Answer **all** questions.

1 (a) Draw a bipartite graph representing the following adjacency matrix. *(2 marks)*

	U	***V***	***W***	***X***	***Y***	***Z***
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(b) Given that initially A is matched to W, B is matched to X, C is matched to V, and E is matched to Y, use the alternating path algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

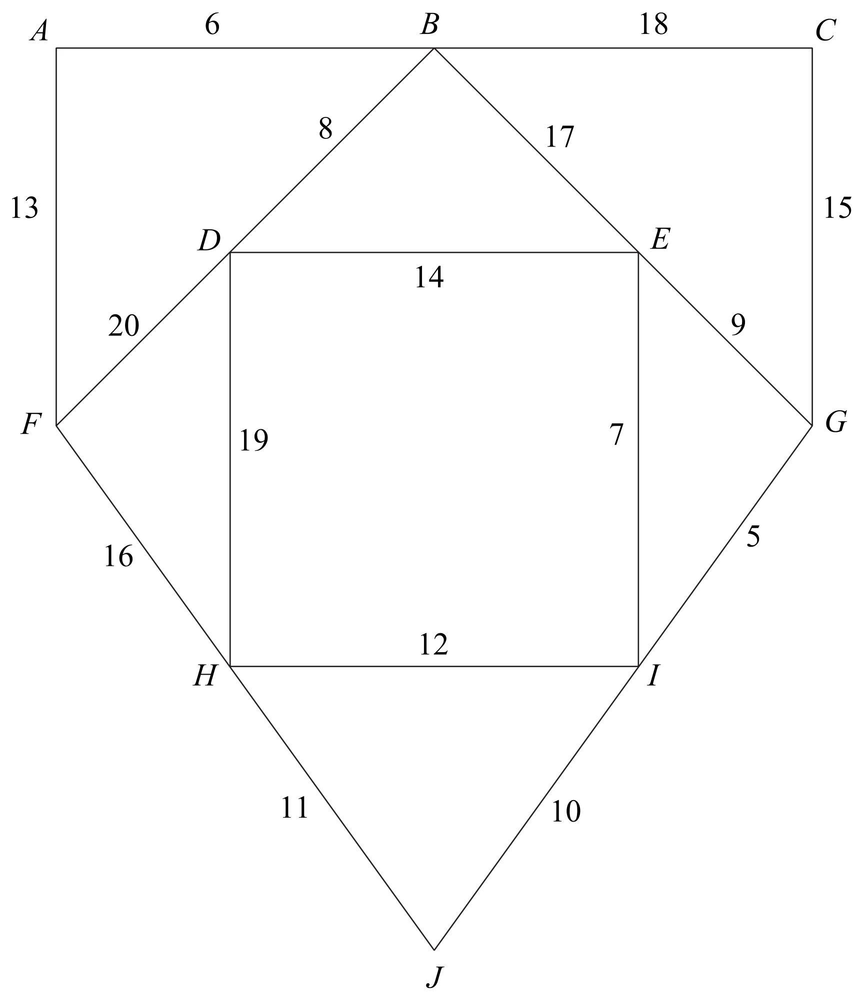
2 Use the quicksort algorithm to rearrange the following numbers into ascending order. Indicate clearly the pivots that you use.

18 23 12 7 26 19 16 24 *(5 marks)*

3 (a) (i) State the number of edges in a minimum spanning tree of a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree of a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, …, J. The numbers on each edge represent the distances, in miles, between pairs of vertices.

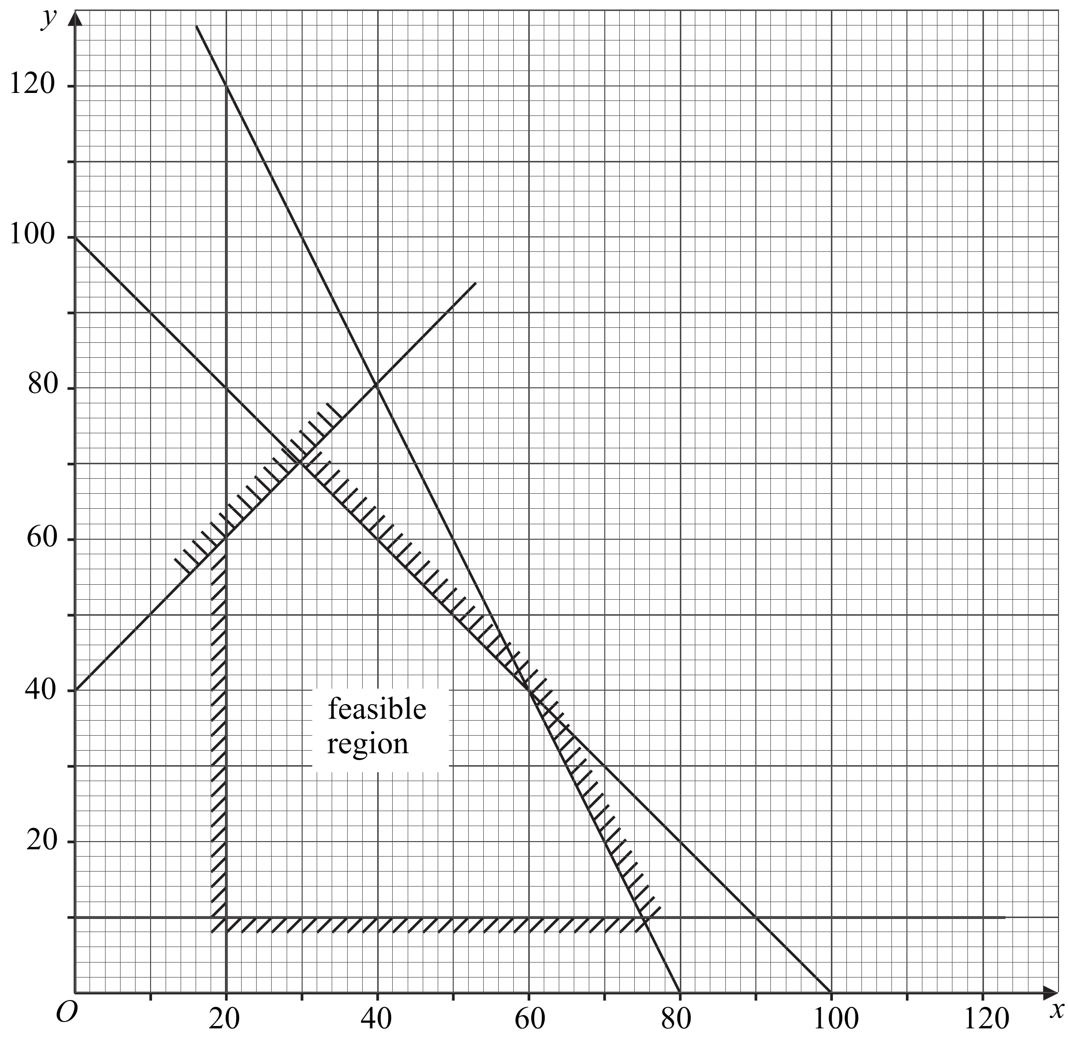


(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your spanning tree. *(1 mark)*

(iii) Draw your spanning tree. *(2 marks)*

4 The diagram shows the feasible region of a linear programming problem.



(a) On the feasible region, find:

(i) the maximum value of $2x + 3y$; *(2 marks)*

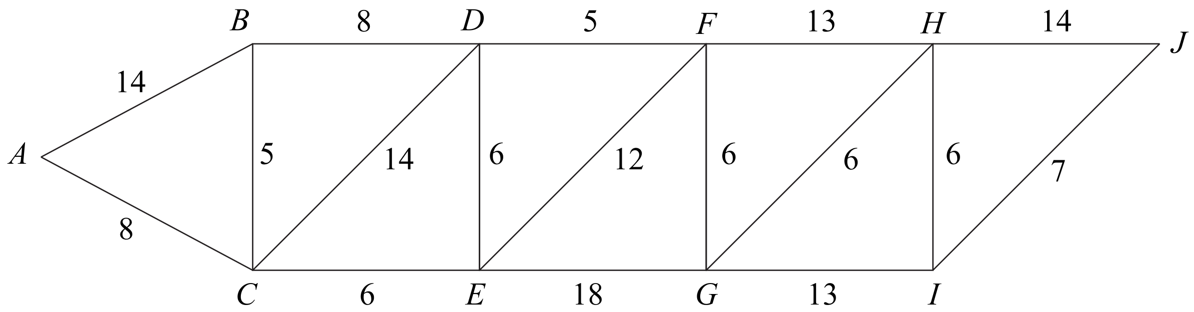
(ii) the maximum value of $3x + 2y$; *(2 marks)*

(iii) the minimum value of $-2x + y$. *(2 marks)*

(b) Find the 5 inequalities that define the feasible region. *(6 marks)*

5 [**Figure 1**, printed on the insert, is provided for use in this question.]

The network shows the times, in minutes, to travel between 10 towns.



(a) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to travel from A to J. *(6 marks)*

(b) State the corresponding route. *(1 mark)*

6 Two algorithms are shown.

Algorithm 1

Line 10	Input P
Line 20	Input R
Line 30	Input T
Line 40	Let $I = (P * R * T)/100$
Line 50	Let $A = P + I$
Line 60	Let $M = A/(12 * T)$
Line 70	Print M
Line 80	Stop

Algorithm 2

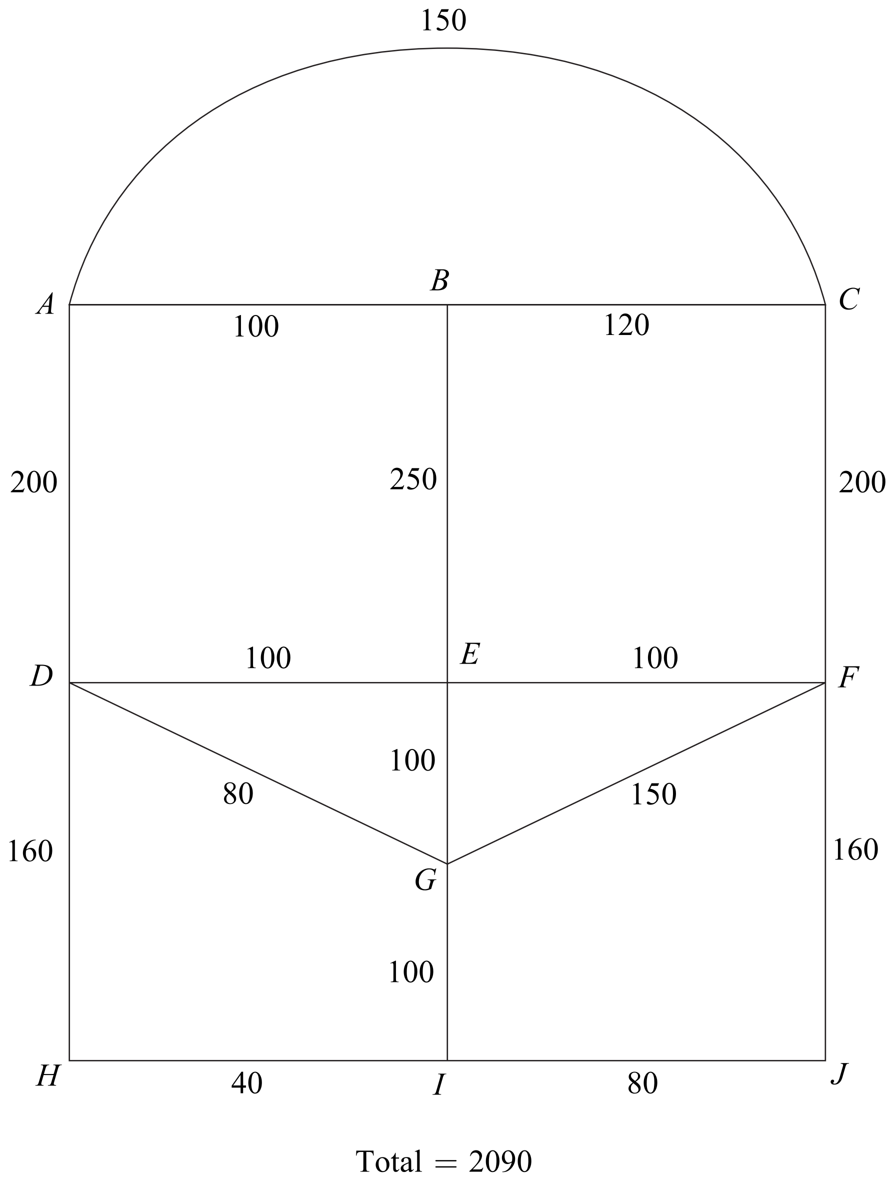
Line 10	Input P
Line 20	Input R
Line 30	Input T
Line 40	Let $A = P$
Line 50	$K = 0$
Line 60	Let $K = K + 1$
Line 70	Let $I = (A * R)/100$
Line 80	Let $A = A + I$
Line 90	If $K < T$ then goto Line 60
Line 100	Let $M = A/(12 * T)$
Line 110	Print M
Line 120	Stop

In the case where the input values are $P = 400$, $R = 5$ and $T = 3$:

(a) trace **Algorithm 1**; *(3 marks)*

(b) trace **Algorithm 2**. *(4 marks)*

7 Stella is visiting Tijuana on a day trip. The diagram shows the lengths, in metres, of the roads near the bus station.



Stella leaves the bus station at A. She decides to walk along all of the roads at least once before returning to A.

(a) Explain why it is not possible to start from A, travel along each road only once and return to A. *(1 mark)*

(b) Find the length of an optimal 'Chinese postman' route around the network, starting and finishing at A. *(5 marks)*

(c) At each of the 9 places B, C, . . ., J, there is a statue. Find the number of times that Stella will pass a statue if she follows her optimal route. *(2 marks)*

8 Salvadore is visiting six famous places in Barcelona: La Pedrera (L), Nou Camp (N), Olympic Village (O), Park Guell (P), Ramblas (R) and Sagrada Familia (S). Owing to the traffic system the time taken to travel between two places may vary according to the direction of travel.

The table shows the times, in minutes, that it will take to travel between the six places.

From \ To	La Pedrera (L)	Nou Camp (N)	Olympic Village (O)	Park Guell (P)	Ramblas (R)	Sagrada Familia (S)
La Pedrera (L)	—	35	30	30	37	35
Nou Camp (N)	25	—	20	21	25	40
Olympic Village (O)	15	40	—	25	30	29
Park Guell (P)	30	35	25	—	35	20
Ramblas (R)	20	30	17	25	—	25
Sagrada Familia (S)	25	35	29	20	30	—

(a) Find the total travelling time for:

(i) the route $LNOL$; *(1 mark)*

(ii) the route $LONL$. *(1 mark)*

(b) Give an example of a Hamiltonian cycle in the context of the above situation. *(1 mark)*

(c) Salvadore intends to travel from one place to another until he has visited all of the places before returning to his starting place.

