Answer **all** questions.

1 (a) Show that the equation

$$x^3 + 2x - 2 = 0$$

has a root between 0.5 and 1. *(2 marks)*

(b) Use linear interpolation once to find an estimate of this root. Give your answer to two decimal places. *(3 marks)*

2 (a) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\int_0^9 \frac{1}{\sqrt{x}}\,dx$; *(3 marks)*

(ii) $\int_0^9 \frac{1}{x\sqrt{x}}\,dx$. *(3 marks)*

(b) Explain briefly why the integrals in part (a) are improper integrals. *(1 mark)*

3 Find the general solution, in **degrees**, for the equation

$$\sin(4x + 10°) = \sin 50°$$ *(5 marks)*

4 A curve has equation

$$y = \frac{6x}{x - 1}$$

(a) Write down the equations of the two asymptotes to the curve. *(2 marks)*

(b) Sketch the curve and the two asymptotes. *(4 marks)*

(c) Solve the inequality

$$\frac{6x}{x - 1} < 3$$ *(4 marks)*

5 (a) (i) Calculate $(2 + i\sqrt{5})(\sqrt{5} - i)$. *(3 marks)*

(ii) Hence verify that $\sqrt{5} - i$ is a root of the equation

$$(2 + i\sqrt{5})\,z = 3z^*$$

where z^* is the conjugate of z. *(2 marks)*

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - i$.

(i) Write down the other root of the equation. *(1 mark)*

(ii) Find the sum and product of the two roots of the equation. *(3 marks)*

(iii) Hence state the values of p and q. *(2 marks)*

6 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = kx^n$$

where k and n are constants.

Experimental evidence has provided the following approximate values:

x	4	17	150	300
y	1.8	5.0	30	50

(a) Complete the table in **Figure 1**, showing values of X and Y, where

$$X = \log_{10} x \quad \text{and} \quad Y = \log_{10} y$$

Give each value to two decimal places. *(3 marks)*

(b) Show that if $y = kx^n$, then X and Y must satisfy an equation of the form

$$Y = aX + b$$ *(3 marks)*

(c) Draw on **Figure 2** a linear graph relating X and Y. *(3 marks)*

(d) Find an estimate for the value of n. *(2 marks)*

Turn over for the next question

7 (a) The transformation T is defined by the matrix **A**, where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(i) Describe the transformation T geometrically. *(2 marks)*

(ii) Calculate the matrix product $\mathbf{A}^2$. *(2 marks)*

(iii) Explain briefly why the transformation T followed by T is the identity transformation. *(1 mark)*

(b) The matrix **B** is defined by

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(i) Calculate $\mathbf{B}^2 - \mathbf{A}^2$. *(3 marks)*

(ii) Calculate $(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A})$. *(3 marks)*

8 A curve has equation $y^2 = 12x$.

(a) Sketch the curve. *(2 marks)*

(b) (i) The curve is translated by 2 units in the positive y direction. Write down the equation of the curve after this translation. *(2 marks)*

(ii) The **original** curve is reflected in the line $y = x$. Write down the equation of the curve after this reflection. *(1 mark)*

(c) (i) Show that if the straight line $y = x + c$, where c is a constant, intersects the curve $y^2 = 12x$, then the x-coordinates of the points of intersection satisfy the equation

$$x^2 + (2c - 12)x + c^2 = 0$$

(3 marks)

(ii) Hence find the value of c for which the straight line is a tangent to the curve. *(2 marks)*

(iii) Using this value of c, find the coordinates of the point where the line touches the curve. *(2 marks)*

