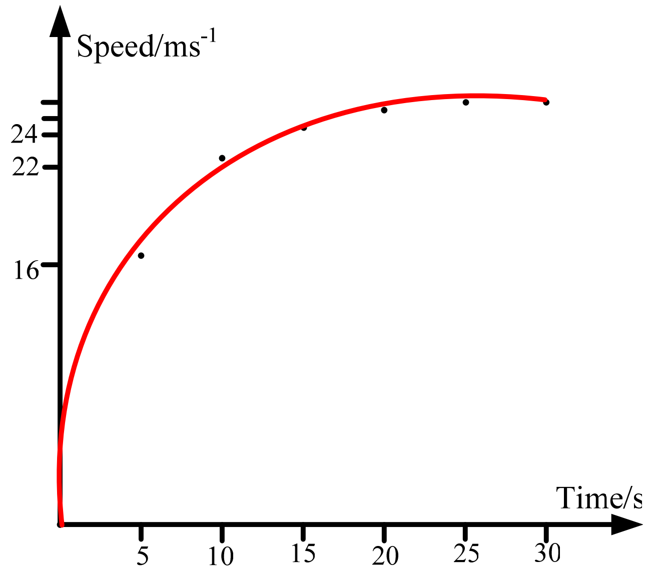


Fig. 8.1 speed—time graph

Note: you must draw the right scales and the six points are correctly plotted, and it is a trend line not a straight line.

(b) Calculate the average acceleration of the car over the first 25 s.

Strategy: $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{26}{25} = 1.04 ms^{-2}$

(c) Use your graph to estimate the distance traveled by the car in the first 25 s.

Strategy: area under the v—t graph represents the distance traveled.
So from the graph, its distance is 510m

(d) Using the axes below, sketch a graph to show how the resultant force acting on the car varies over the first 30 s of motion.

Solution:

From table 8.1, the rate of change of speed decreases to zero, thus the resultant force decreases to zero. As shown in **Fig. 8.2**.

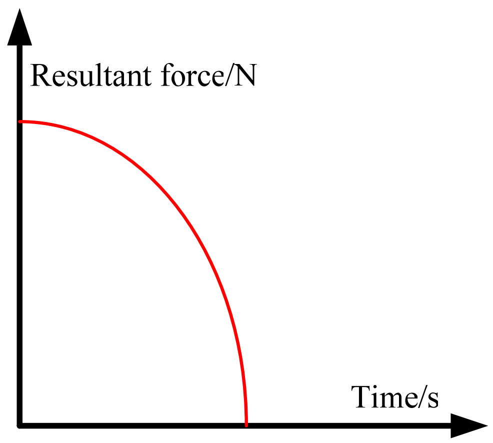


Fig. 8.2 resultant force—time graph

(e) Explain the shape of the graph you have sketched in part (d), with reference to the graph you plotted in part (a).

Because the first graph shows that the gradient of the car decreases, which means that the acceleration of the car decreases, and by the Newton's second law, $F = ma$, the force, F, decreases, and as the acceleration is changing in the first 25s, so the force is also changing, so the graph of the force is not a straight line.

9. A supertanker of mass $4.0 \times 10^{8} kg$, cruising at an initial speed of $4.5 m/s$, takes one hour to come to rest.

(a) Assuming that the force slowing the tanker down is constant, calculate

(i) The deceleration of the tanker,

Solution:

The force slowing the tanker down is constant, so the tanker decelerates uniformly. Therefore, deceleration of the tanker is given by

$$a = \frac{0-4.5}{t} = \frac{-4.5}{1\times 60\times 60} = 1.25\times 10^{-3} m/s^2$$

(ii) The distance travelled by the tanker while slowing to a stop.

Solution:

The average speed is given by

$$\bar{v} = \frac{0+4.5}{2} = 2.25 m/s$$

So the distance traveled: $s = \bar{v}t = 2.25\times 1\times 60\times 60 = 8100m$

(b) Sketch, using the axes below, a distance-time graph representing the motion of the tanker until it stops.

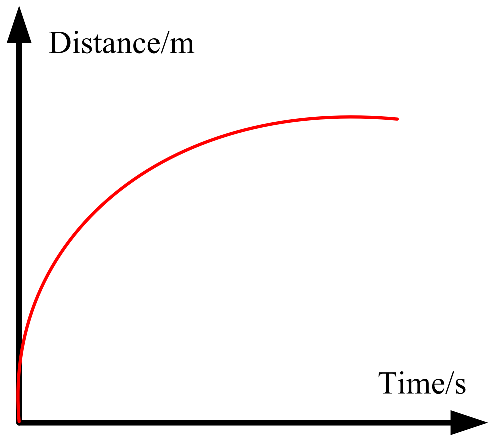


Fig. 9.1 Distance—time graph

(c) Explain the shape of the graph you have sketched in part (b).