(i) Show that, using the nearest neighbour algorithm starting from Sagrada Familia (S), the total travelling time for Salvadore is 145 minutes. *(3 marks)*

(ii) Explain why your answer to part (c)(i) is an upper bound for the minimum travelling time for Salvadore. *(2 marks)*

(iii) Salvadore starts from Sagrada Familia (S) and then visits Ramblas (R). Given that he visits Nou Camp (N) before Park Guell (P), find an improved upper bound for the total travelling time for Salvadore. *(3 marks)*

Turn over for the next question

Turn over ▶

9 A factory makes three different types of widget: plain, bland and ordinary. Each widget is made using three different machines: A, B and C.

Each plain widget needs 5 minutes on machine A, 12 minutes on machine B and 24 minutes on machine C.

Each bland widget needs 4 minutes on machine A, 8 minutes on machine B and 12 minutes on machine C.

Each ordinary widget needs 3 minutes on machine A, 10 minutes on machine B and 18 minutes on machine C.

Machine A is available for 3 hours a day, machine B for 4 hours a day and machine C for 9 hours a day.

The factory must make:

more plain widgets than bland widgets;

more bland widgets than ordinary widgets.

At least 40% of the total production must be plain widgets.

Each day, the factory makes x plain, y bland and z ordinary widgets.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, writing your answers with simplified integer coefficients. *(8 marks)*

END OF QUESTIONS

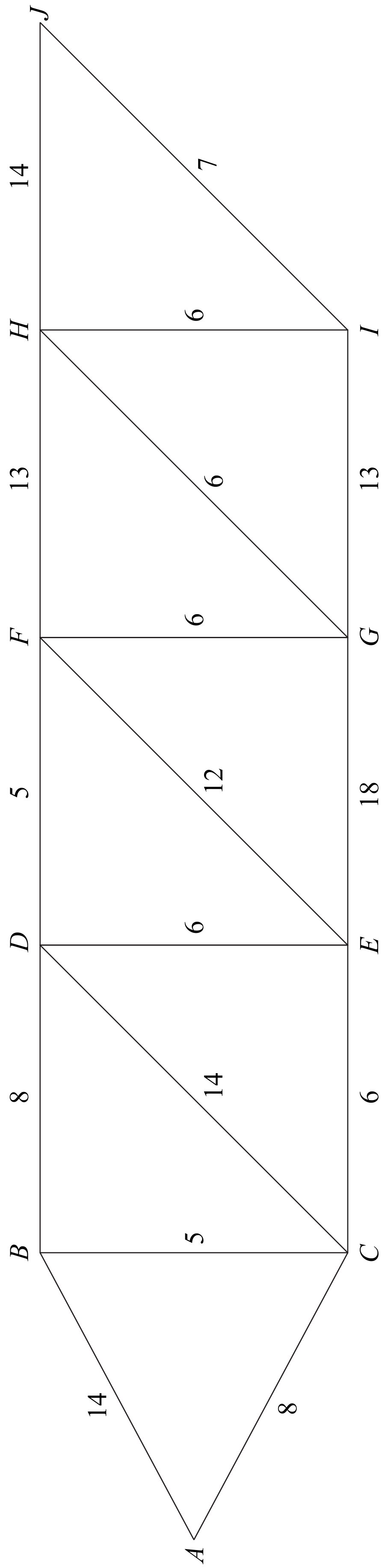
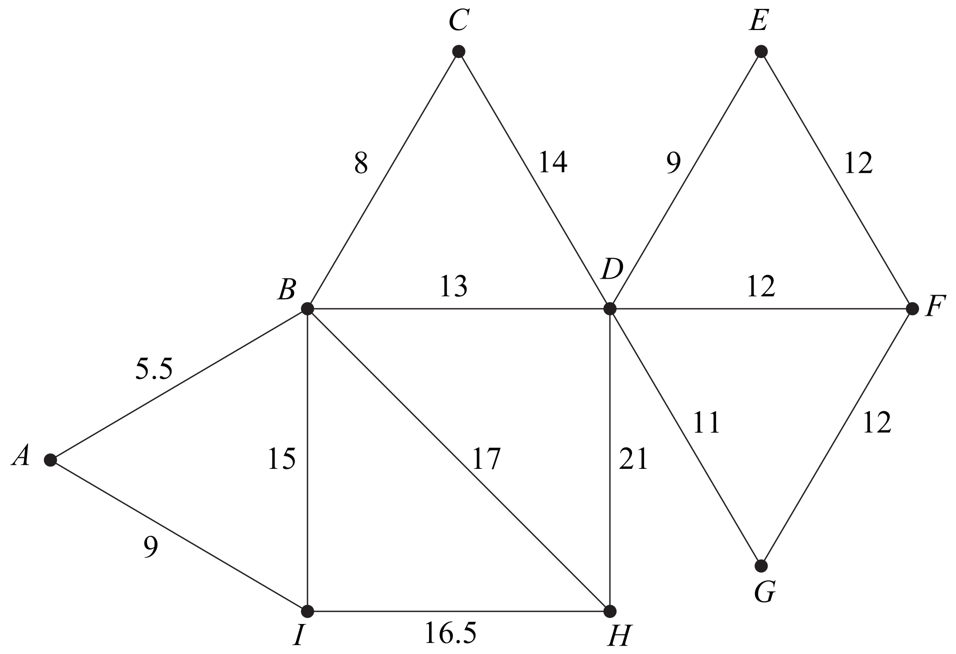


Figure 1 (for Question 5)

Answer **all** questions.

1 The following network shows the lengths, in miles, of roads connecting nine villages.



(a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(b) Find the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

(d) State the number of other spanning trees that are of the same length as your answer in part (a). *(1 mark)*

2 Five people A, B, C, D and E are to be matched to five tasks R, S, T, U and V.

The table shows the tasks that each person is able to undertake.

Person	**Tasks**
A	R, V
B	R, T
C	T, V
D	U, V
E	S, U

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task V, B to task R, C to task T, and E to task U.

Demonstrate, by using an alternating path from this initial matching, how each person can be matched to a task. *(4 marks)*

3 Mark is driving around the one-way system in Leicester. The following table shows the times, in minutes, for Mark to drive between four places: A, B, C and D. Mark decides to start from A, drive to the other three places and then return to A.

Mark wants to keep his driving time to a minimum.

From \ **To**	***A***	***B***	***C***	***D***
A	–	8	6	11
B	14	–	13	25
C	14	9	–	17
D	26	10	18	–

(a) Find the length of the tour $ABCDA$. *(2 marks)*

(b) Find the length of the tour $ADCBA$. *(1 mark)*

(c) Find the length of the tour using the nearest neighbour algorithm starting from A. *(4 marks)*

(d) Write down which of your answers to parts (a), (b) and (c) gives the best upper bound for Mark's driving time. *(1 mark)*

4 (a) A student is using a bubble sort to rearrange seven numbers into ascending order.

Her correct solution is as follows:

Initial list	18	17	13	26	10	14	24
After 1st pass	17	13	18	10	14	24	26
After 2nd pass	13	17	10	14	18	24	26
After 3rd pass	13	10	14	17	18	24	26
After 4th pass	10	13	14	17	18	24	26
After 5th pass	10	13	14	17	18	24	26

Write down the number of comparisons and swaps on each of the five passes.

(6 marks)

(b) Find the maximum number of comparisons and the maximum number of swaps that might be needed in a bubble sort to rearrange seven numbers into ascending order.

(2 marks)

5 A student is using the following algorithm with different values of A and B.

Line 10	Input A, B
Line 20	Let $C = 0$ and let $D = 0$
Line 30	Let $C = C + A$
Line 40	Let $D = D + B$
Line 50	If $C = D$ then go to Line 110
Line 60	If $C > D$ then go to Line 90
Line 70	Let $C = C + A$
Line 80	Go to Line 50
Line 90	Let $D = D + B$
Line 100	Go to Line 50
Line 110	Print C
Line 120	End

(a) (i) Trace the algorithm in the case where $A = 2$ and $B = 3$. *(3 marks)*

(ii) Trace the algorithm in the case where $A = 6$ and $B = 8$. *(3 marks)*

(b) State the purpose of the algorithm. *(1 mark)*

(c) Write down the final value of C in the case where $A = 200$ and $B = 300$. *(1 mark)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

Dino is to have a rectangular swimming pool at his villa.

He wants its width to be at least 2 metres and its length to be at least 5 metres.

He wants its length to be at least twice its width.

He wants its length to be no more than three times its width.

Each metre of the width of the pool costs £1000 and each metre of the length of the pool costs £500.

He has £9000 available.

Let the width of the pool be x metres and the length of the pool be y metres.

(a) Show that one of the constraints leads to the inequality

$$2x + y \leqslant 18$$ *(1 mark)*

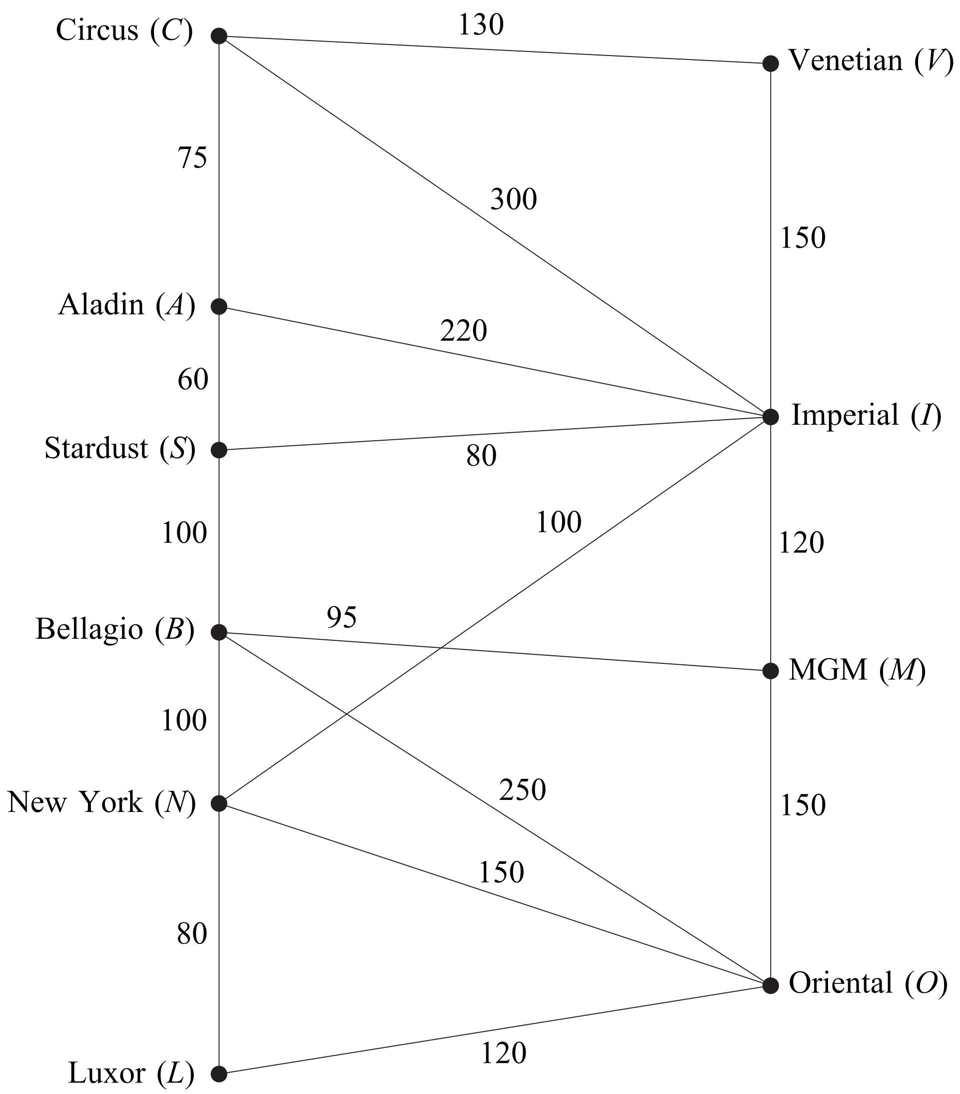
(b) Find four further inequalities. *(3 marks)*

(c) On **Figure 1**, draw a suitable diagram to show the feasible region. *(6 marks)*

(d) Use your diagram to find the maximum width of the pool. State the corresponding length of the pool. *(3 marks)*

7 [**Figure 2**, printed on the insert, is provided for use in this question.]

The network shows the times, in seconds, taken by Craig to walk along walkways connecting ten hotels in Las Vegas.



The total of all the times in the diagram is 2280 seconds.

(a) (i) Craig is staying at the Circus (C) and has to visit the Oriental (O).

Use Dijkstra's algorithm on **Figure 2** to find the minimum time to walk from C to O. *(6 marks)*

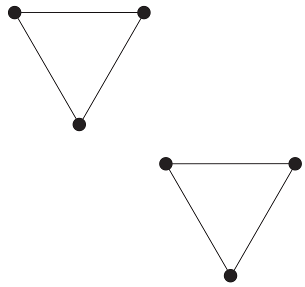
(ii) Write down the corresponding route. *(1 mark)*

(b) (i) Find, by inspection, the shortest time to walk from A to M. *(1 mark)*

(ii) Craig intends to walk along all the walkways. Find the minimum time for Craig to walk along every walkway and return to his starting point. *(6 marks)*

Turn over for the next question

8 (a) The diagram shows a graph **G** with 9 vertices and 9 edges.



(i) State the minimum number of edges that need to be added to **G** to make a connected graph. Draw an example of such a graph. *(2 marks)*

(ii) State the minimum number of edges that need to be added to **G** to make the graph Hamiltonian. Draw an example of such a graph. *(2 marks)*

(iii) State the minimum number of edges that need to be added to **G** to make the graph Eulerian. Draw an example of such a graph. *(2 marks)*

(b) A complete graph has n vertices and is Eulerian.

