Answer **all** questions.

1 (a) Simplify $(\sqrt{5}+2)(\sqrt{5}-2)$. *(2 marks)*

(b) Express $\sqrt{8}+\sqrt{18}$ in the form $n\sqrt{2}$, where n is an integer. *(2 marks)*

2 The point A has coordinates $(1, 1)$ and the point B has coordinates $(5, k)$.

The line AB has equation $3x+4y=7$.

(a) (i) Show that $k=-2$. *(1 mark)*

(ii) Hence find the coordinates of the mid-point of AB. *(2 marks)*

(b) Find the gradient of AB. *(2 marks)*

(c) The line AC is perpendicular to the line AB.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the line AC. *(1 mark)*

(iii) Given that the point C lies on the x-axis, find its x-coordinate. *(2 marks)*

3 (a) (i) Express x^2-4x+9 in the form $(x-p)^2+q$, where p and q are integers. *(2 marks)*

(ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation $y=x^2-4x+9$. *(2 marks)*

(b) The line L has equation $y+2x=12$ and the curve C has equation $y=x^2-4x+9$.

(i) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2-2x-3=0$$ *(1 mark)*

(ii) Hence find the coordinates of the points of intersection of L and C. *(4 marks)*

4 The quadratic equation $x^2 + (m+4)x + (4m+1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. *(3 marks)*

(b) Hence find the possible values of m. *(2 marks)*

5 A circle with centre C has equation $x^2 + y^2 - 8x + 6y = 11$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) The point O has coordinates $(0, 0)$.

(i) Find the length of CO. *(2 marks)*

(ii) Hence determine whether the point O lies inside or outside the circle, giving a reason for your answer. *(2 marks)*

6 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that $x - 2$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Hence express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Sketch the curve with equation $y = x^3 + x^2 - 10x + 8$, showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) *(4 marks)*

7 The volume, V m^3, of water in a tank at time t seconds is given by

$$V = \frac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geqslant 0$$

(a) Find:

(i) $\dfrac{dV}{dt}$; *(3 marks)*

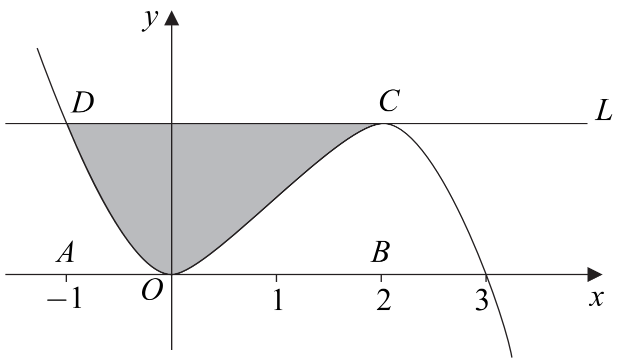
(ii) $\dfrac{d^2V}{dt^2}$. *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in m^3 s^{-1}, when $t = 2$. *(2 marks)*

(c) (i) Verify that V has a stationary value when $t = 1$. *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

(i) find $\frac{dy}{dx}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

END OF QUESTIONS

Practice 2

Leave blank

1. Find $\int(3x^2 + 4x^5 - 7)\,dx$.

(4)

Q1

(Total 4 marks)

Leave blank

2. (a) Write down the value of $16^{\frac{1}{4}}$.

(1)

(b) Simplify $(16x^{12})^{\frac{3}{4}}$.

(2)

Q2

(Total 3 marks)

Leave blank

3. Simplify

$$\frac{5-\sqrt{3}}{2+\sqrt{3}},$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(4)

Q3

(Total 4 marks)

Leave blank

4. The point A (–6, 4) and the point B (8, –3) lie on the line L.

(a) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer. **(3)**

Q4

(Total 7 marks)

Leave blank

5. (a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$ where p and q are constants. **(2)**

Given that $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$, $\quad x > 0$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term. **(4)**

Leave blank

Question 5 continued

Q5

(Total 6 marks)

Leave blank

6.

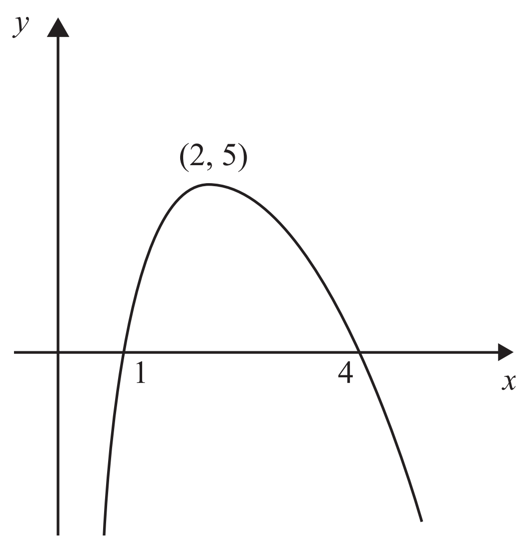


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points (1, 0) and (4, 0). The maximum point on the curve is (2, 5).
In separate diagrams sketch the curves with the following equations.
On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(a) $y = 2f(x)$,

(3)

(b) $y = f(-x)$.