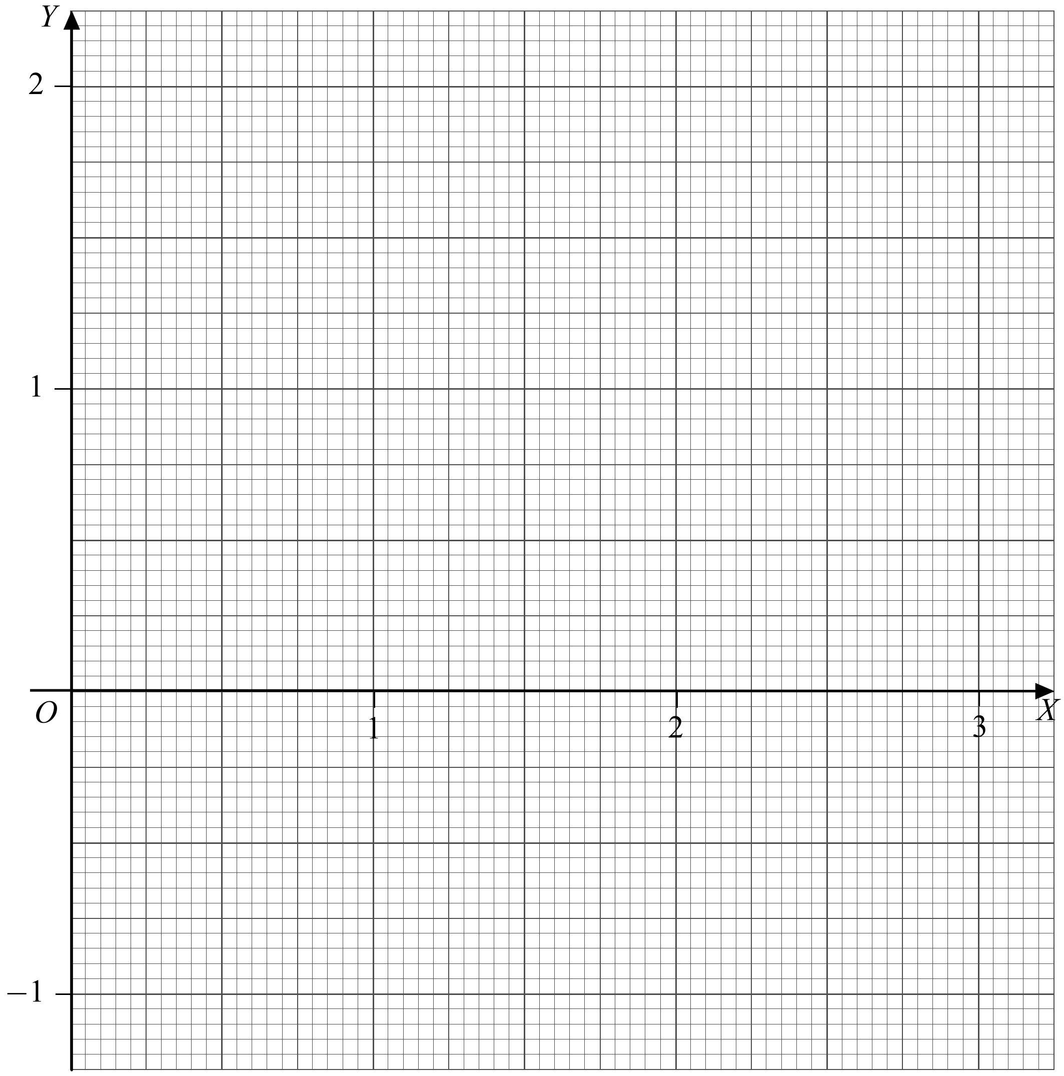
(iv) In the case where $c = 4$, determine whether the line intersects the curve or not. *(3 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 6)

X	0.60			2.48
Y	0.26			1.70

Figure 2 (for use in Question 6)



Practice 2

Leave blank

1. Solve the differential equation

$$\frac{dy}{dx} - 3y = x$$

to obtain y as a function of x.

(5)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

Turn over

Leave blank

2. Given that $x = -\frac{1}{2}$ is the real solution of the equation

$$2x^3 - 11x^2 + 14x + 10 = 0,$$

find the two complex solutions of this equation.

(6)

Leave blank

Question 2 continued

(Total 6 marks)

Q2

Leave blank

3. (a) Simplify the expression $\dfrac{(x+3)(x+9)}{x-1}-(3x-5)$, giving your answer in the form

$\dfrac{a(x+b)(x+c)}{x-1}$, where a, b and c are integers.

(4)

(b) Hence, or otherwise, solve the inequality

$$\frac{(x+3)(x+9)}{x-1} > 3x-5.$$

(4)

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4.

$$f(x) = 3x^2 + x - \tan\left(\frac{x}{2}\right) - 2, \qquad -\pi < x < \pi.$$

The equation f(x) = 0 has a root α in the interval [0.7, 0.8].

(a) Use linear interpolation, on the values at the end points of this interval, to obtain an approximation to α. Give your answer to 3 decimal places.

(4)

(b) Taking 0.75 as a first approximation to α, apply the Newton–Raphson procedure once to f(x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

Leave blank

Question 4 continued

Q4

(Total 8 marks)

Turn over

Leave blank

5. (a) Express $\dfrac{5r+4}{r(r+1)(r+2)}$ in partial fractions.

(4)

(b) Hence, or otherwise, show that

$$\sum_{r=1}^{n} \frac{5r+4}{r(r+1)(r+2)} = \frac{7n^2+11n}{2(n+1)(n+2)}.$$

(5)

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 9 marks)

Turn over

Leave blank

6. $z = -2 + \mathrm{i}.$

(a) Express in the form $a + \mathrm{i}b$

(i) $\frac{1}{z}$,

(ii) z^2.

(4)

(b) Show that $|z^2 - z| = 5\sqrt{2}$.

(2)

(c) Find arg $(z^2 - z)$.