(i) State the condition that n must satisfy. *(1 mark)*

(ii) In addition, the number of edges in a Hamiltonian cycle for the graph is the same as the number of edges in an Eulerian trail. State the value of n. *(1 mark)*

END OF QUESTIONS

Figure 1 (for use in Question 6)

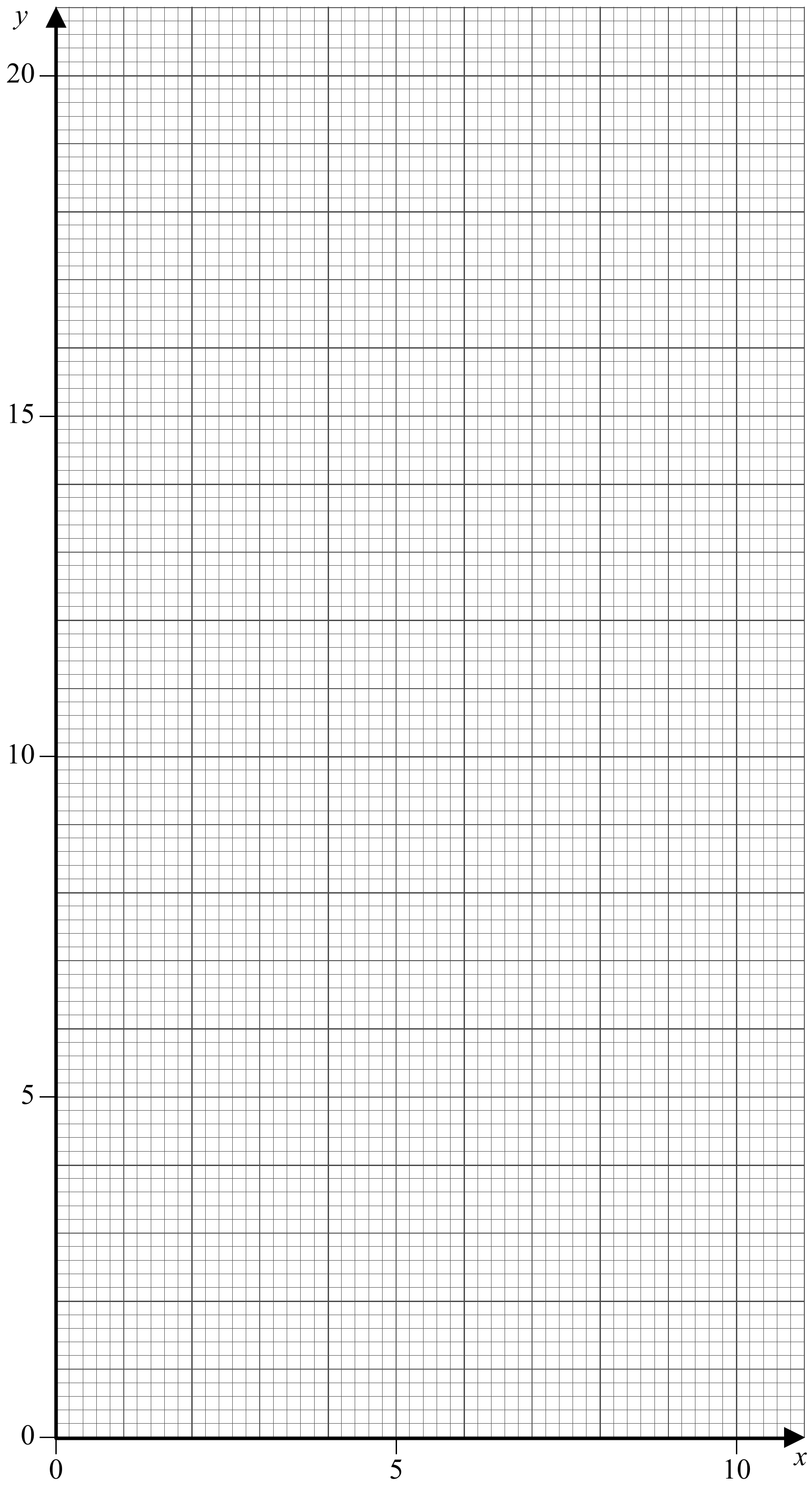
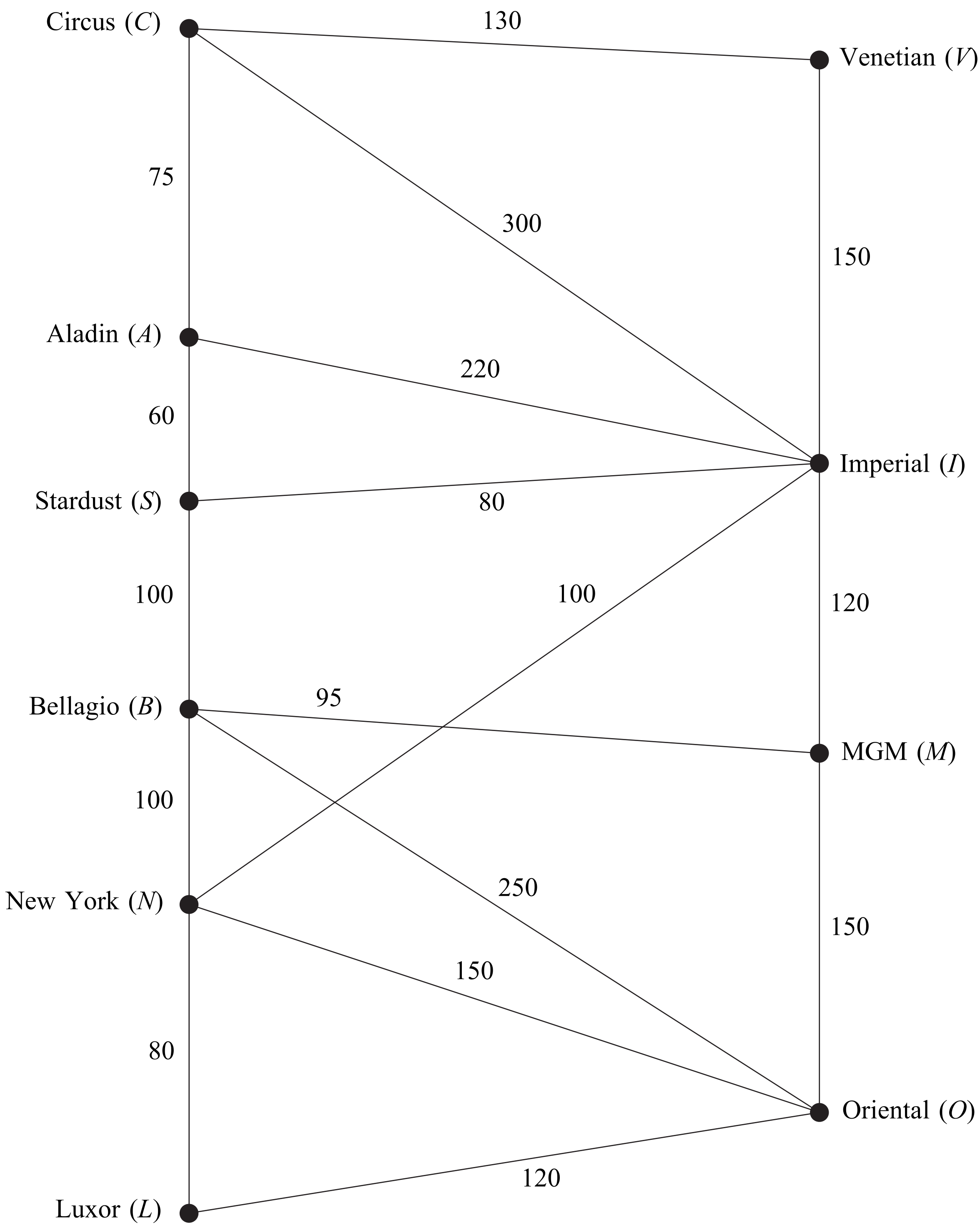


Figure 2 (for use in Question 7)



1.

Max Lauren John Hannah Kieran Tara Richard Imogen

(a) Use a quick sort to produce a list of these names in ascending alphabetical order.
You must make your pivots clear.

(5)

(b) Use the binary search algorithm on your list from part (a) to try to locate the name 'Hugo'.

(4)

(Total 9 marks)

2.

	A	**B**	**C**	**D**	**E**	**F**
A	-	24	-	-	23	22
B	24	-	18	19	17	20
C	-	18	-	11	14	-
D	-	19	11	-	13	-
E	23	17	14	13	-	21
F	22	20	-	-	21	-

The table shows the distances, in metres, between six vertices, **A**, **B**, **C**, **D**, **E** and **F**, in a network.

(a) Draw the weighted network using the vertices given in Diagram 1 in the answer booklet.

(3)

(b) Use Kruskal's algorithm to find a minimum spanning tree. You should list the edges in the order that you consider them and state whether you are adding them to your minimum spanning tree.

(3)

(c) Draw your tree on Diagram 2 in the answer booklet and find its total weight.

(2)

(Total 8 marks)

---

3. (a) Draw the activity network described in this precedence table, using activity on arc and exactly two dummies.

(5)

Activity	Immediately preceding activities
A	-
B	-
C	-
D	**B**
E	**B, C**
F	**B, C**
G	**F**
H	**F**
I	**G, H**
J	**I**

(b) Explain why each of the two dummies is necessary.

(2)

(Total 7 marks)

4.

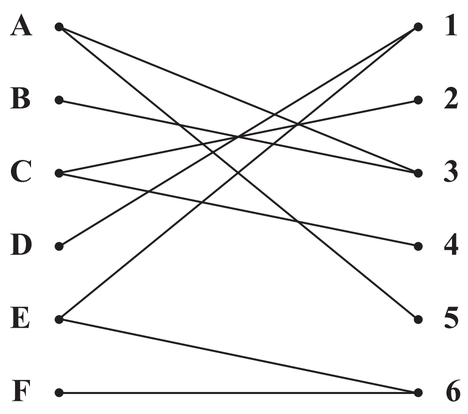


Figure 1

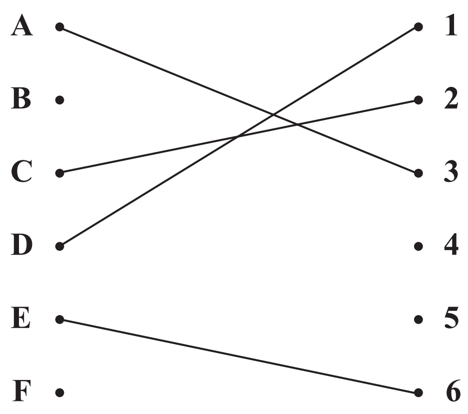


Figure 2

Figure 1 shows the possible allocations of six people, A, B, C, D, E and F, to six tasks, 1, 2, 3, 4, 5 and 6.
Figure 2 shows an initial matching.

(a) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You must list the alternating path used, and your improved matching.
(3)

(b) Explain why it is not possible to find a complete matching.
(2)

D now has task 2 added to their possible allocation.

(c) Using the improved matching found in part (a) as the new initial matching, use the maximum matching algorithm to find a complete matching. You must list the alternating path used and your complete matching.
(3)

(Total 8 marks)

5.

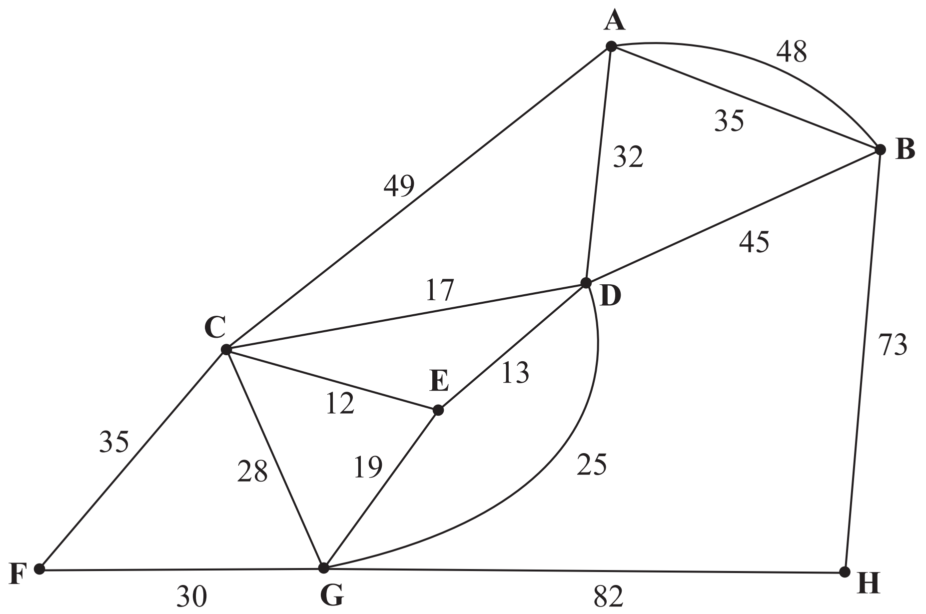


Figure 3

(The total weight of the network in Figure 3 is 543 km.)

Figure 3 models a network of railway tracks that have to be inspected. The number on each arc is the length, in km, of that section of railway track.
Each track must be traversed at least once and the length of the inspection route must be minimised.
The inspection route must start and finish at the same vertex.

(a) Use an appropriate algorithm to find the length of the shortest inspection route. You should make your method and working clear.

(5)

It is now permitted to start and finish the inspection at two distinct vertices.

(b) State which two vertices should be chosen to minimise the length of the new route. Give a reason for your answer.

(3)

(Total 8 marks)

6.

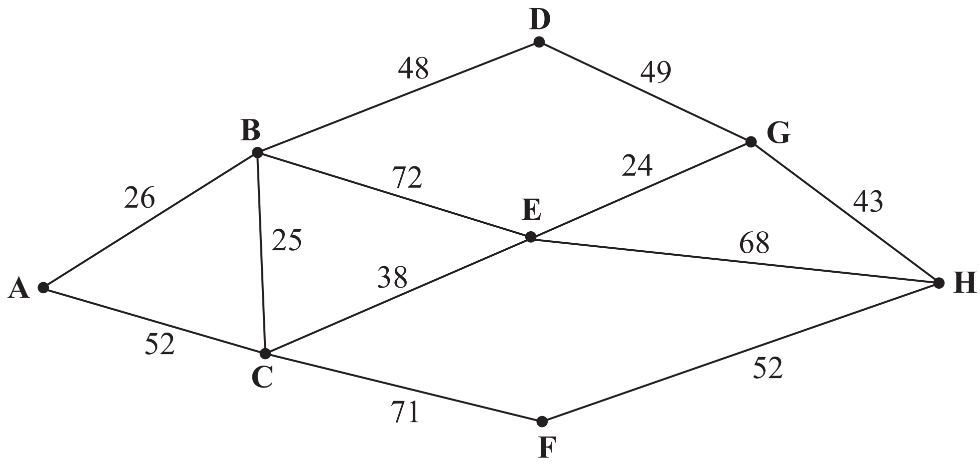


Figure 4

Figure 4 shows a network of roads through eight villages, A, B, C, D, E, F, G and H. The number on each arc is the length of that road in km.

(a) Use Dijkstra's algorithm to find the shortest route from A to H. State your shortest route and its length.

(5)

There is a fair in village C and you cannot drive through the village. A shortest route from A to H which avoids C needs to be found.

(b) State this new minimal route and its length.

(2)

(Total 7 marks)

7. A linear programming problem is modelled by the following constraints

$$8x + 3y \leqslant 480$$
$$8x + 7y \geqslant 560$$
$$y \geqslant 4x$$
$$x, y \geqslant 0$$

(a) Use the grid provided in your answer book to represent these inequalities graphically. Hence determine the feasible region and label it R.

(6)

The objective function, F, is given by

$$F = 3x + y$$

(b) Making your method clear, determine

(i) the minimum value of the function F and the coordinates of the optimal point,

(ii) the maximum value of the function F and the coordinates of the optimal point.

(6)

(Total 12 marks)

8.