Solution:

Because the speed is decreasing, the gradient of the curve decreases in the distance-time graph.

10. (a) A cheetah accelerating uniformly from rest reaches a speed of $29m/s$ in 2.0 s and then maintains this speed for 15 s. Calculate

(i) Its acceleration,

Solution:

Using $a = \frac{v-u}{t} = \frac{29-0}{2} = 14.5m/s^2$

(ii) The distance it travels while accelerating,

Solution:

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 14.5 \times 2^2 = 29m$

(iii) The distance it travels while it is moving at constant speed.

Solution:

$s = vt = 29 \times 15 = 435m$

(b) The cheetah and an antelope are both at rest and 100 m apart. The cheetah starts to chase the antelope. The antelope takes 0.50 s to react. It then accelerates uniformly for 2.0 s to a speed of $25m/s$ and then maintains this speed. **Fig. 10.1** shows the speed-time graph for the cheetah.

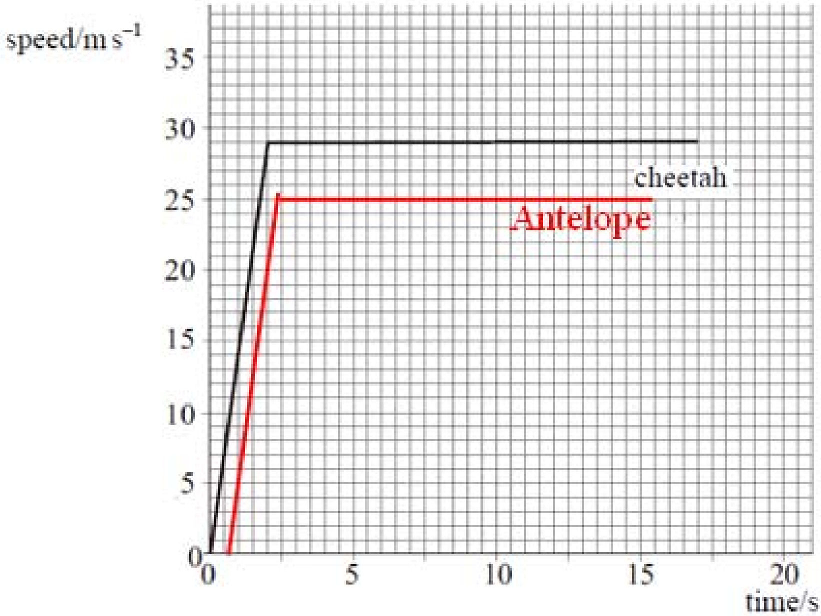


Fig. 10.1 speed—time graph for cheetah and antelope

(i) Using the same axes plot the speed-time graph for the antelope during the chase.

Solution:

The antelope takes 0.50 s to react and accelerates uniformly for 2.0 s to a speed of 25 m/s. thus we can get the speed-time graph beginning with 0.50 s.

(ii) Calculate the distance covered by the antelope in the 17 s after the cheetah started to run.

Solution:

The antelope accelerates from rest, and reaches to a speed of 25 m/s in 2 s. then maintains this speed. Thus the distance is given by

$s = \frac{v+u}{2} \times 2 + 25 \times (17 - 2 - 0.5) = 12.5 \times 2 + 25 \times 14.5 = 387.5m$

(iii) How far apart are the cheetah and the antelope after 17 s?

Solution:

From (a), the distance of cheetah is $s_1 = 435 + 29 = 464m$

And at the beginning, they are 100 m apart. Thus

$$\Delta s = s + 100 - s_1 = 387.5 + 100 - 464 = 23.5m$$

11. Figure 11.1 shows a distance-time graphs for two runners, A and B, in a 100 m race.

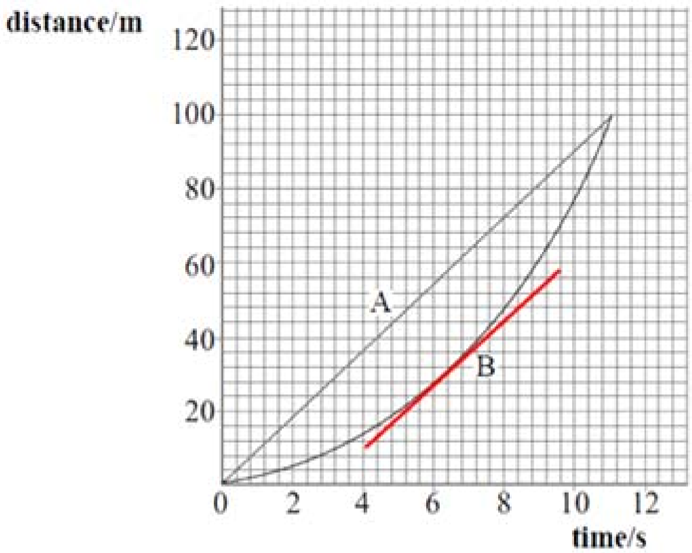


Fig. 11.1 distance—time graph for two runners

(a) Explain how the graph shows that athlete B accelerates throughout the race.

Solution:

The gradient is changing (increasing)

(b) Estimate the maximum distance between the athletes.

Solution:

When B's speed is the same as A's, it has the maximum distance between the athletes. From the graph is the gradient of B curve is the same that of A.

From the graph, the maximum distance is 25 m.

(c) Calculate the speed of athlete A during the race.

Solution:

For A, it has a distance in time 11 s, thus

$$speed = \frac{distance}{time} = \frac{100m}{11s} = 9.1m/s$$

(d) The acceleration of athlete B is uniform for the duration of the race.

(i) State what is meant by uniform acceleration.

(ii) Calculate the acceleration of athlete B.

Solution:

(i) The acceleration keeps the same or the velocity increases uniformly with time.

(ii) For B, its initial velocity is u = 0 m/s, distance s = 100 m, time taken t = 11 s.

Thus, using $s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$, gives

$$a = \frac{2s}{t^2} = \frac{2 \times 100}{11^2} = 1.7m/s^2$$

12.An aircraft accelerates horizontally from rest and takes off when its speed is 82 m s^{-1}. The mass of the aircraft is $5.6 \times 10^4 kg$ and its engines provide a constant thrust of $1.9 \times 10^5 N$.

(a) Calculate

(i) The initial acceleration of the aircraft,

Solution:

(i) Initially, the resultant force $F = 1.9 \times 10^5 N$, from Newton's second law: $F = ma$, we can get that

$$a = \frac{F}{m} = \frac{1.9 \times 10^5 N}{5.6 \times 10^4 kg} = 3.4m/s^{-2}$$

(ii) The minimum length of runway required, assuming the acceleration is constant.

Solution: let the minimum length of the runway required L. thus

$$v^2 - u^2 = 2aL$$

Therefore

$$L = \frac{v^2 - u^2}{2a} = \frac{82^2 - 0}{2 \times 3.4} = 989m$$

(b) In practice, the acceleration is unlikely to be constant. State a reason for this and explain what effect this will have on the minimum length of runway required.

Solution:

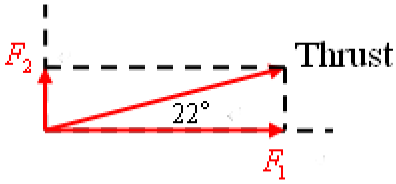
In practice, the air resistance increases with speed, hence the runway will be longer.

(c) After taking off, the aircraft climbs at an angle of 22° to the ground. The thrust from the engines remains at $1.9\times10^5 N$. Calculate

(i) The horizontal component of the thrust,

(ii) The vertical component of the thrust.

Solution:



The thrust $T = 1.9\times10^5 N$

The horizontal component of the thrust is given by

$F_1 = T\cos 22^0 = 1.76\times10^5 N$

The vertical component of the thrust is given by

$F_2 = T\sin 22^0 = 0.71\times10^5 N$

13. Figure 13.1 shows how the velocity, v, of a car varies with time, t.

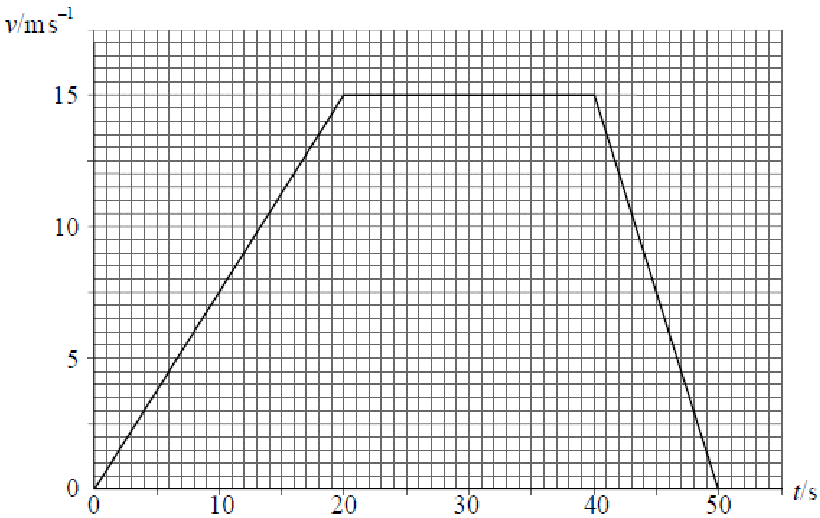


Fig. 13.1 velocity—time graph

(a) Describe the motion of the car for the 50 s period.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

Solution:

0—20 s: the car uniformly accelerates to a velocity of 15 m/s.

20—40 s: the car moves with constant velocity 15 m/s.

40—50 s: the car uniformly decelerates from a velocity of 15 m/s to 0 m/s.

(b) The mass of the car is 1200 kg. Calculate for the first 20 s of motion, (b)
(i) the change in momentum of the car,
(b) (ii) the rate of change of momentum,
(b) (iii) the distance travelled.

Solution: for the first 20 s of motion

(i) At t = o s, the initial velocity is u = 0 m/s; at t = 20 s, the final velocity is v = 15 m/s. thus the change in momentum of the car is given by

Therefore,

$$\Delta p = mv - mu = (1200kg)\times 15m/s - 0 = 1.8\times 10^4 kg\cdot m/s$$

(ii)

$$\text{The rate of change of momentum} = \frac{change\ \ in\ \ momentum}{time\ taken} = \frac{1.8\times 10^4 kg\cdot m/s}{20} = 0.9\times 10^3 kg\cdot m/s^2$$

(iii) The area under a velocity—time graph measures the displacement traveled.

Thus the area for the first 20 s is given by

$$A = \frac{1}{2}\times 20\times 15 = 150$$

Therefore the distance traveled is 150 m.

14. A car is travelling on a level road at a speed of 15.0 m s^{-1} towards a set of traffic lights when the lights turn red. The driver applies the brakes 0.5 s after seeing the lights turn red and stops the car at the traffic lights.

Table 14.1 shows how the speed of the car changes from when the traffic lights turn red.

Table 14.1

Time/s	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Speed/ms^{-1}	15.0	15.0	12.5	10.0	7.5	5.0	2.5	0.0

(a) Draw a graph of speed on the y-axis against time on the x-axis on the grid provided (**Fig. 14.1**).

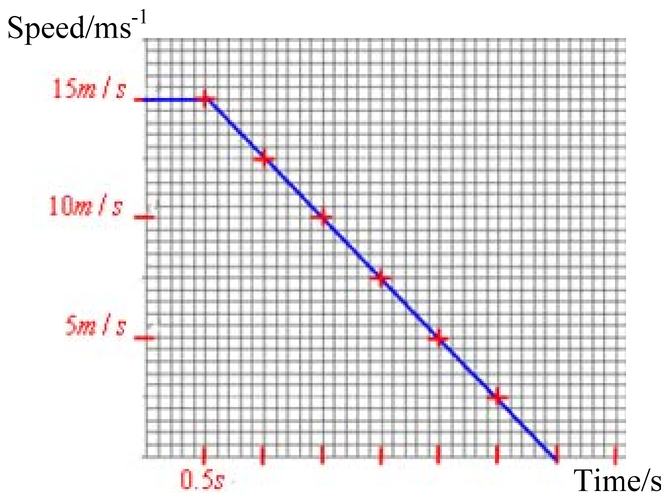


Fig. 14.1 speed—time graph

(b) (i) State and explain what feature of the graph shows that the car's deceleration was uniform.

Solution:

Deceleration is uniform because the graph is a decreasing straight line. And the gradient of the line represents the deceleration.

(b) (ii) Use your graph to calculate the distance the car travelled after the lights turned red to when it stopped.