(2)

(d) Display z and $z^2 - z$ on a single Argand diagram.

(2)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total 10 marks)

Leave blank

7. (a) Find the general solution of the differential equation

$$3\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = x^2.$$

(8)

(b) Find the particular solution for which, at $x = 0$, $y = 2$ and $\frac{dy}{dx} = 3$.

(6)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

(Total 14 marks)

Q7

Leave blank

8. **Figure 1**

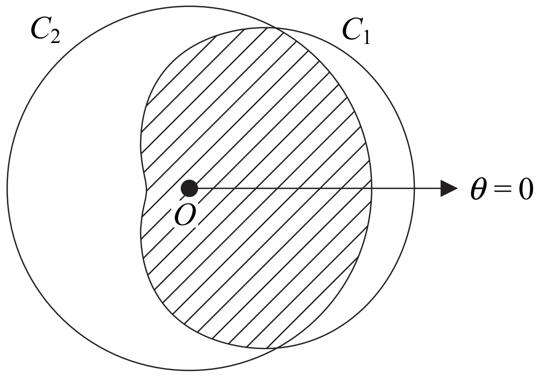


Figure 1 shows the curve C_1 which has polar equation $r = a(3 + 2\cos\theta)$, $0 \leqslant \theta < 2\pi$, and the circle C_2 with equation $r = 4a$, $0 \leqslant \theta < 2\pi$, where a is a positive constant.

(a) Find, in terms of a, the polar coordinates of the points where the curve C_1 meets the circle C_2.

(4)

The regions enclosed by the curves C_1 and C_2 overlap and this common region R is shaded in the figure.

(b) Find, in terms of a, an exact expression for the area of the shaded region R.

(8)

(c) In a single diagram, copy the two curves in Figure 1 and also sketch the curve C_3 with polar equation $r = 2a\cos\theta$, $0 \leqslant \theta < 2\pi$. Show clearly the coordinates of the points of intersection of C_1, C_2 and C_3 with the initial line, $\theta = 0$.

(3)

Leave blank

Question 8 continued

Turn over

Leave blank

Question 8 continued

Leave blank

Question 8 continued

(Total 15 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 (a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + i)^3$ in the form $a + bi$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + i$ satisfies the equation

$$x^3 + 2x - 4i = 0$$ *(2 marks)*

2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{BA}$. *(3 marks)*

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\mathbf{A}$; *(2 marks)*

(ii) $\mathbf{B}$; *(2 marks)*

(iii) $\mathbf{BA}$. *(2 marks)*

3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = 1$. *(3 marks)*

(c) Find the value of $\alpha^4 + \beta^4$. *(3 marks)*

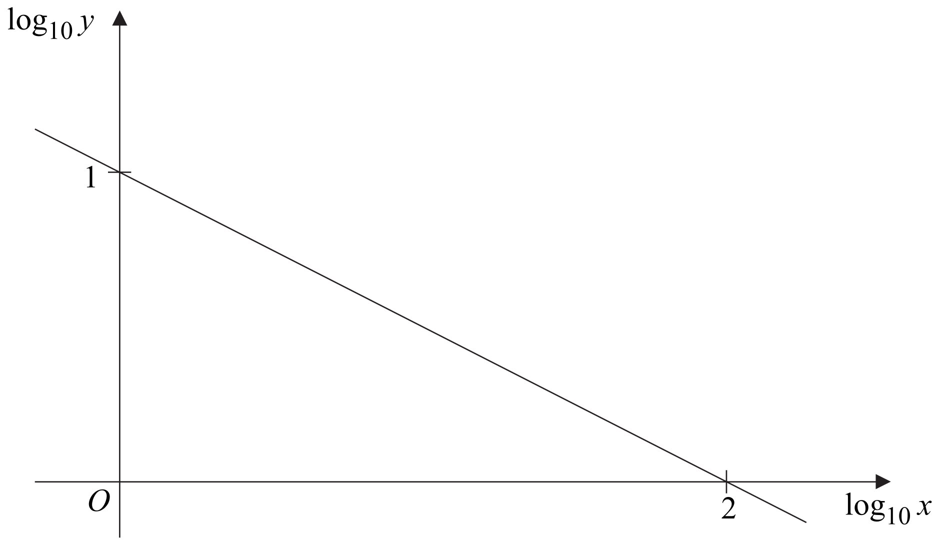
4 The variables x and y are related by an equation of the form

$$y = ax^b$$

where a and b are constants.

(a) Using logarithms to base 10, reduce the relation $y = ax^b$ to a linear law connecting $\log_{10} x$ and $\log_{10} y$. *(2 marks)*

(b) The diagram shows the linear graph that results from plotting $\log_{10} y$ against $\log_{10} x$.