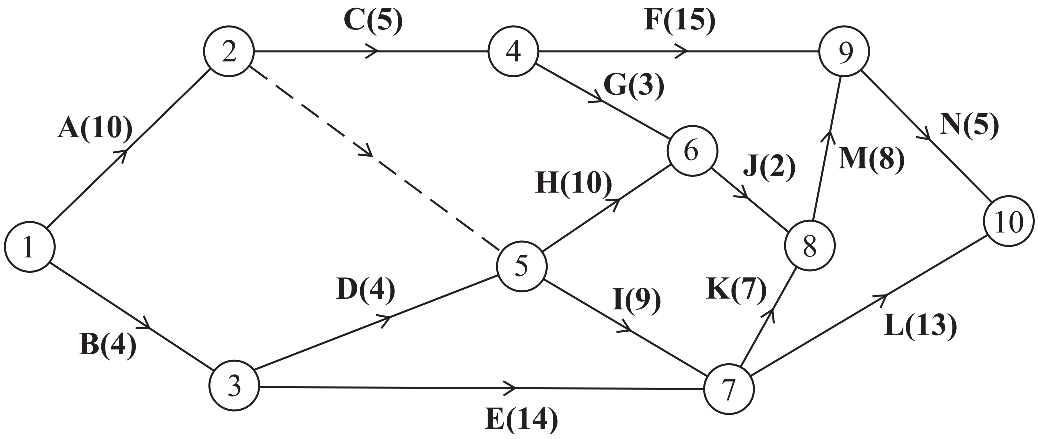


Figure 5

The network in Figure 5 shows the activities involved in a process. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, taken to complete the activity.

(a) Calculate the early time and the late time for each event, showing them on the diagram in the answer book.

(4)

(b) Determine the critical activities and the length of the critical path.

(3)

(c) Calculate the total float on activities F and G. You **must** make the numbers you used in your calculation clear.

(3)

(d) On the grid in the answer book, draw a cascade (Gantt) chart for the process.

(4)

Given that each task requires just one worker,

(e) use your cascade chart to determine the minimum number of workers required to complete the process in the minimum time. Explain your reasoning clearly.

(2)

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer book for Practice 3

Leave blank

1.

Leave blank

(Question 1 continued)

Q1

(Total 9 marks)

Turn over

Leave blank

2.

	A	**B**	**C**	**D**	**E**	**F**
A	-	24	-	-	23	22
B	24	-	18	19	17	20
C	-	18	-	11	14	-
D	-	19	11	-	13	-
E	23	17	14	13	-	21
F	22	20	-	-	21	-

(a)

Diagram 1

Leave blank

(Question 2 continued)

(b)

(c)

Diagram 2

Total weight of tree ____________

(Total 8 marks)

Q2

Leave blank

3. (a)

Leave blank

(Question 3 continued)

(b)

Q3

(Total 7 marks)

Turn over

Leave blank

4.

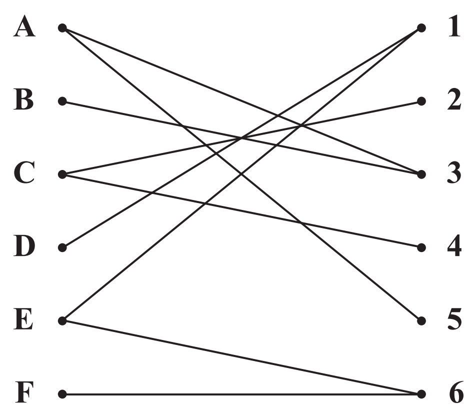


Figure 1

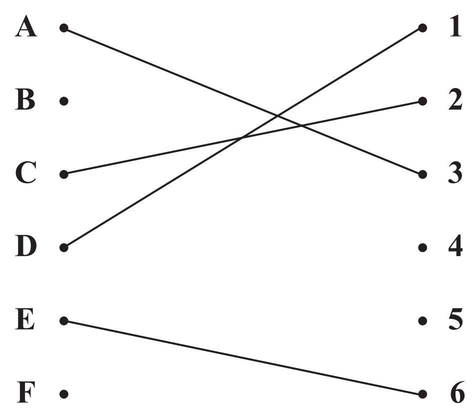


Figure 2

Leave blank

A •	• 1	A •	• 1
B •	• 2	B •	• 2
C •	• 3	C •	• 3
D •	• 4	D •	• 4
E •	• 5	E •	• 5
F •	• 6	F •	• 6

Q4

(Total 8 marks)

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5.

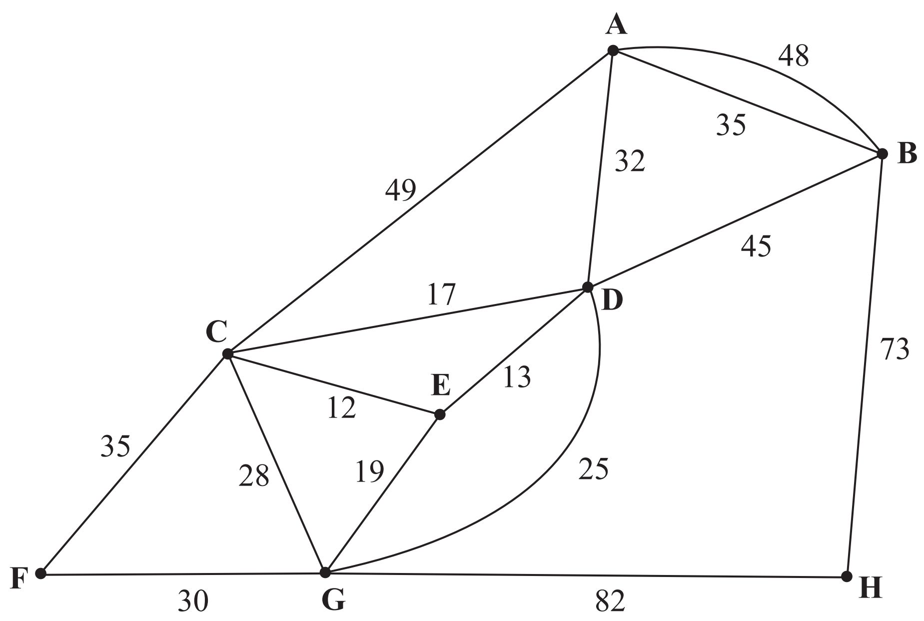


Figure 3

(The total weight of the network in Figure 3 is 543 km.)

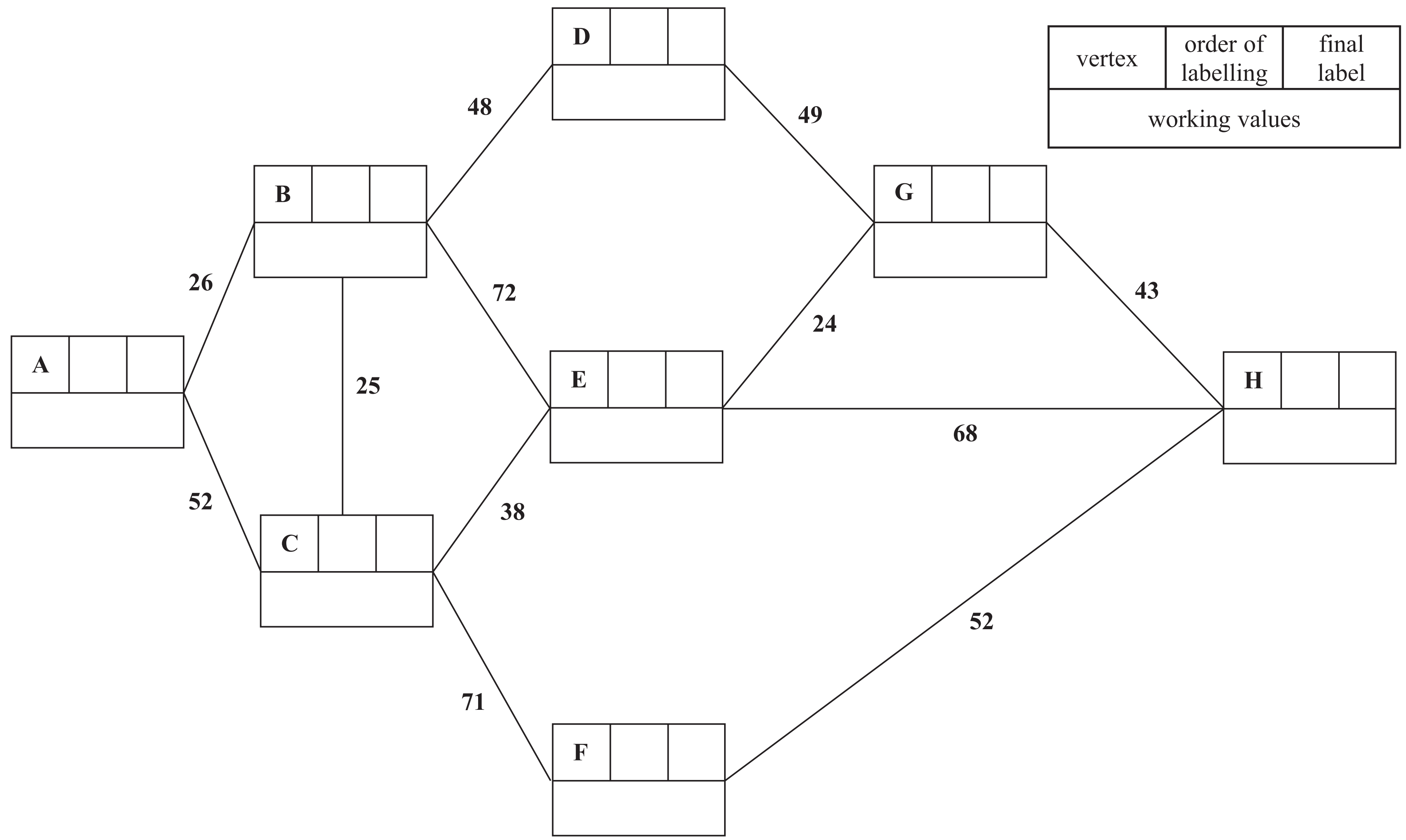
Leave blank

(Question 5 continued)

(Total 8 marks)

Q5

6. (a)



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(Question 6 continued)

shortest route: ______________________ length: ______________

(b) ______________________

shortest route (avoiding C): ______________________

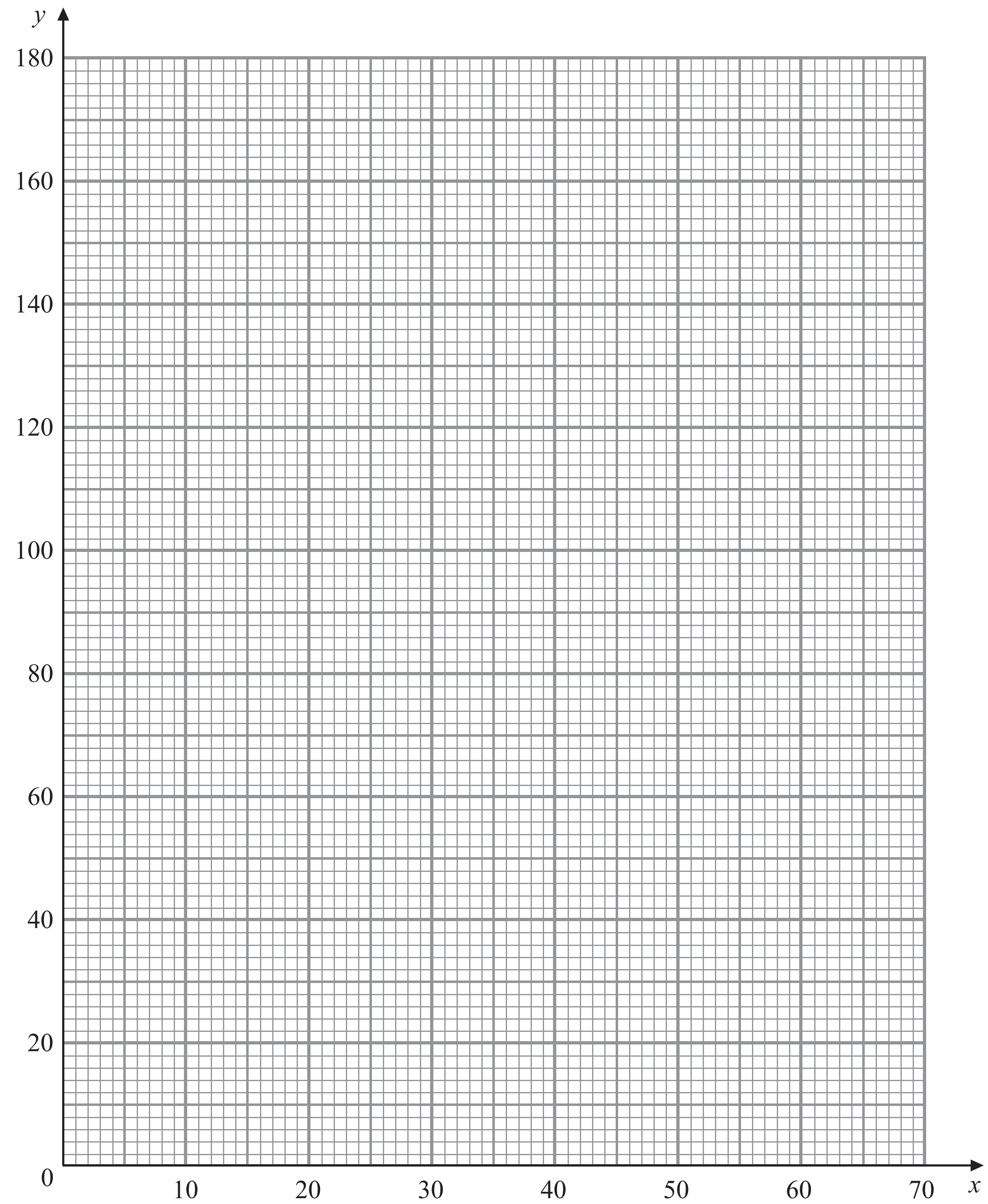
length: ______________

(Total 7 marks)

Q6

Leave blank

7.