(3)

The maximum point on the curve with equation $y = f(x + a)$ is on the y-axis.

(c) Write down the value of the constant a.

(1)

Leave blank

Question 6 continued

Q6

(Total 7 marks)

Leave blank

7. A sequence is given by:

$$x_1 = 1,$$
$$x_{n+1} = x_n(p + x_n),$$

where p is a constant $(p \neq 0)$.

(a) Find x_2 in terms of p.

(1)

(b) Show that $x_3 = 1 + 3p + 2p^2$.

(2)

Given that $x_3 = 1$,

(c) find the value of p,

(3)

(d) write down the value of x_{2008} .

(2)

Leave blank

Question 7 continued

Q7

(Total 8 marks)

Turn over

Leave blank

8. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

Leave blank

Question 8 continued

Q8

(Total 7 marks)

Turn over

Leave blank

9. The curve C has equation $y = \mathrm{f}(x)$, $x > 0$, and $\mathrm{f}'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $\mathrm{f}(x)$ and simplify your answer.
(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.
(4)

Leave blank

Question 9 continued

(Total 10 marks)

Q9

Leave blank

10. The curve C has equation

$$y = (x+3)(x-1)^2 .$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

Leave blank

Question 10 continued

Turn over

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 12 marks)

Turn over

Leave blank

11. The first term of an arithmetic sequence is 30 and the common difference is –1.5

(a) Find the value of the 25th term.
(2)

The rth term of the sequence is 0.

(b) Find the value of r.
(2)

The sum of the first n terms of the sequence is S_n.

(c) Find the largest positive value of S_n.
(3)

Leave blank

Question 11 continued

Leave blank

Question 11 continued

Leave blank

Question 11 continued

Q11

(Total 7 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 - 4x^2 - 7x + k$$

where k is a constant.

(a) (i) Given that $x + 2$ is a factor of $\mathrm{p}(x)$, show that $k = 10$. *(2 marks)*

(ii) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 3$. *(2 marks)*

(c) Sketch the curve with equation $y = x^3 - 4x^2 - 7x + 10$, indicating the values where the curve crosses the x-axis and the y-axis. (You are **not** required to find the coordinates of the stationary points.) *(4 marks)*

2 The line AB has equation $3x + 5y = 8$ and the point A has coordinates $(6, -2)$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) Hence find an equation of the straight line which is perpendicular to AB and which passes through A. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 3$ at the point B. Find the coordinates of B. *(3 marks)*

(c) The point C has coordinates $(2, k)$ and the distance from A to C is 5. Find the **two** possible values of the constant k. *(3 marks)*

3 (a) Express $\dfrac{\sqrt{5} + 3}{\sqrt{5} - 2}$ in the form $p\sqrt{5} + q$, where p and q are integers. *(4 marks)*

(b) (i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer. *(1 mark)*

(ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form. *(3 marks)*

4 A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) Show that the circle does **not** intersect the x-axis. *(2 marks)*

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q.

(i) Show that the x-coordinates of P and Q satisfy the equation

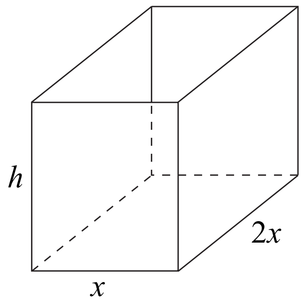
$$x^2 + 3x - 10 = 0$$ *(3 marks)*

(ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q. *(2 marks)*

(iii) Hence find the coordinates of the midpoint of PQ. *(2 marks)*

Turn over for the next question

5 The diagram shows an **open-topped** water tank with a horizontal rectangular base and four vertical faces. The base has width x metres and length $2x$ metres, and the height of the tank is h metres.



The combined internal surface area of the base and four vertical faces is $54\,\text{m}^2$.