Find the values of a and b. *(4 marks)*

5 A curve has equation

$$y = \frac{x}{x^2 - 1}$$

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) Sketch the curve.

(You are given that the curve has no stationary points.) *(4 marks)*

(c) Solve the inequality

$$\frac{x}{x^2 - 1} > 0$$ *(3 marks)*

6 (a) (i) Expand $(2r-1)^2$. *(1 mark)*

(ii) Hence show that

$$\sum_{r=1}^{n} (2r-1)^2 = \tfrac{1}{3} n(4n^2 - 1)$$ *(5 marks)*

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. *(4 marks)*

7 The function f is defined for all real numbers by

$$\mathrm{f}(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(a) Find the general solution of the equation $\mathrm{f}(x) = 0$. *(3 marks)*

(b) The quadratic function g is defined for all real numbers by

$$\mathrm{g}(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $\mathrm{g}(x)$ gives a good approximation to $\mathrm{f}(x)$ for small values of x.

(i) Show that $\mathrm{g}(0.05)$ and $\mathrm{f}(0.05)$ are identical when rounded to four decimal places. *(2 marks)*

(ii) A chord joins the points on the curve $y = \mathrm{g}(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*

(iii) Using your answer to part (b)(ii), find the value of $\mathrm{g}'(0)$. *(1 mark)*

8 A curve C has equation

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

(a) Find the y-coordinates of the points on C for which $x = 10$, giving each answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(b) Sketch the curve C, indicating the coordinates of any points where the curve intersects the coordinate axes. *(3 marks)*

(c) Write down the equation of the tangent to C at the point where C intersects the positive x-axis. *(1 mark)*

(d) (i) Show that, if the line $y = x - 4$ intersects C, the x-coordinates of the points of intersection must satisfy the equation

$$16x^2 - 200x + 625 = 0$$ *(3 marks)*

(ii) Solve this equation and hence state the relationship between the line $y = x - 4$ and the curve C. *(2 marks)*

END OF QUESTIONS

Answer **all** questions.

1 It is given that $z_1 = 2 + \mathrm{i}$ and that $z_1{}^*$ is the complex conjugate of z_1.

Find the real numbers x and y such that

$$x + 3\mathrm{i}y = z_1 + 4\mathrm{i}z_1{}^*$$

(4 marks)

2 A curve satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2^x$$

Starting at the point $(1, 4)$ on the curve, use a step-by-step method with a step length of 0.01 to estimate the value of y at $x = 1.02$. Give your answer to six significant figures. *(5 marks)*

3 Find the general solution of the equation

$$\tan 4\left(x - \frac{\pi}{8}\right) = 1$$

giving your answer in terms of π. *(5 marks)*

4 (a) Find

$$\sum_{r=1}^{n} (r^3 - 6r)$$

expressing your answer in the form

$$kn(n+1)(n+p)(n+q)$$

where k is a fraction and p and q are integers. *(5 marks)*

(b) It is given that

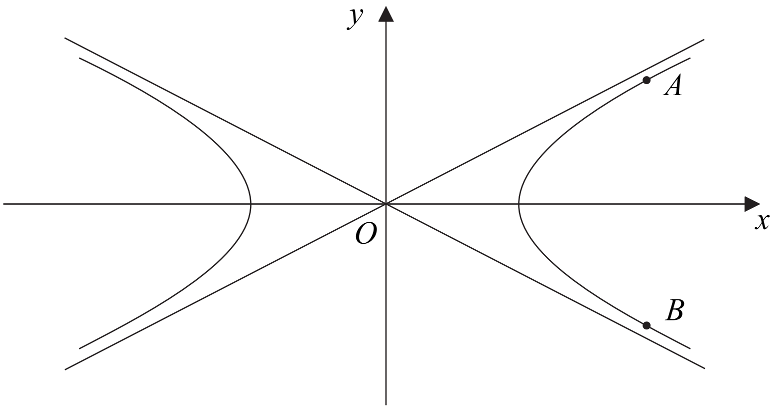
$$S = \sum_{r=1}^{1000} (r^3 - 6r)$$

Without calculating the value of S, show that S is a multiple of 2008. *(2 marks)*

5 The diagram shows the hyperbola

$$\frac{x^2}{4} - y^2 = 1$$

and its asymptotes.



(a) Write down the equations of the two asymptotes. *(2 marks)*

(b) The points on the hyperbola for which $x = 4$ are denoted by A and B.

Find, in surd form, the y-coordinates of A and B. *(2 marks)*

(c) The hyperbola and its asymptotes are translated by two units in the positive y direction.