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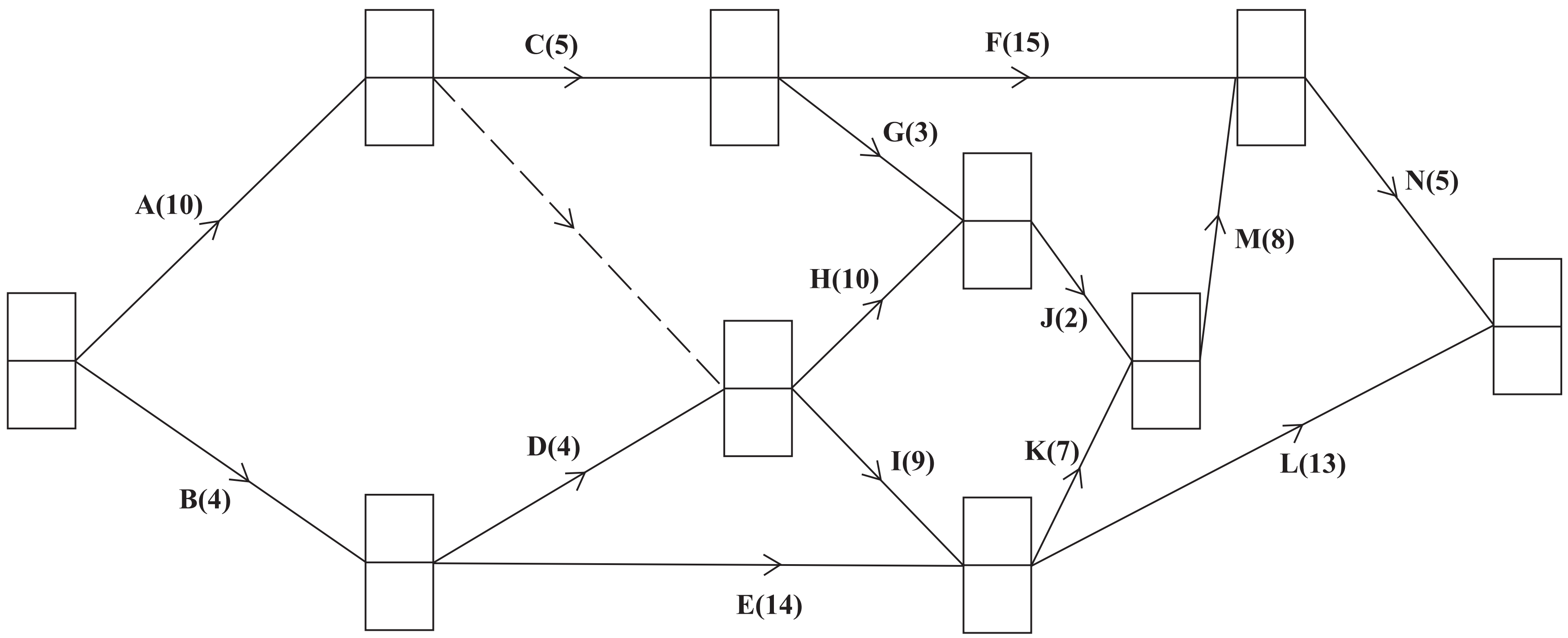
(Question 7 continued)

(Total 12 marks)

Q7

8. (a)



(b) Critical activities: ____________________

Length of critical path: ____________________

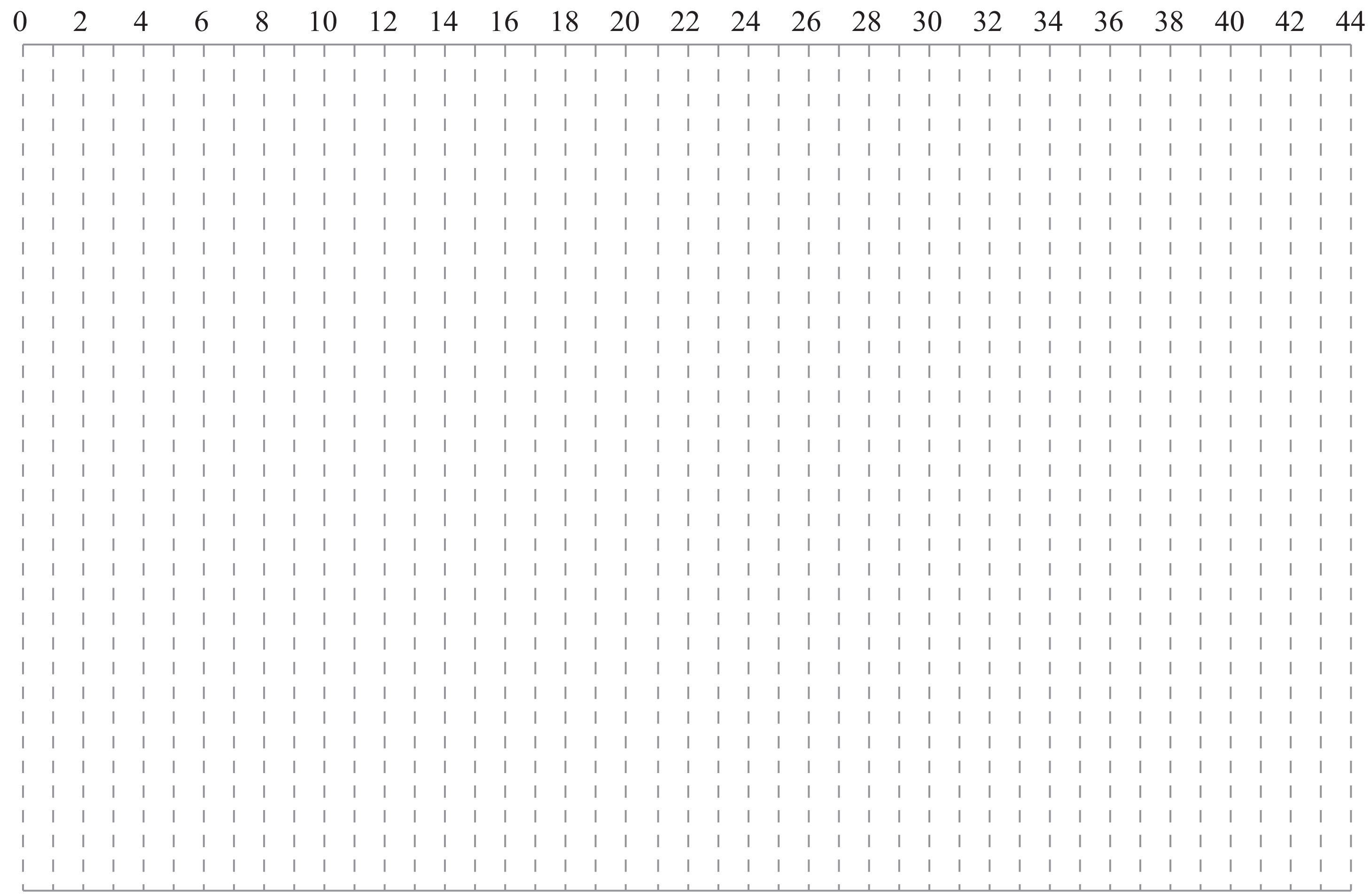
(c) Total float on activity F: ____________________

Total float on activity G: ____________________

Leave blank

(Question 8 continued)

(d)



(e)

Leave blank

Q8

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 Five people, A, B, C, D and E, are to be matched to five tasks, J, K, L, M and N. The table shows the tasks that each person is able to undertake.

Person	**Task**
A	J, N
B	J, L
C	L, N
D	M, N
E	K, M

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Initially, A is matched to task N, B to task J, C to task L, and E to task M.

Complete the alternating path $D-M$ ..., from this initial matching, to demonstrate how each person can be matched to a task. *(3 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by

$$x + y \leqslant 30$$
$$2x + y \leqslant 40$$
$$y \geqslant 5$$
$$x \geqslant 4$$
$$y \geqslant \tfrac{1}{2}x$$

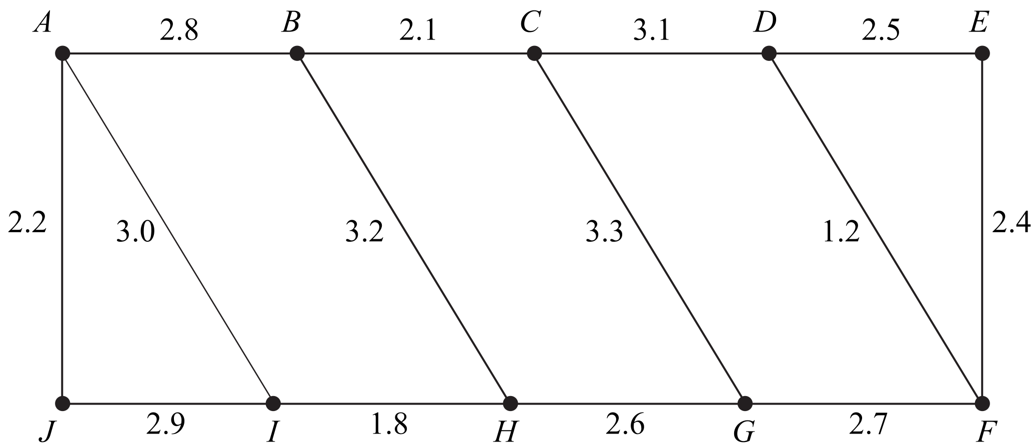
(a) On **Figure 1**, draw a suitable diagram to represent these inequalities and indicate the feasible region. *(5 marks)*

(b) Use your diagram to find the maximum value of F, on the feasible region, in the case where:

(i) $F = 3x + y$; *(2 marks)*

(ii) $F = x + 3y$. *(2 marks)*

3 The diagram shows 10 bus stops, A, B, C, ..., J, in Geneva. The number on each edge represents the distance, in kilometres, between adjacent bus stops.



The city council is to connect these bus stops to a computer system which will display waiting times for buses at each of the 10 stops. Cabling is to be laid between some of the bus stops.

(a) Use Kruskal's algorithm, showing the order in which you select the edges, to find a minimum spanning tree for the 10 bus stops. *(5 marks)*

(b) State the minimum length of cabling needed. *(1 mark)*

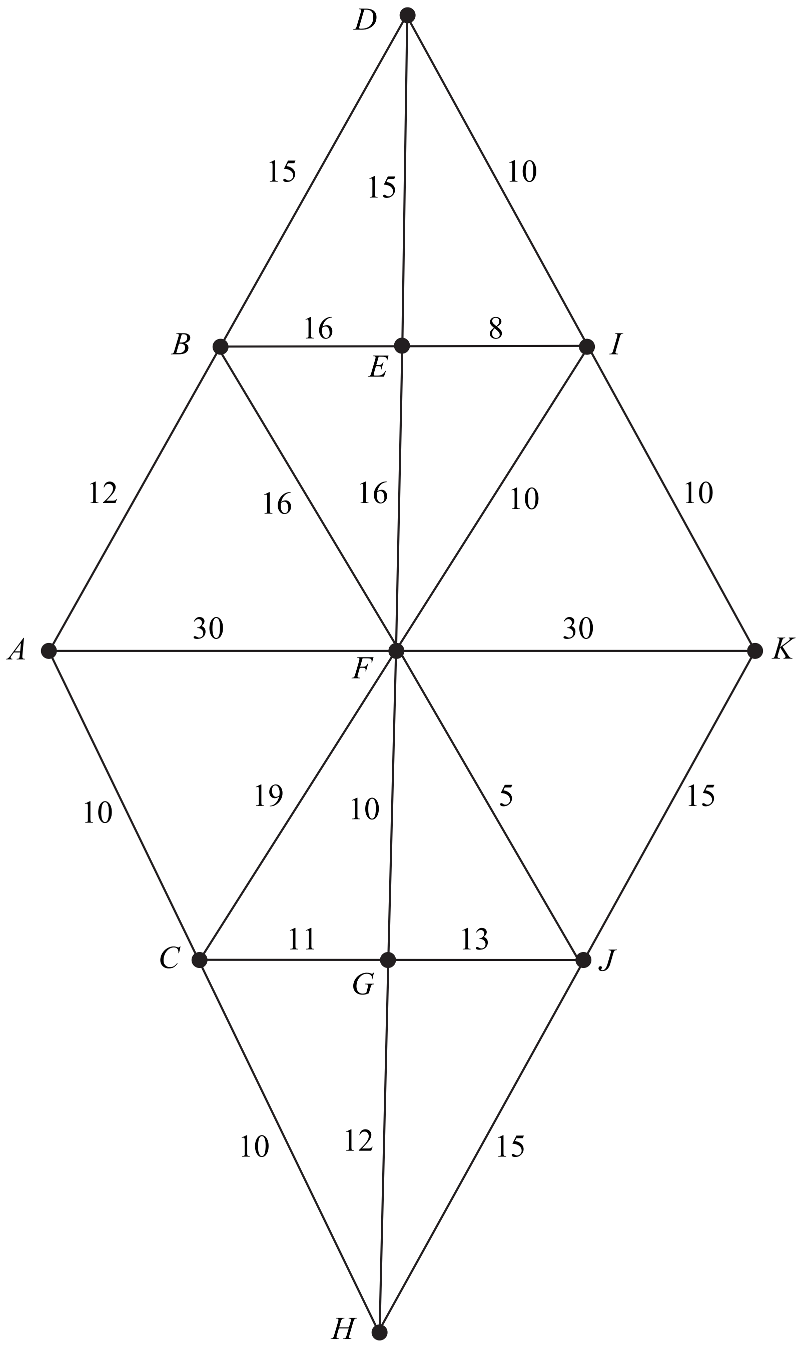
(c) Draw your minimum spanning tree. *(2 marks)*

(d) If Prim's algorithm, starting from A, had been used to find the minimum spanning tree, state which edge would have been the final edge to complete the minimum spanning tree. *(2 marks)*

Turn over for the next question

4 [**Figure 2**, printed on the insert, is provided for use in this question.]

The network shows 11 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



The total of all of the times is 308 minutes.

(a) (i) Use Dijkstra's algorithm on **Figure 2** to find the minimum time to travel from A to K. *(6 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) Find the length of an optimum Chinese postman route around the network, starting and finishing at A. (The minimum time to travel from D to H is 40 minutes.) *(5 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

(a) James is solving a travelling salesperson problem.

(i) He finds the following upper bounds: 43, 40, 43, 41, 55, 43, 43.

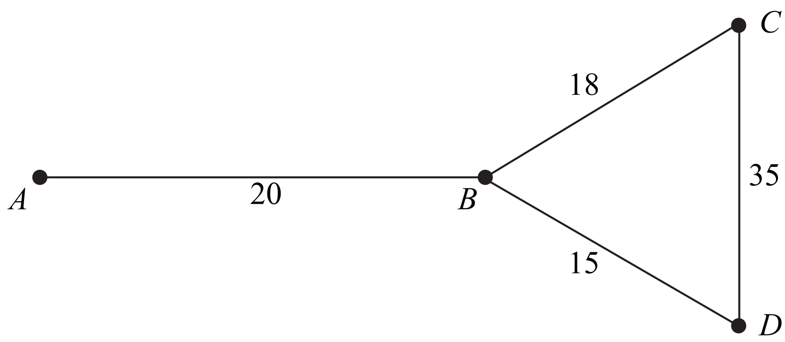
Write down the best upper bound. *(1 mark)*

(ii) James finds the following lower bounds: 33, 40, 33, 38, 33, 38, 38.

Write down the best lower bound. *(1 mark)*

(b) Karen is solving a different travelling salesperson problem and finds an upper bound of 55 and a lower bound of 45. Write down an interpretation of these results. *(1 mark)*

(c) The diagram below shows roads connecting 4 towns, A, B, C and D. The numbers on the edges represent the lengths of the roads, in kilometres, between adjacent towns.



Xiong lives at town A and is to visit each of the other three towns before returning to town A. She wishes to find a route that will minimise her travelling distance.

(i) Complete **Figure 3**, on the insert, to show the shortest distances, in kilometres, between **all** pairs of towns. *(2 marks)*

(ii) Use the nearest neighbour algorithm on **Figure 3** to find an upper bound for the minimum length of a tour of this network that starts and finishes at A. *(3 marks)*

(iii) Hence find the actual route that Xiong would take in order to achieve a tour of the same length as that found in part (c)(ii). *(2 marks)*

6 A student is solving cubic equations that have three different positive integer solutions.

The algorithm that the student is using is as follows:

Line 10	Input A, B, C, D
Line 20	Let $K = 1$
Line 30	Let $N = 0$
Line 40	Let $X = K$
Line 50	Let $Y = AX^3 + BX^2 + CX + D$
Line 60	If $Y \neq 0$ then go to Line 100
Line 70	Print X, "is a solution"
Line 80	Let $N = N + 1$
Line 90	If $N = 3$ then go to Line 120
Line 100	Let $K = K + 1$
Line 110	Go to Line 40
Line 120	End

(a) Trace the algorithm in the case where the input values are:

(i) $A = 1$, $B = -6$, $C = 11$ and $D = -6$; *(4 marks)*

(ii) $A = 1$, $B = -10$, $C = 29$ and $D = -20$. *(4 marks)*

(b) Explain where and why this algorithm will fail if $A = 0$. *(2 marks)*

7 The numbers 17, 3, 16 and 4 are to be sorted into ascending order.

The following four methods are to be compared: bubble sort, shuttle sort, Shell sort and quick sort (with the first number used as the pivot).