(a) (i) Show that $x^2 + 3xh = 27$. *(2 marks)*

(ii) Hence express h in terms of x. *(1 mark)*

(iii) Hence show that the volume of water, $V\ \text{m}^3$, that the tank can hold when full is given by

$$V = 18x - \frac{2x^3}{3}$$

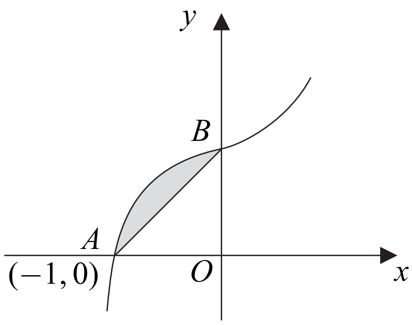
(1 mark)

(b) (i) Find $\frac{dV}{dx}$. *(2 marks)*

(ii) Verify that V has a stationary value when $x = 3$. *(2 marks)*

(c) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 3$. *(2 marks)*

6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.



The curve cuts the x-axis at the point $A\,(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. *(3 marks)*

(ii) Find $\int (3x^5 + 2x + 5)\ \mathrm{d}x$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A\,(-1, 0)$. *(3 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

7 The quadratic equation $(k + 1)x^2 + 12x + (k - 4) = 0$ has real roots.

(a) Show that $k^2 - 3k - 40 \leqslant 0$. *(3 marks)*

(b) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS

Practice 4

Leave blank

1. (a) Write down the value of $125^{\frac{1}{3}}$.

(1)

(b) Find the value of $125^{-\frac{2}{3}}$.

(2)

Q1

(Total 3 marks)

Leave blank

2. Find $\int (12x^5 - 8x^3 + 3)\,dx$, giving each term in its simplest form.

(4)

Q2

(Total 4 marks)

Leave blank

3. Expand and simplify $(\sqrt{7} + 2)(\sqrt{7} - 2)$.

(2)

Q3

(Total 2 marks)

Leave blank

4. A curve has equation $y = \mathrm{f}(x)$ and passes through the point (4, 22).

Given that

$$\mathrm{f}'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $\mathrm{f}(x)$, giving each term in its simplest form.

(5)

Leave blank

Question 4 continued

Q4

(Total 5 marks)

Leave blank

5.

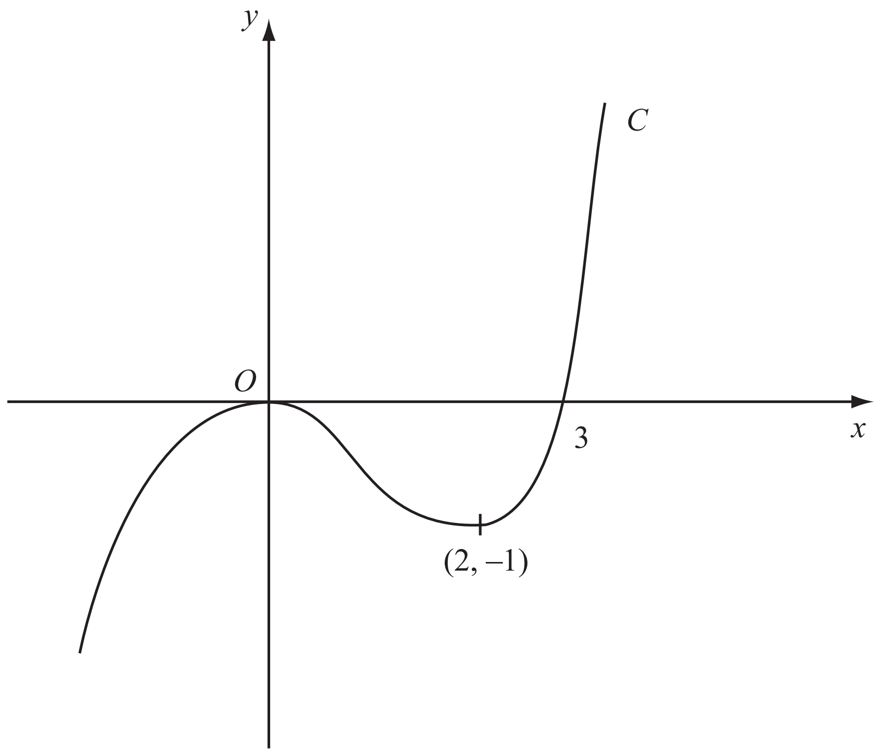


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$. There is a maximum at (0, 0), a minimum at (2, –1) and C passes through (3, 0).