Write down:

(i) the y-coordinates of the image points of A and B under this translation; *(1 mark)*

(ii) the equations of the hyperbola and the asymptotes after the translation. *(3 marks)*

Turn over for the next question

6 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \sqrt{3} & 3 \\ 3 & -\sqrt{3} \end{bmatrix}$$

(a) (i) Show that

$$\mathbf{M}^2 = p\mathbf{I}$$

where p is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(3 marks)*

(ii) Show that the matrix $\mathbf{M}$ can be written in the form

$$q \begin{bmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{bmatrix}$$

where q is a real number. Give the value of q in surd form. *(3 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement and a reflection.

Find:

(i) the scale factor of the enlargement; *(1 mark)*

(ii) the equation of the mirror line of the reflection. *(1 mark)*

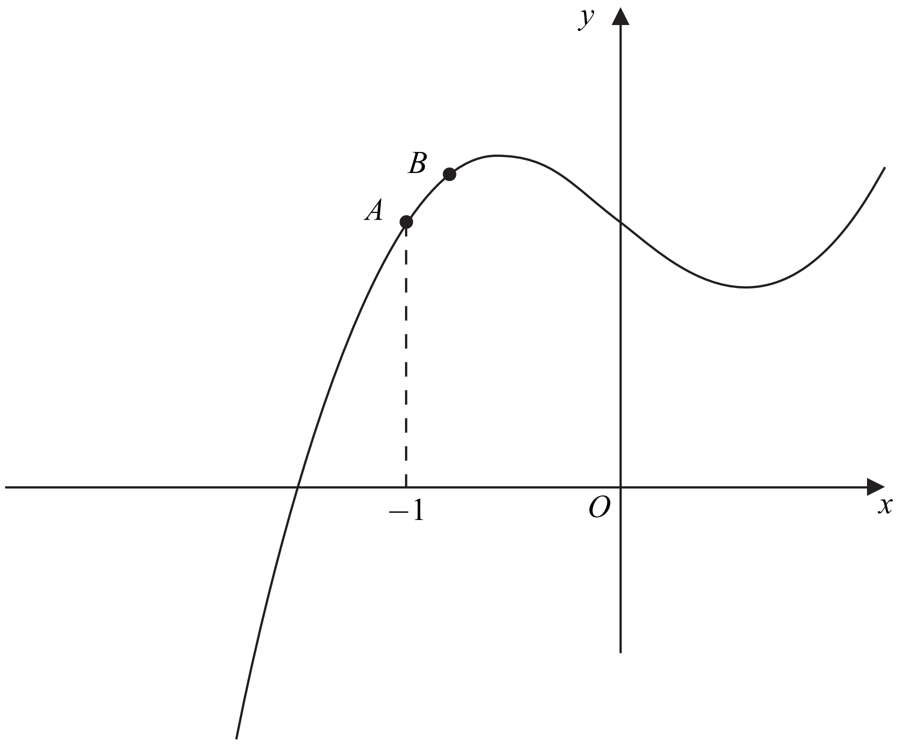
(c) Describe fully the geometrical transformation represented by $\mathbf{M}^4$. *(2 marks)*

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows the curve

$$y = x^3 - x + 1$$

The points A and B on the curve have x-coordinates -1 and $-1 + h$ respectively.



(a) (i) Show that the y-coordinate of the point B is

$$1 + 2h - 3h^2 + h^3$$

(3 marks)

(ii) Find the gradient of the chord AB in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(3 marks)*

(iii) Explain how your answer to part (a)(ii) can be used to find the gradient of the tangent to the curve at A. State the value of this gradient. *(2 marks)*

(b) The equation $x^3 - x + 1 = 0$ has one real root, α.

(i) Taking $x_1 = -1$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2, to α. *(2 marks)*

(ii) On **Figure 1**, draw a straight line to illustrate the Newton-Raphson method as used in part (b)(i). Show the points $(x_2, 0)$ and $(\alpha, 0)$ on your diagram. *(2 marks)*

8 (a) (i) It is given that α and β are the roots of the equation

$$x^2 - 2x + 4 = 0$$

Without solving this equation, show that α^3 and β^3 are the roots of the equation