A student uses each of the four methods and produces the correct solutions below. Each solution shows the order of the numbers after each pass.

Solution 1	17	3	16	4
	3	17	16	4
	3	16	17	4
	3	4	16	17
Solution 2	17	3	16	4
	16	3	17	4
	3	4	16	17
Solution 3	17	3	16	4
	3	16	4	17
	3	16	4	17
	3	4	16	17
Solution 4	17	3	16	4
	3	16	4	17
	3	4	16	17
	3	4	16	17

(a) Write down which of the four solutions is the bubble sort, the shuttle sort, the Shell sort and the quick sort. *(3 marks)*

(b) For each of the four solutions, write down the number of comparisons and swaps (exchanges) on the first pass. *(8 marks)*

Turn over for the next question

8 Each day, a factory makes three types of hinge: basic, standard and luxury. The hinges produced need three different components: type A, type B and type C.

Basic hinges need 2 components of type A, 3 components of type B and 1 component of type C.

Standard hinges need 4 components of type A, 2 components of type B and 3 components of type C.

Luxury hinges need 3 components of type A, 4 components of type B and 5 components of type C.

Each day, there are 360 components of type A available, 270 of type B and 450 of type C.

Each day, the factory must use at least 720 components in total.

Each day, the factory must use at least 40% of the total components as type A.

Each day, the factory makes x basic hinges, y standard hinges and z luxury hinges.

In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, find five inequalities, each involving x, y and z, which must be satisfied. Simplify each inequality where possible. *(8 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 2)

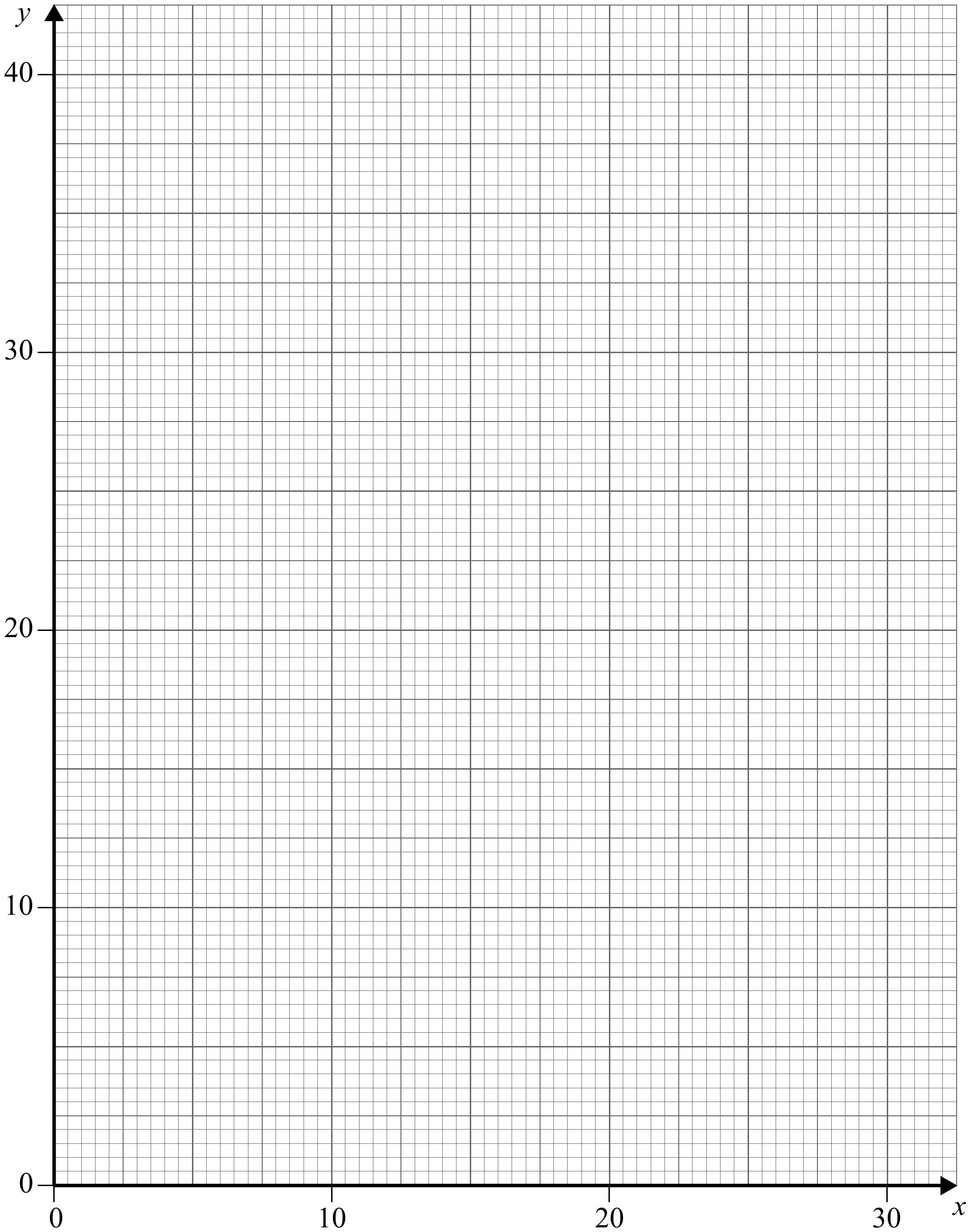


Figure 2 (for use in Question 4)

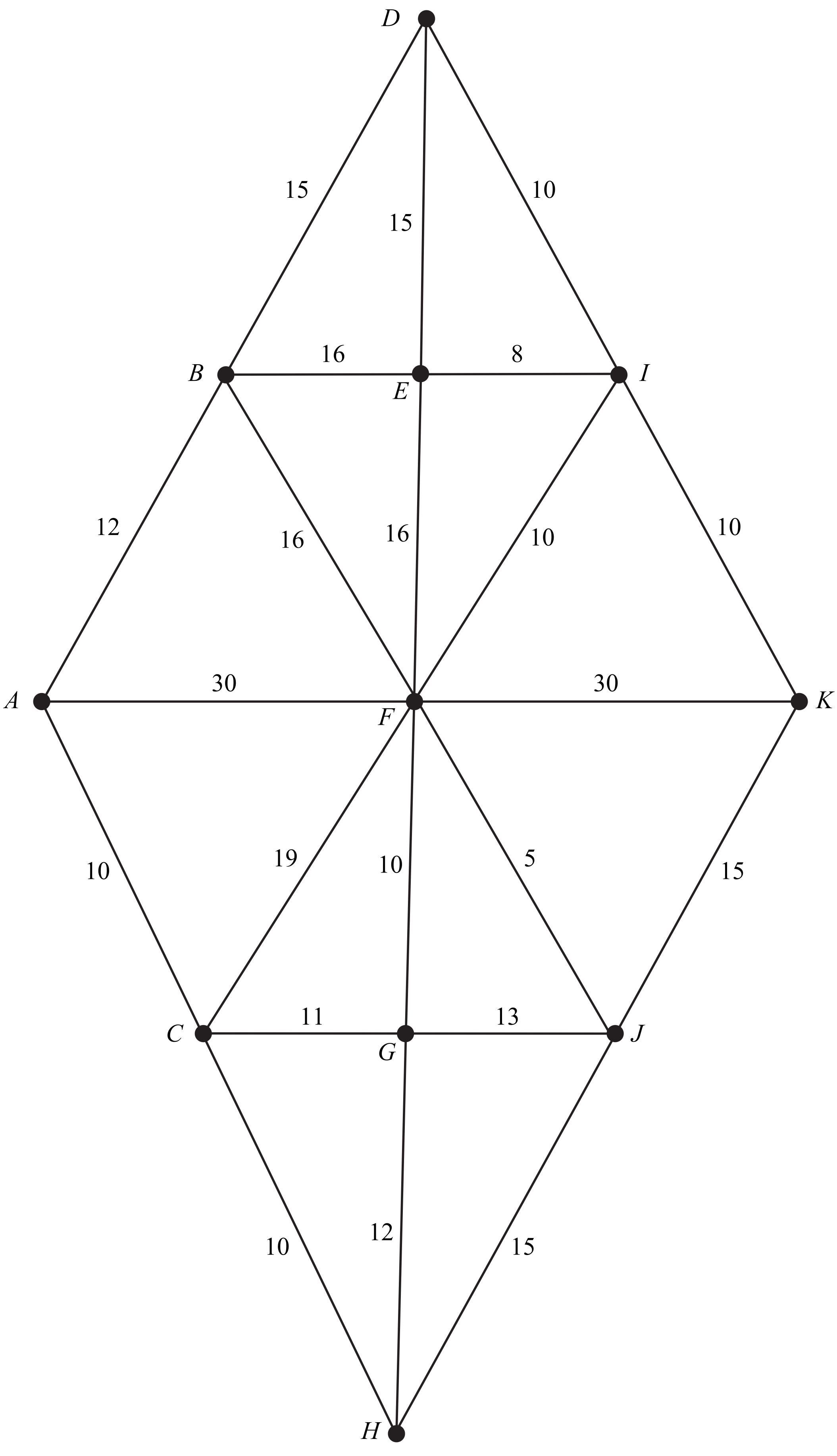


Figure 3 (for use in Question 5)

	A	B	C	D
A	—		38	
B		—		
C	38		—	
D				—

Answer **all** questions.

1 Six girls, Alfonsa (A), Bianca (B), Claudia (C), Desiree (D), Erika (E) and Flavia (F), are going to a pizza restaurant. The restaurant provides a special menu of six different pizzas: Margherita (M), Neapolitana (N), Pepperoni (P), Romana (R), Stagioni (S) and Viennese (V).

The table shows the pizzas that each girl likes.

Girl	**Pizza**
Alfonsa (A)	Margherita (M), Pepperoni (P), Stagioni (S)
Bianca (B)	Neapolitana (N), Romana (R)
Claudia (C)	Neapolitana (N), Viennese (V)
Desiree (D)	Romana (R), Stagioni (S)
Erika (E)	Pepperoni (P), Stagioni (S), Viennese (V)
Flavia (F)	Romana (R)

(a) Show this information on a bipartite graph. *(2 marks)*

(b) Each girl is to eat a different pizza. Initially, the waiter brings six different pizzas and gives Alfonsa the Pepperoni, Bianca the Romana, Claudia the Neapolitana and Erika the Stagioni. The other two pizzas are put in the middle of the table.

From this initial matching, use the maximum matching algorithm to obtain a complete matching so that every girl gets a pizza that she likes. List your complete matching. *(5 marks)*

2 (a) Use a bubble sort to rearrange the following numbers into ascending order.

13 16 10 11 4 12 6 7 *(5 marks)*

(b) State the number of comparisons and the number of swaps (exchanges) for each of the first three passes. *(3 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The feasible region of a linear programming problem is represented by the following:

$$x \geqslant 0,\ y \geqslant 0$$
$$x + 4y \leqslant 36$$
$$4x + y \leqslant 68$$
$$y \leqslant 2x$$
$$y \geqslant \tfrac{1}{4}x$$

(a) On **Figure 1**, draw a suitable diagram to represent the inequalities and indicate the feasible region. *(6 marks)*

(b) Use your diagram to find the maximum value of P, stating the corresponding coordinates, on the feasible region, in the case where:

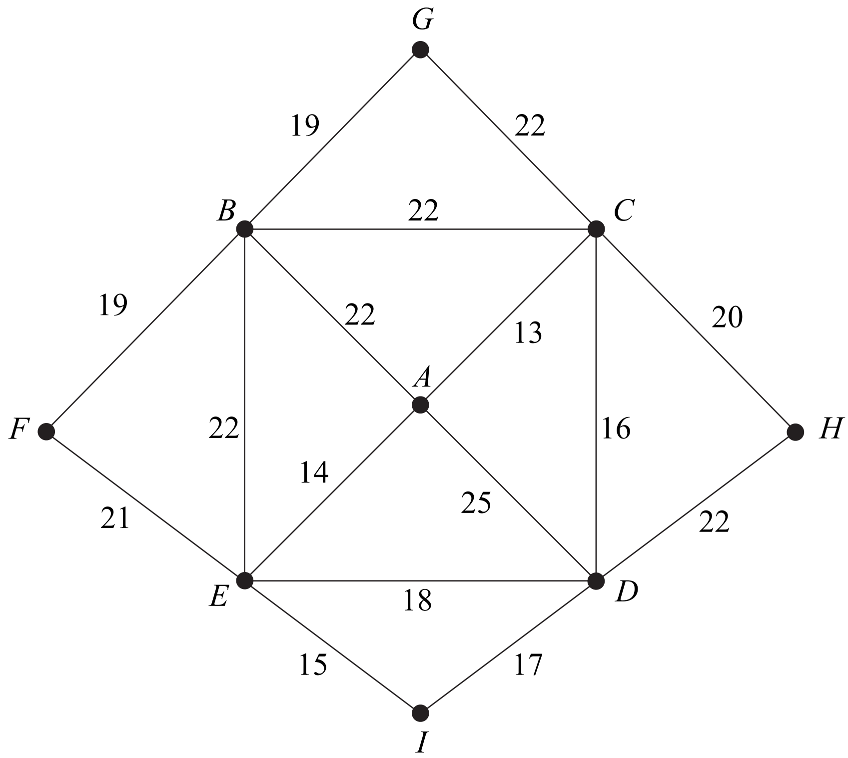
(i) $P = x + 5y$; *(2 marks)*

(ii) $P = 5x + y$. *(2 marks)*

Turn over for the next question

4 In Paris, there is a park where there are statues of famous people; there are many visitors each day to this park. Lighting is to be installed at nine places, A, B, …, I, in the park. The places have to be connected either directly or indirectly by cabling, to be laid alongside the paths, as shown in the diagram.

The diagram shows the length of each path, in metres, connecting adjacent places.



Total length of paths = 307 metres

(a) (i) Use Prim's algorithm, starting from A, to find the minimum length of cabling required. *(5 marks)*

(ii) State this minimum length. *(1 mark)*

(iii) Draw the minimum spanning tree. *(2 marks)*

(b) A security guard walks along all the paths before returning to his starting place. Find the length of an optimal Chinese postman route for the guard. *(6 marks)*

5 There is a one-way system in Manchester. Mia is parked at her base, B, in Manchester and intends to visit four other places, A, C, D and E, before returning to her base. The following table shows the distances, in kilometres, for Mia to drive between the five places A, B, C, D and E. Mia wants to keep the total distance that she drives to a minimum.