Solution:

Distance traveled = area under the line (0s to 3.5s).

$$\text{Area } = \frac{1}{2} \times (0.5 + 3.5) \times 15 = 30$$

Therefore, distance traveled = 30 m.

15. Galileo used an inclined plane, similar to the one shown in **Fig. 15.1**, to investigate the motion of falling objects.

(a) Explain why using an inclined plane rather than free fall would produce data which is valid when investigating the motion of a falling object.

Solution:

Freefall is too quick; Galileo had no accurate method to time freefall.

(b) In a demonstration of Galileo's investigation, the number of swings of a pendulum was used to time a trolley after it was released from rest. A block was positioned to mark the distance that the trolley had travelled after a chosen whole number of swings.

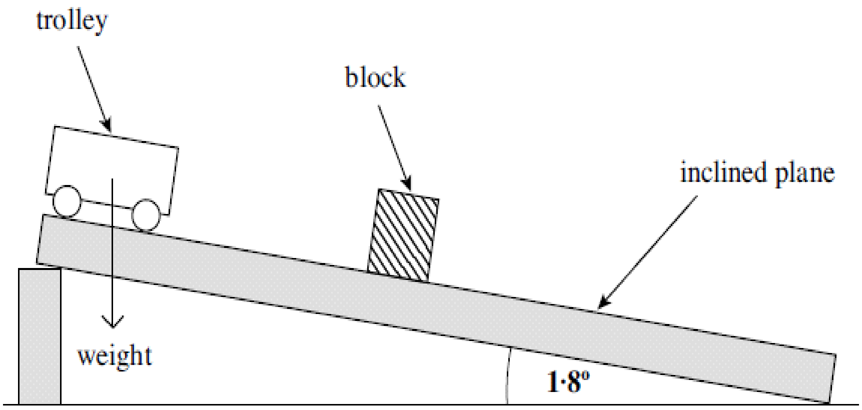


Fig. 15.1

The mass of the trolley in **Fig. 15.1** is 0.20 kg and the slope is at an angle of 1.8° to the horizontal.

(b) (i) Show that the component of the weight acting along the slope is about 0.06 N.

Solution:

The component of weight acting along the slope is given by

$W_1 = W\sin 1.8^0 = 0.2 \times 9.81 \times 0.031 = 0.06N$

(b) (ii) Calculate the initial acceleration down the slope.

Solution:

The initial resultant force along the slope equals to W_1, thus

$$a = \frac{W_1}{m} = \frac{0.06}{0.2} = 0.3m/s^{-2}$$

(c) In this experiment, the following data was obtained. A graph of the data (**Fig. 15.2**) is shown below it.

Time/pendulum swings	Distance travelled/m
1	0.29
2	1.22
3	2.70
4	4.85

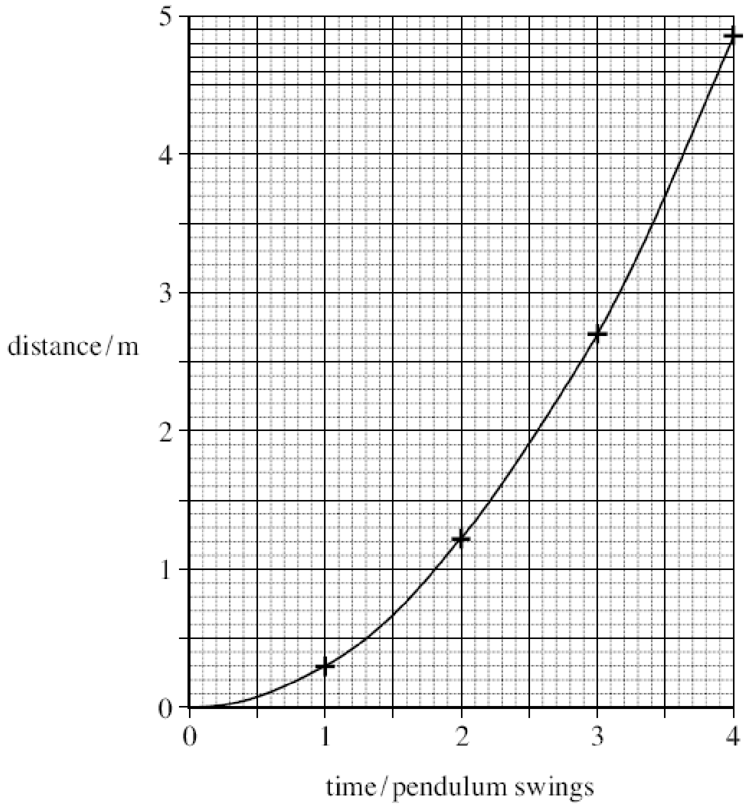


Fig. 15.2 distance—time graph

(c) From **Fig. 15.2**, state what you would conclude about the motion of the trolley?
Give a reason for your answer.