$$x^2 + 16x + 64 = 0$$ *(6 marks)*

(ii) State, giving a reason, whether the roots of the equation

$$x^2 + 16x + 64 = 0$$

are real and equal, real and distinct, or non-real. *(2 marks)*

(b) Solve the equation

$$x^2 - 2x + 4 = 0$$ *(2 marks)*

(c) Use your answers to parts (a) and (b) to show that

$$(1 + \mathrm{i}\sqrt{3})^3 = (1 - \mathrm{i}\sqrt{3})^3$$ *(2 marks)*

9 A curve C has equation

$$y = \frac{2}{x(x-4)}$$

(a) Write down the equations of the three asymptotes of C. *(3 marks)*

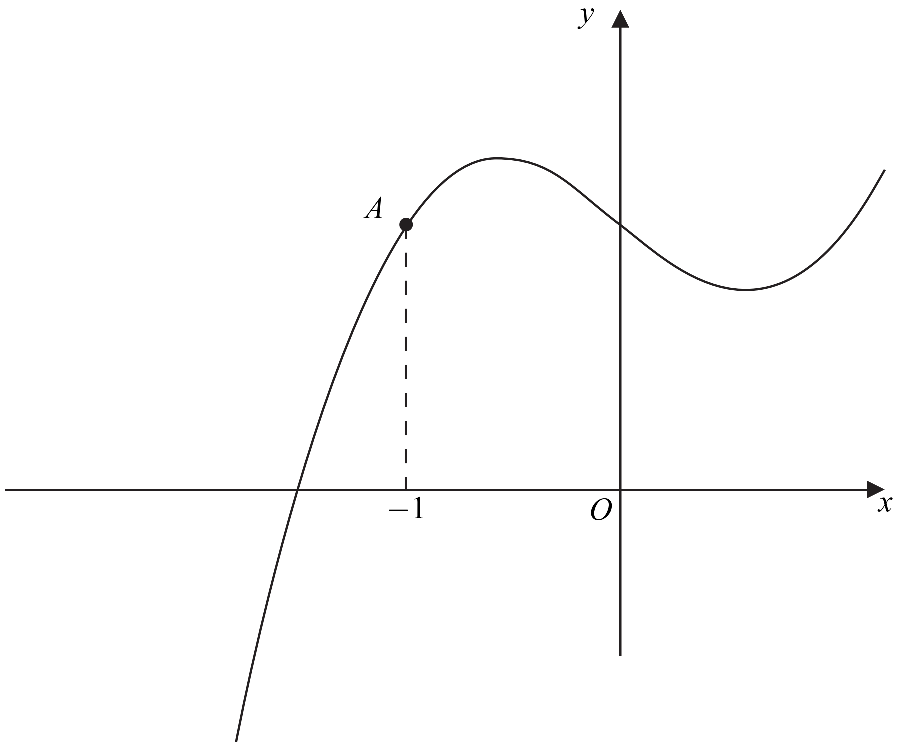
(b) The curve C has one stationary point. By considering an appropriate quadratic equation, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(6 marks)*

(c) Sketch the curve C. *(3 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 7)



Answer **all** questions.

1 A curve passes through the point $(0,\ 1)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{1+x^2}$$

Starting at the point $(0,\ 1)$, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 0.4$. Give your answer to five decimal places. *(5 marks)*

2 The complex number $2 + 3\mathrm{i}$ is a root of the quadratic equation

$$x^2 + bx + c = 0$$

where b and c are real numbers.

(a) Write down the other root of this equation. *(1 mark)*

(b) Find the values of b and c. *(4 marks)*

3 Find the general solution of the equation

$$\tan\left(\frac{\pi}{2} - 3x\right) = \sqrt{3}$$ *(5 marks)*

4 It is given that

$$S_n = \sum_{r=1}^{n}(3r^2 - 3r + 1)$$

(a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = n^3$. *(5 marks)*

(b) Hence show that $\sum_{r=n+1}^{2n} (3r^2 - 3r + 1) = kn^3$ for some integer k. *(2 marks)*

5 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

where k is a constant.

(a) Find, in terms of k:

(i) $\mathbf{A} + \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{A}^2$. *(2 marks)*

(b) Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{B}^2$. *(4 marks)*

(c) It is now given that $k = 1$.

(i) Describe the geometrical transformation represented by the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) The matrix **A** represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(3 marks)*

6 A curve has equation

$$y = \frac{(x-1)(x-3)}{x(x-2)}$$

(a) (i) Write down the equations of the three asymptotes of this curve. *(3 marks)*

(ii) State the coordinates of the points at which the curve intersects the x-axis. *(1 mark)*

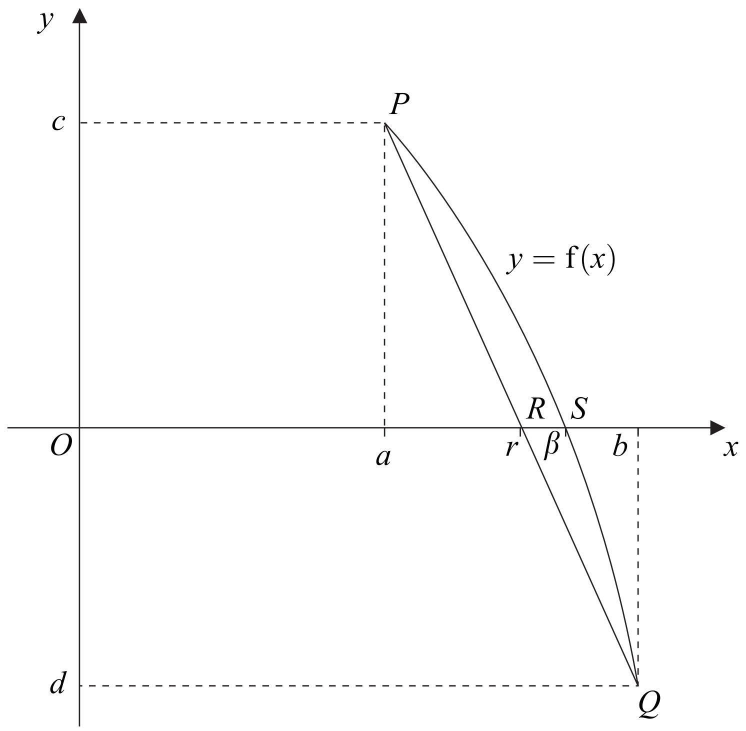
(iii) Sketch the curve.