From \ To	***A***	***B***	***C***	***D***	***E***
A	–	1.7	1.9	1.8	2.1
B	3.1	–	2.5	1.8	3.7
C	3.1	2.9	–	2.7	4.2
D	2.0	2.8	2.1	–	2.3
E	2.2	3.6	1.9	1.7	–

(a) Find the length of the tour $BECDAB$. *(1 mark)*

(b) Find the length of the tour obtained by using the nearest neighbour algorithm starting from B. *(4 marks)*

(c) Write down which of your answers to parts (a) and (b) would be the better upper bound for the total distance that Mia drives. *(1 mark)*

(d) On a particular day, the council decides to reverse the one-way system. For this day, find the length of the tour obtained by using the nearest neighbour algorithm starting from B. *(4 marks)*

Turn over for the next question

Turn over ▶

6 A student is finding a numerical approximation for the area under a curve.

The algorithm that the student is using is as follows:

Line 10	Input A, B, N
Line 20	Let $T = 0$
Line 30	Let $D = A$
Line 40	Let $H = (B - A)/N$
Line 50	Let $E = H/2$
Line 60	Let $T = T + A^3 + B^3$
Line 70	Let $D = D + H$
Line 80	If $D = B$ then go to line 110
Line 90	Let $T = T + 2D^3$
Line 100	Go to line 70
Line 110	Print 'Area = ', $T \times E$
Line 120	End

Trace the algorithm in the case where the input values are:

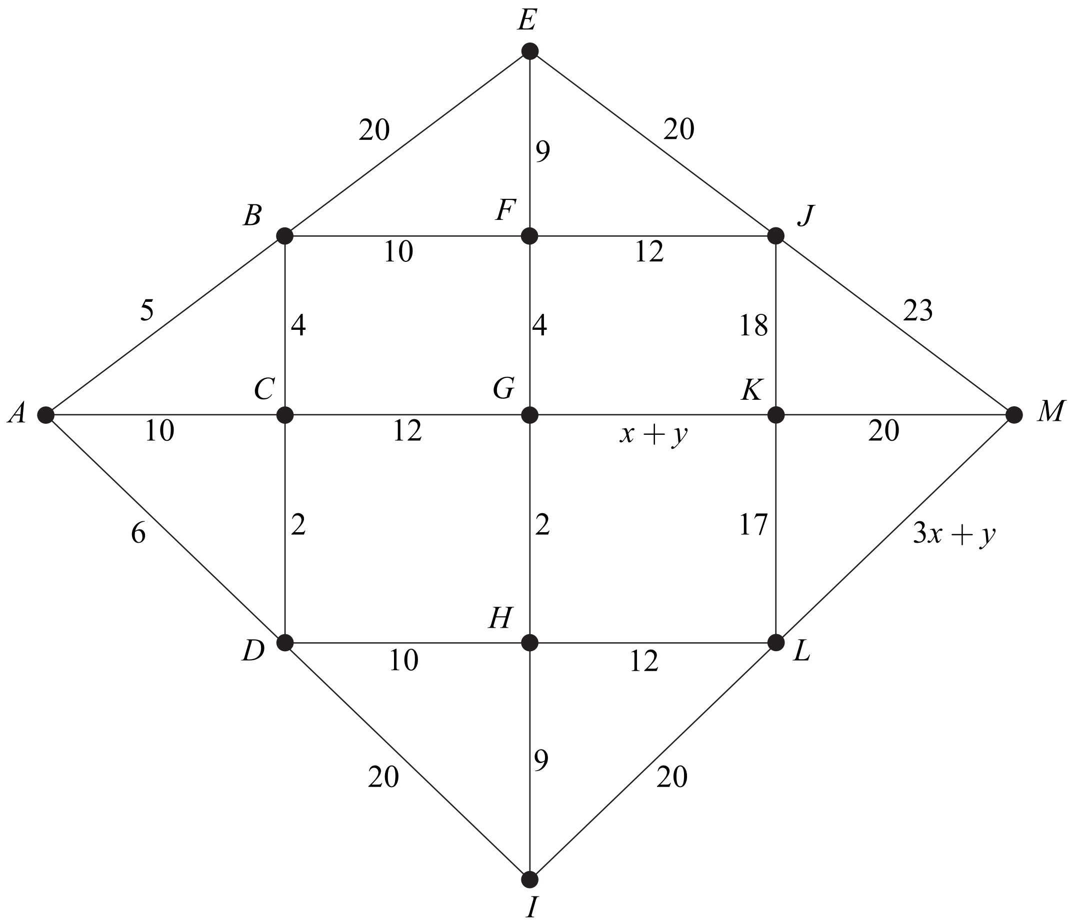
(a) $A = 1,\ B = 5,\ N = 2;$ *(4 marks)*

(b) $A = 1,\ B = 5,\ N = 4.$ *(4 marks)*

7 [**Figure 2**, printed on the insert, is provided for use in this question.]

The following network has 13 vertices and 24 edges connecting some pairs of vertices. The number on each edge is its weight.

The weights on the edges GK and LM are functions of x and y, where $x > 0$, $y > 0$ and $10 < x + y < 27$.



There are three routes from A to M of the same minimum total weight.

(a) Use Dijkstra's algorithm on **Figure 2** to find this minimum total weight. *(7 marks)*

(b) Find the values of x and y. *(3 marks)*

Turn over for the next question

8 A factory packs three different kinds of novelty box: red, blue and green. Each box contains three different types of toy: A, B and C.

Each red box has 2 type A toys, 3 type B toys and 4 type C toys.

Each blue box has 3 type A toys, 1 type B toy and 3 type C toys.

Each green box has 4 type A toys, 5 type B toys and 2 type C toys.

Each day, the maximum number of each type of toy available to be packed is 360 type A, 300 type B and 400 type C.

Each day, the factory must pack more type A toys than type B toys.

Each day, the total number of type A and type B toys that are packed must together be at least as many as the number of type C toys that are packed.

Each day, at least 40% of the total toys that are packed must be type C toys.

Each day, the factory packs x red boxes, y blue boxes and z green boxes.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, simplifying your answers. *(8 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 3)

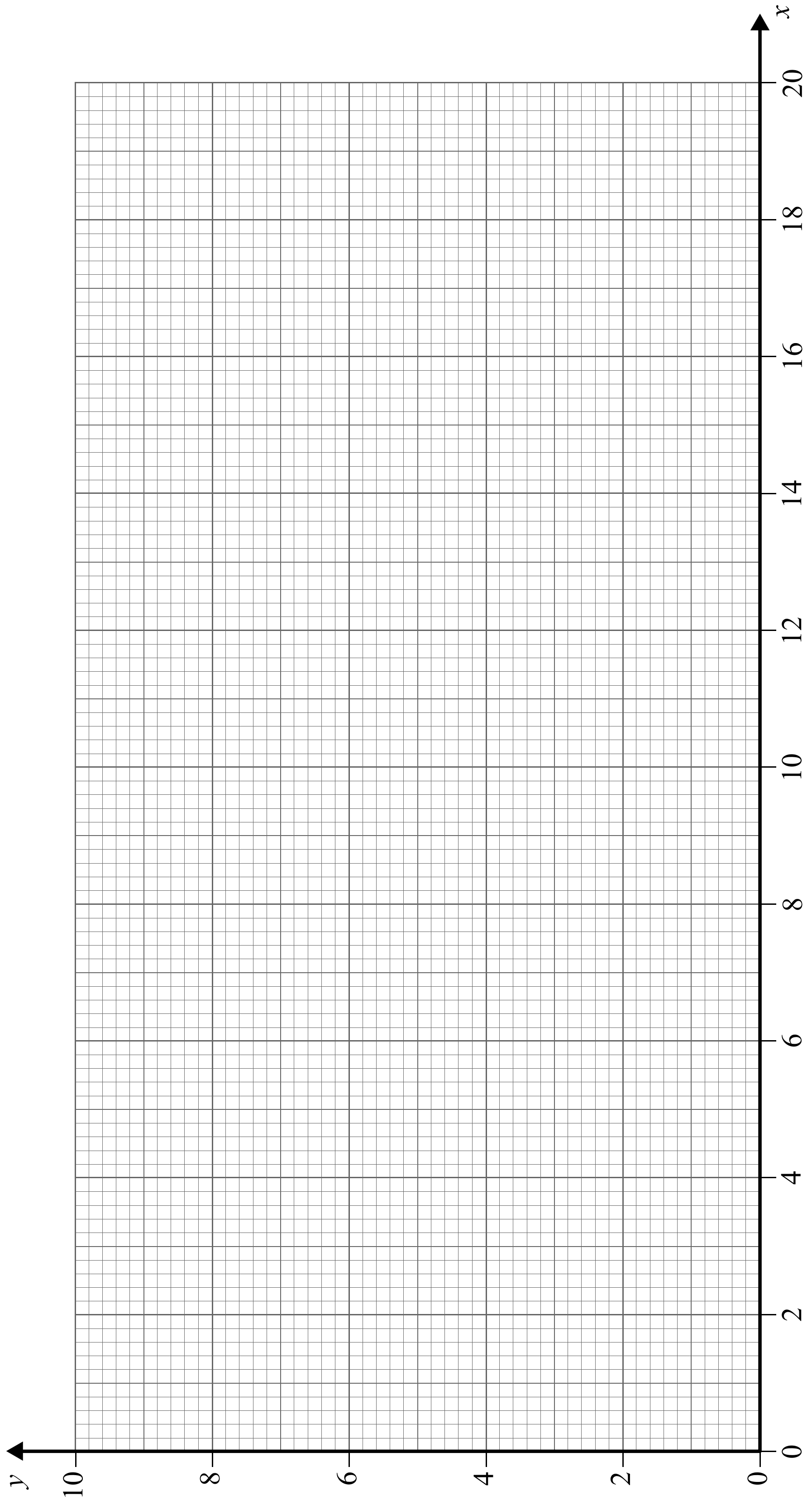
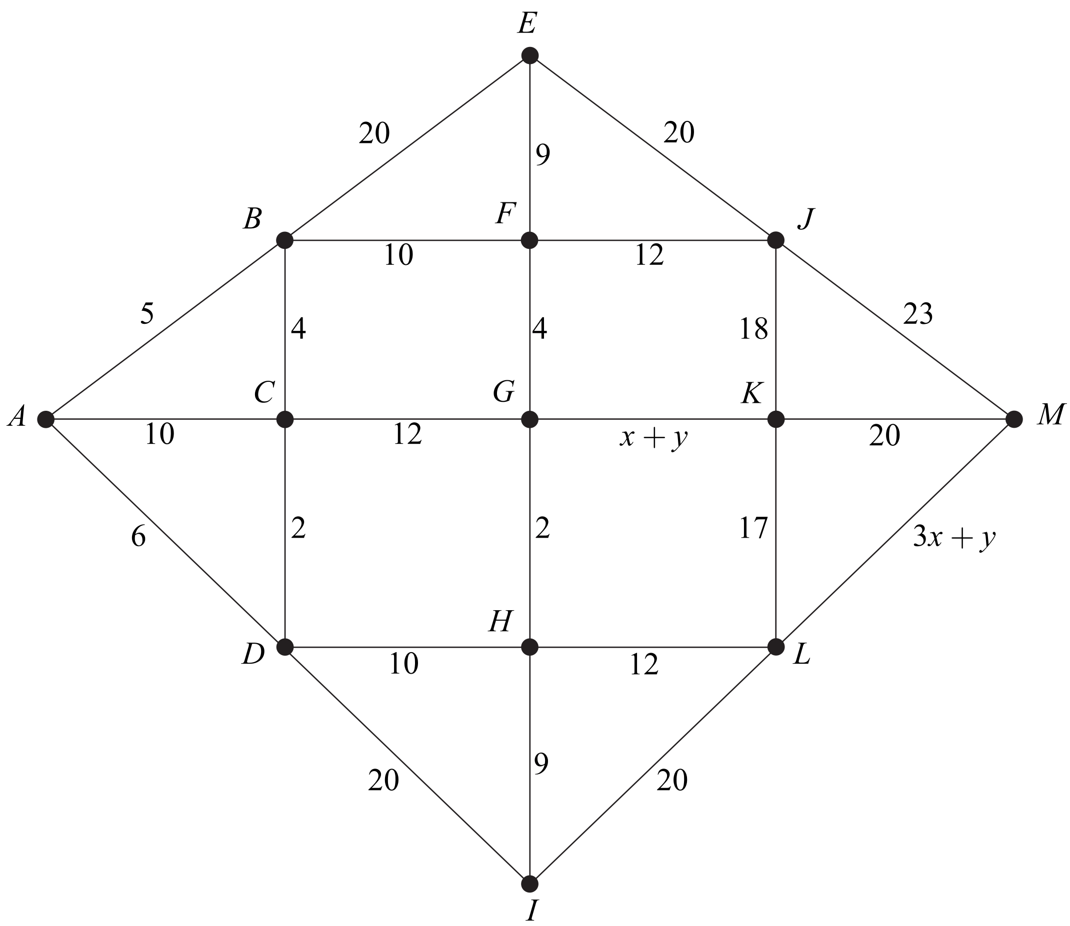


Figure 2 (for use in Question 7)



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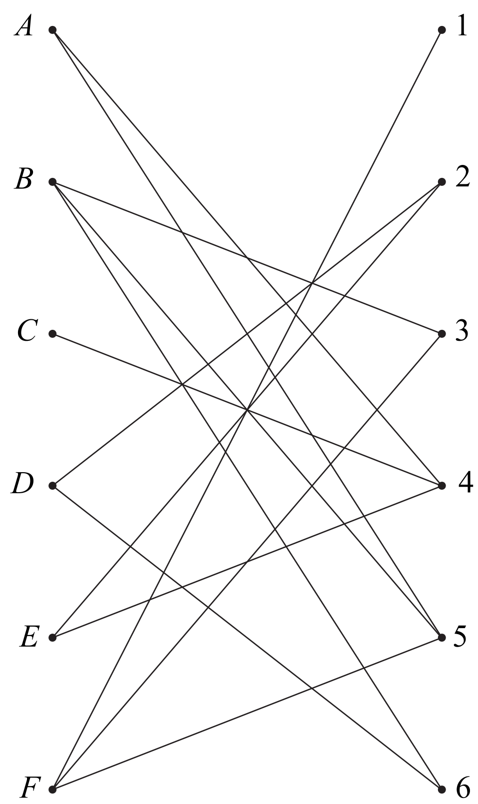
Answer **all** questions in the spaces provided.

1 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.



(a) Represent this information in an adjacency matrix. *(2 marks)*

(b) Initially, B is assigned to task 5, D to task 2, E to task 4 and F to task 3.

Demonstrate, by using an algorithm from this initial matching, how each person can be allocated to a task. *(5 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Turn over ▶

2 A student is using a quicksort algorithm to rearrange a set of numbers into ascending order. She uses the first number in each list (or sublist) as the pivot.

Her correct solution for the first three passes is as follows.