Solution:

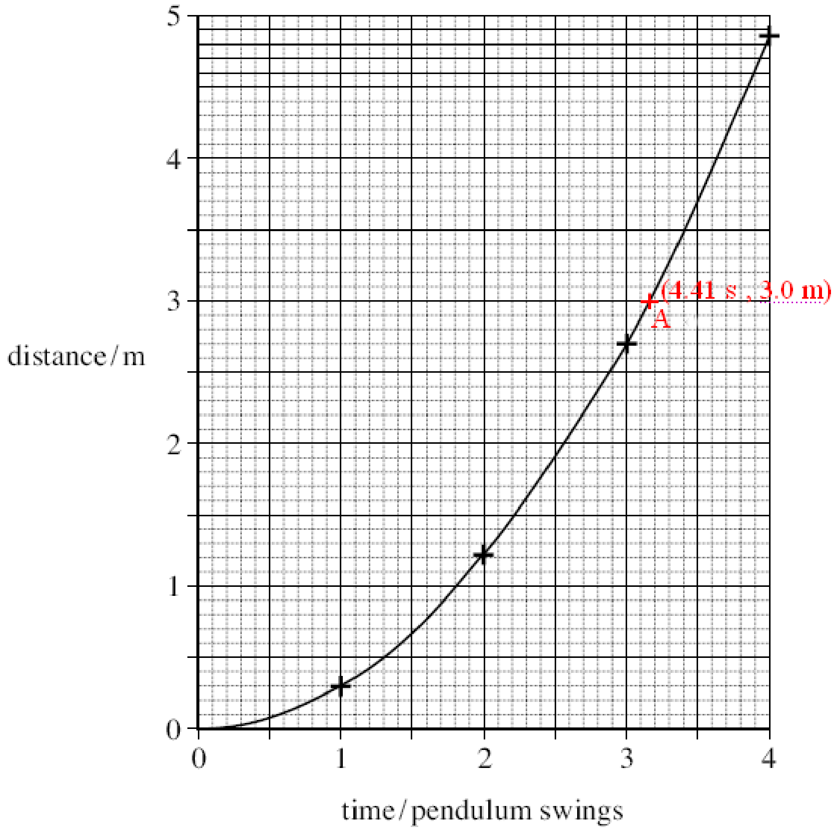
The gradient of the curve increases as time increasing. Thus the speed of the trolley is increasing.

(d) Each complete pendulum swing had a period of 1.4 s. Use the distance-time graph above to find the speed of the trolley after it had travelled 3.0 m.

Solution:

From **Fig. 15.2**, the time taken for traveling 3.0 m is given by

$$t = 3 \times 1.4 + 1.5 \times \frac{1.4}{10} = 4.41s$$



And initial speed $u = 0m/s$, thus

$s = \frac{u+v}{2} \times t = \frac{vt}{2}$, gives

Speed, $v = \frac{2s}{t} = \frac{2 \times 3.0m}{4.41s} = 1.36m/s$

16. A steel ball of mass 0.15kg released from rest in a liquid, falls a distance of 0.20m in 5.0s. Assuming the ball reaches terminal speed within a fraction of a second, calculate

(i) Its terminal speed,

(ii) The drag force on it when it falls at terminal speed.

Strategy: as the ball reaches terminal speed within a fraction of a second, so the ball falls a distance of 0.20m in 5.0s with the constant terminal speed, let the terminal speed V.

So (i) $s = Vt \Rightarrow 0.2 = V \times 5$

$V = 0.04 \text{ m s}^{-1}$

(ii) When the ball falls at terminal speed, the drag force on it is equal and opposite to its weight.

So drag force, F = weight = mg = $0.15 \times 9.8 = 1.47$ N

17. Explain why a raindrop falling vertically through still air reaches a constant velocity.

Answers: Because as the falling of the raindrop, its speed is increasing; and the air resistance of the raindrop is increasing with the increasing speed, so the resultant force of the raindrop decreases, by the Newton's second law, $F = ma$, its acceleration decreases. So when the speed reaches to a certain value, the resultant force is equal to zero, then the raindrop reaches a constant velocity.