(You are given that the curve has no stationary points.) *(4 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{(x-1)(x-3)}{x(x-2)} < 0$$ *(2 marks)*

7 The points $P(a, c)$ and $Q(b, d)$ lie on the curve with equation $y = \mathrm{f}(x)$. The straight line PQ intersects the x-axis at the point $R(r, 0)$. The curve $y = \mathrm{f}(x)$ intersects the x-axis at the point $S(\beta, 0)$.



(a) Show that

$$r = a + c\left(\frac{b-a}{c-d}\right)$$ *(4 marks)*

(b) Given that

$$a = 2,\ b = 3 \text{ and } \mathrm{f}(x) = 20x - x^4$$

(i) find the value of r; *(3 marks)*

(ii) show that $\beta - r \approx 0.18$. *(3 marks)*

8 For each of the following improper integrals, find the value of the integral **or** explain why it does not have a value:

(a) $\int_1^{\infty} x^{-\frac{3}{4}}\, \mathrm{d}x$; *(3 marks)*

(b) $\int_1^{\infty} x^{-\frac{5}{4}}\, \mathrm{d}x$; *(3 marks)*

(c) $\int_1^{\infty} \left(x^{-\frac{3}{4}} - x^{-\frac{5}{4}}\right) \mathrm{d}x$. *(1 mark)*

9 A hyperbola H has equation

$$x^2 - \frac{y^2}{2} = 1$$

(a) Find the equations of the two asymptotes of H, giving each answer in the form $y = mx$. *(2 marks)*

(b) Draw a sketch of the two asymptotes of H, using roughly equal scales on the two coordinate axes. Using the same axes, sketch the hyperbola H. *(3 marks)*

(c) (i) Show that, if the line $y = x + c$ intersects H, the x-coordinates of the points of intersection must satisfy the equation

$$x^2 - 2cx - (c^2 + 2) = 0$$ *(4 marks)*

(ii) Hence show that the line $y = x + c$ intersects H in two distinct points, whatever the value of c. *(2 marks)*

(iii) Find, in terms of c, the y-coordinates of these two points. *(3 marks)*

END OF QUESTIONS

Answer **all** questions.

1 The quadratic equation

$$3x^2 - 6x + 1 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^3 + \beta^3 = 6$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. *(4 marks)*

2 The complex number z is defined by

$$z = 1 + \mathrm{i}$$

(a) Find the value of z^2, giving your answer in its simplest form. *(2 marks)*

(b) Hence show that $z^8 = 16$. *(2 marks)*

(c) Show that $(z^*)^2 = -z^2$. *(2 marks)*

3 Find the general solution of the equation

$$\sin\left(4x + \frac{\pi}{4}\right) = 1$$ *(4 marks)*

4 It is given that

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$$

and that $\mathbf{I}$ is the 2×2 identity matrix.

(a) Show that $(\mathbf{A} - \mathbf{I})^2 = k\mathbf{I}$ for some integer k. *(3 marks)*

(b) Given further that

$$\mathbf{B} = \begin{bmatrix} 1 & 3 \\ p & 1 \end{bmatrix}$$

find the integer p such that

$$(\mathbf{A} - \mathbf{B})^2 = (\mathbf{A} - \mathbf{I})^2$$ *(4 marks)*

5 (a) Explain why $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,dx$ is an improper integral. *(1 mark)*