Initial list	10	7	4	22	13	16	19	5
After 1st pass	7	4	5	10	22	13	16	19
After 2nd pass	4	5	7	10	13	16	19	22
After 3rd pass	4	5	7	10	13	16	19	22

(a) State the pivots used for the 2nd pass. *(2 marks)*

(b) Write down the number of comparisons on each of the three passes. *(3 marks)*

(c) Explain whether the student has completed the algorithm. *(1 mark)*

QUESTION PART REFERENCE

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Turn over ▶

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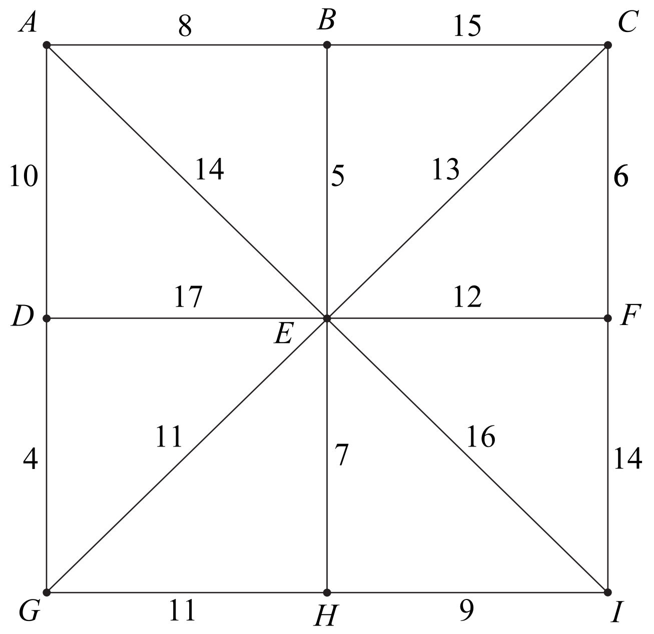
3 The following network shows the lengths, in miles, of roads connecting nine villages, A, B, …, I.



(a) (i) Use Prim's algorithm starting from E, showing the order in which you select the edges, to find a minimum spanning tree for the network. *(4 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

(b) On a particular day, village B is cut off, so its connecting roads cannot be used.

Find the length of a minimum spanning tree for the remaining eight villages. *(2 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Turn over ▶

4 The network below shows some paths on an estate. The number on each edge represents the time taken, in minutes, to walk along a path.

(a) (i) Use Dijkstra's algorithm on the network to find the minimum walking time from A to J. *(6 marks)*

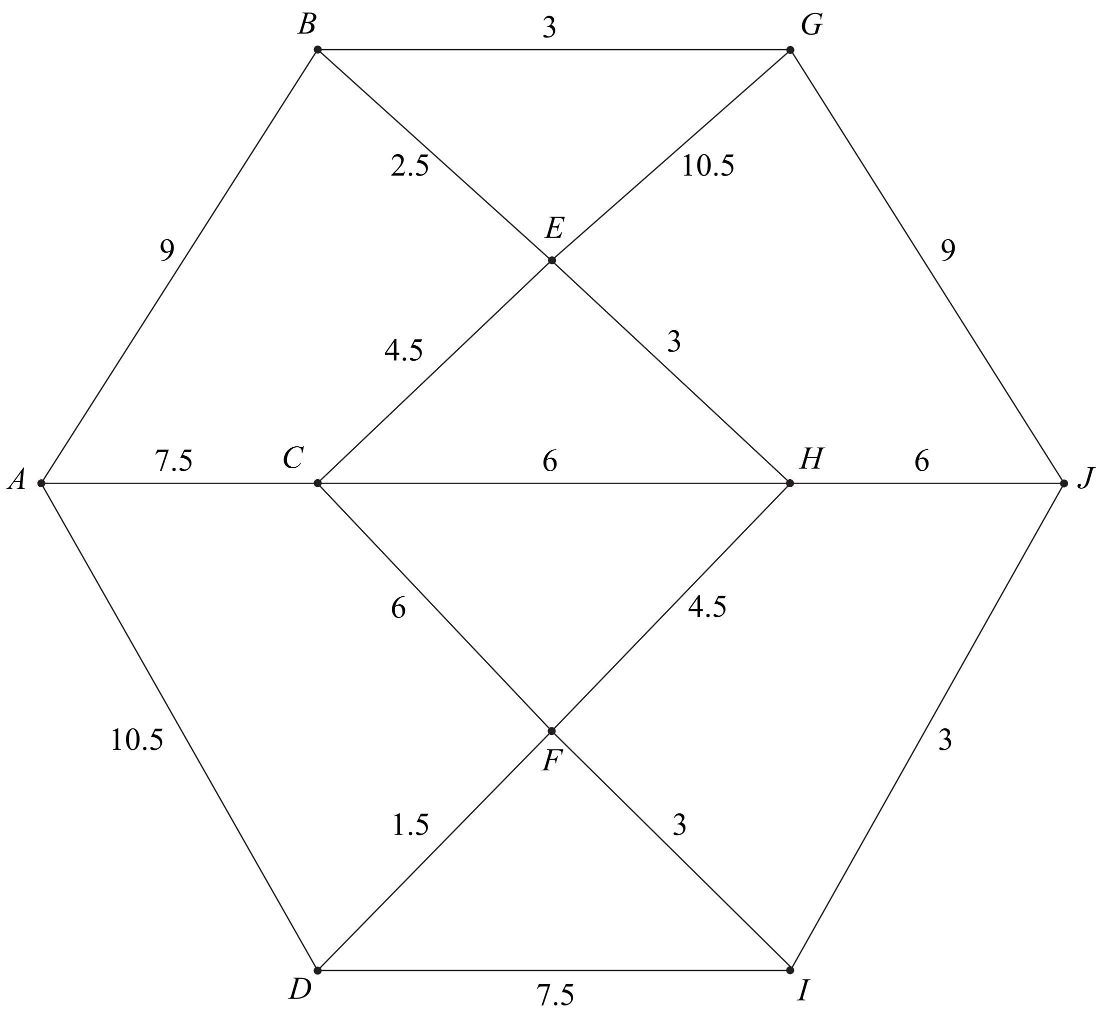
(ii) Write down the corresponding route. *(1 mark)*

(b) A new subway is constructed connecting C to G directly. The time taken to walk along this subway is x minutes. The minimum time taken to walk from A to G is now reduced, but the minimum time taken to walk from A to J is not reduced.

Find the range of possible values for x. *(3 marks)*

QUESTION PART REFERENCE

(a)(i)



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QUESTION PART REFERENCE

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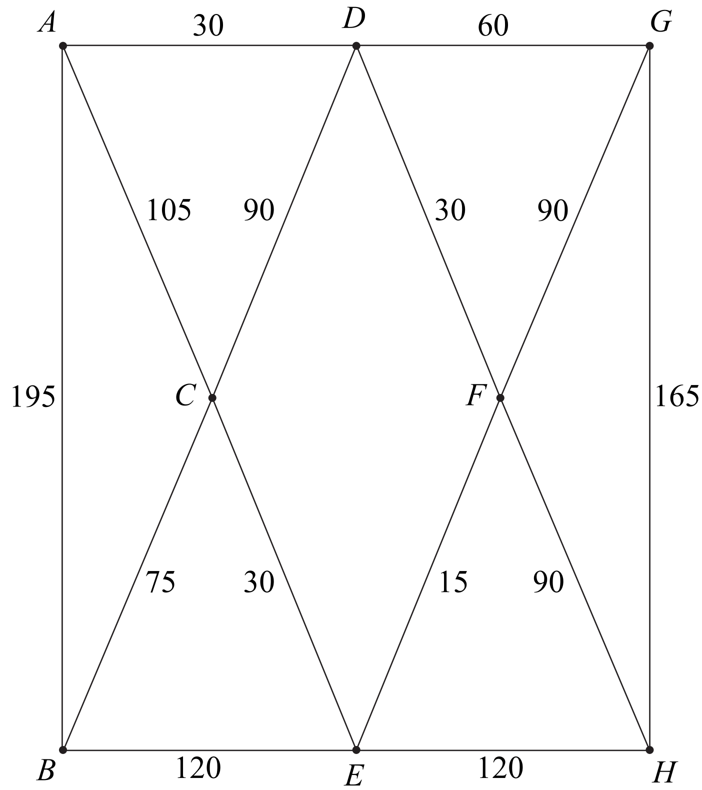
5 Norris delivers newspapers to houses on an estate. The network shows the streets on the estate. The number on each edge shows the length of the street, in metres.

Norris starts from the newsagents located at vertex A, and he must walk along all the streets at least once before returning to the newsagents.



The total length of the streets is 1215 metres.

(a) Give a reason why it is not possible to start at A, walk along each street once only, and return to A. *(1 mark)*

(b) Find the length of an optimal Chinese postman route around the estate, starting and finishing at A. *(5 marks)*

(c) For an optimal Chinese postman route, state:

(i) the number of times that the vertex F would occur; *(1 mark)*

(ii) the number of times that the vertex H would occur. *(1 mark)*

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QUESTION PART REFERENCE

Turn over ▶

6 (a) The complete graph K_n has every one of its n vertices connected to each of the other vertices by a single edge.

(i) Find the total number of edges in the graph K_5. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree for the graph K_5. *(1 mark)*

(iii) State the number of edges in a Hamiltonian cycle for the graph K_5. *(1 mark)*

(b) A simple graph G has six vertices and nine edges, and G is Eulerian. Draw a sketch to show a possible graph G. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

7 Fred delivers bread to five shops, A, B, C, D and E. Fred starts his deliveries at shop B, and travels to each of the other shops once before returning to shop B. Fred wishes to keep his travelling time to a minimum.

The table shows the travelling times, in minutes, between the shops.

	A	***B***	***C***	***D***	***E***
A	–	3	11	15	5
B	3	–	18	12	4
C	11	18	–	5	16
D	15	12	5	–	10
E	5	4	16	10	–

(a) Find the travelling time for the tour $BACDEB$. *(1 mark)*

(b) Use the nearest neighbour algorithm, starting at B, to find another upper bound for the travelling time for Fred's tour. *(3 marks)*

(c) By deleting C, find a lower bound for the travelling time for Fred's tour. *(4 marks)*

(d) Sketch a network showing the edges that give you the lower bound in part **(c)** and comment on its significance. *(2 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Turn over ▶

8 A student is tracing the following algorithm with positive integer values of A and B.

The function INT gives the integer part of a number, eg INT(2.3) = 2 and INT(3.8) = 3 .

Line 10	Let $X = 0$
Line 20	Input A, B
Line 30	If INT($A/2$) = $A/2$ then go to Line 50
Line 40	Let $X = X + B$
Line 50	If $A = 1$ then go to Line 90
Line 60	Let A = INT($A/2$)
Line 70	Let $B = 2 \times B$
Line 80	Go to Line 30
Line 90	Print X
Line 100	End

(a) Trace the algorithm in the case where the input values are $A = 20$ and $B = 8$. *(4 marks)*

(b) State the purpose of the algorithm. *(1 mark)*

(c) Another student changed Line 50 to

Line 50 If $A = 1$ then go to Line 80

Explain what would happen if this algorithm were traced. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

Turn over ▶

9 Herman is packing some hampers. Each day, he packs three types of hamper: basic, standard and luxury.

Each basic hamper has 6 tins, 9 packets and 6 bottles.

Each standard hamper has 9 tins, 6 packets and 12 bottles.

Each luxury hamper has 9 tins, 9 packets and 18 bottles.

Each day, Herman has 600 tins and 600 packets available, and he must use at least 480 bottles.

Each day, Herman packs x basic hampers, y standard hampers and z luxury hampers.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find three inequalities in x, y and z that model the above constraints, simplifying each inequality. *(4 marks)*

(b) On a particular day, Herman packs the same number of standard hampers as luxury hampers.

(i) Show that your answers in part **(a)** become

$$x + 3y \leqslant 100$$

$$3x + 5y \leqslant 200$$

$$x + 5y \geqslant 80$$

(2 marks)

(ii) On the grid opposite, draw a suitable diagram to represent Herman's situation, indicating the feasible region. *(4 marks)*

(iii) Use your diagram to find the maximum total number of hampers that Herman can pack on that day. *(2 marks)*

(iv) Find the number of each type of hamper that Herman packs that corresponds to your answer to part **(b)(iii)**. *(1 mark)*

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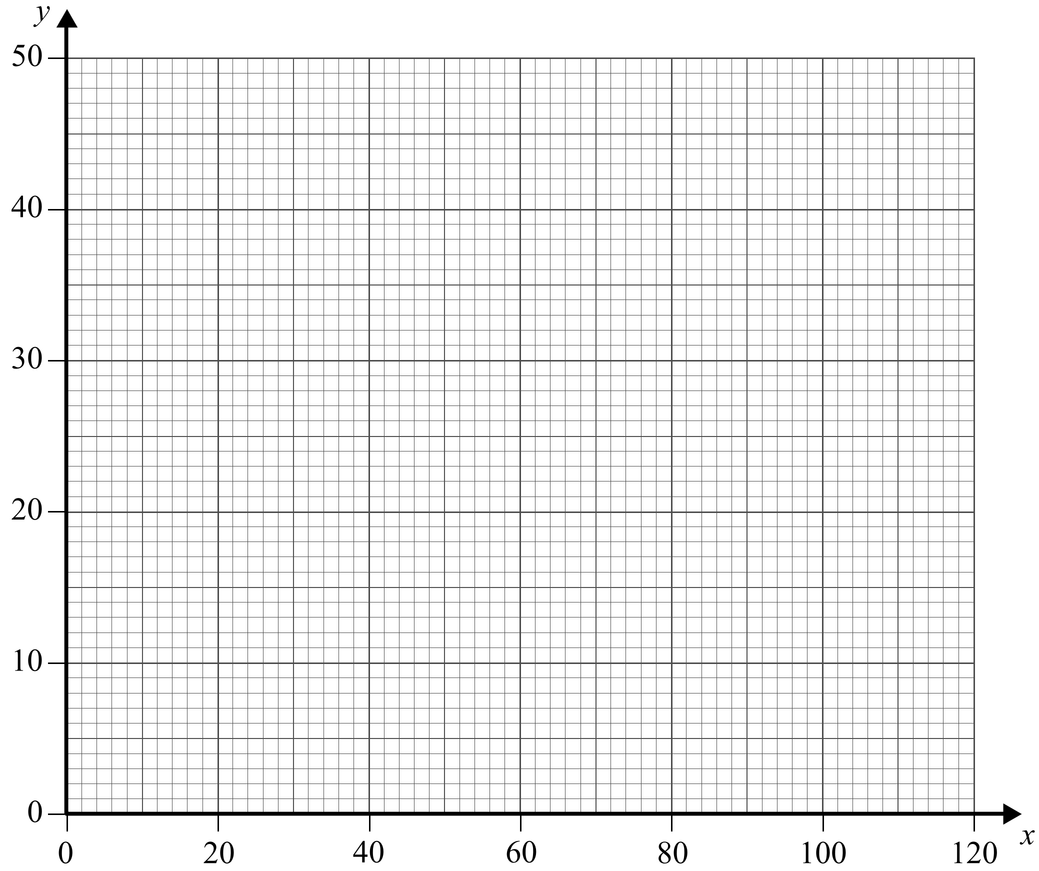
(b)(ii)



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QUESTION PART REFERENCE

END OF QUESTIONS