(b) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

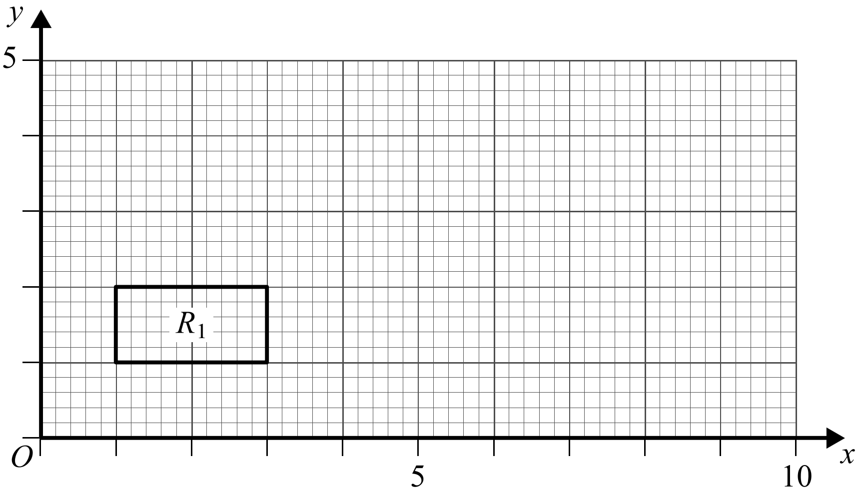
(i) $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,dx$; *(3 marks)*

(ii) $\int_0^{\frac{1}{16}} x^{-\frac{5}{4}}\,dx$. *(3 marks)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows a rectangle R_1.



(a) The rectangle R_1 is mapped onto a second rectangle, R_2, by a transformation with matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.

(i) Calculate the coordinates of the vertices of the rectangle R_2. *(2 marks)*

(ii) On **Figure 1**, draw the rectangle R_2. *(1 mark)*

(b) The rectangle R_2 is rotated through 90° clockwise about the origin to give a third rectangle, R_3.

(i) On **Figure 1**, draw the rectangle R_3. *(2 marks)*

(ii) Write down the matrix of the rotation which maps R_2 onto R_3. *(1 mark)*

(c) Find the matrix of the transformation which maps R_1 onto R_3. *(2 marks)*

7 A curve C has equation $y = \dfrac{1}{(x-2)^2}$.

(a) (i) Write down the equations of the asymptotes of the curve C. *(2 marks)*

(ii) Sketch the curve C. *(2 marks)*

(b) The line $y = x - 3$ intersects the curve C at a point which has x-coordinate α.

(i) Show that α lies within the interval $3 < x < 4$. *(2 marks)*

(ii) Starting from the interval $3 < x < 4$, use interval bisection twice to obtain an interval of width 0.25 within which α must lie. *(3 marks)*

8 (a) Show that

$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r$$

can be expressed in the form

$$kn(n+1)(an^2 + bn + c)$$

where k is a rational number and a, b and c are integers. *(4 marks)*

(b) Show that there is exactly one positive integer n for which

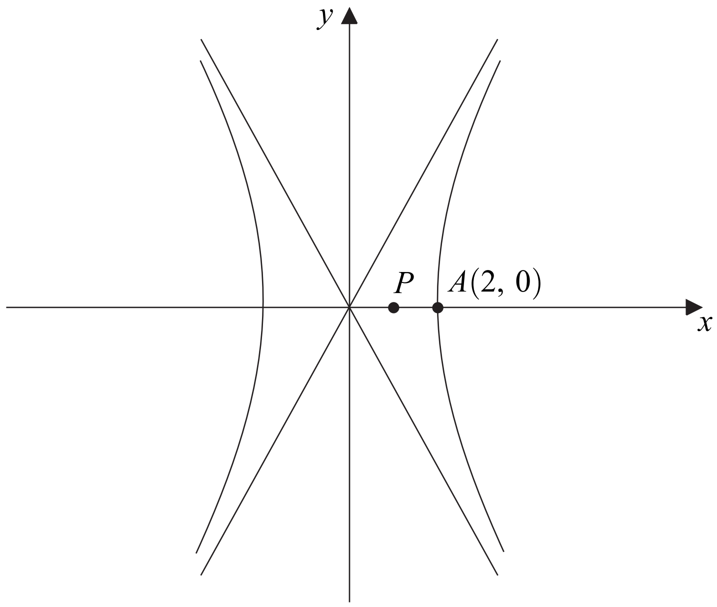
$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r = 8\sum_{r=1}^{n} r^2$$ *(5 marks)*

Turn over for the next question

9 The diagram shows the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and its asymptotes.



The constants a and b are positive integers.

The point A on the hyperbola has coordinates (2, 0).

The equations of the asymptotes are $y = 2x$ and $y = -2x$.

(a) Show that $a = 2$ and $b = 4$. *(4 marks)*

(b) The point P has coordinates (1, 0). A straight line passes through P and has gradient m. Show that, if this line intersects the hyperbola, the x-coordinates of the points of intersection satisfy the equation

$$(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0$$ *(4 marks)*

(c) Show that this equation has equal roots if $3m^2 = 16$. *(3 marks)*

(d) There are two tangents to the hyperbola which pass through P. Find the coordinates of the points at which these tangents touch the hyperbola.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 6)