On separate diagrams sketch the curve with equation

(a) $y = f(x + 3)$,

(3)

(b) $y = f(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

Leave blank

Question 5 continued

Q5

(Total 6 marks)

Turn over

Leave blank

6. Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

Leave blank

Question 6 continued

(Total 6 marks)

Q6

Turn over

Leave blank

7. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

Leave blank

Question 7 continued

Q7

(Total 7 marks)

Turn over

Leave blank

8. The point P (1, a) lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$,

(ii) $y = \frac{2}{x}$.

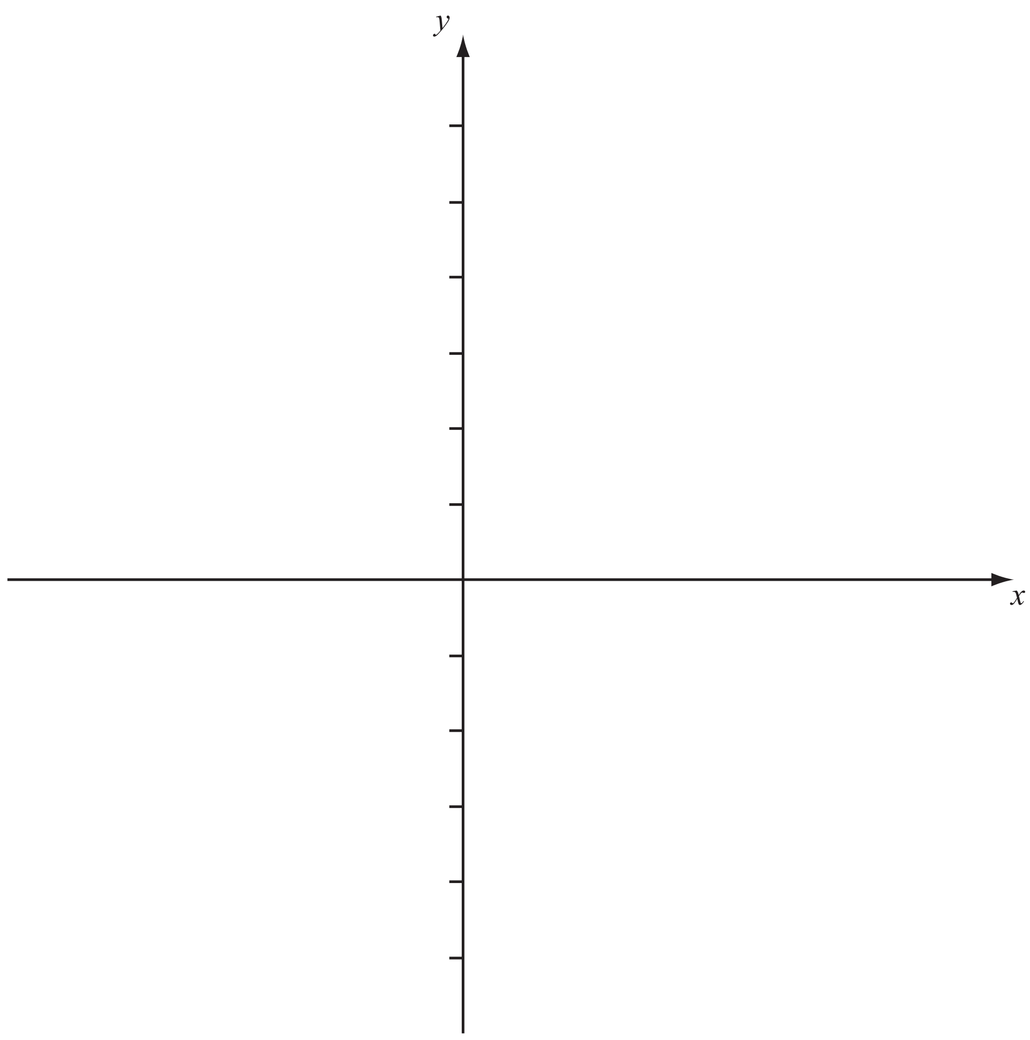
On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

(1)



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Question 8 continued

Q8

(Total 7 marks)

Leave blank

9. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d.
(2)

(b) Show that $a = -17.5$ and find the value of d.
(2)

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(d) Hence find the value of n.
(3)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total 11 marks)

Leave blank

10. The line l_1 passes through the point A (2, 5) and has gradient $-\frac{1}{2}$.

(a) Find an equation of l_1, giving your answer in the form $y = mx + c$.
(3)

The point B has coordinates (–2, 7).

(b) Show that B lies on l_1.
(1)

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.
(3)

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(d) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$
(4)

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Leave blank

Question 10 continued

Q10

(Total 11 marks)

Turn over

Leave blank

11. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0 .$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.
(6)

(b) Find an equation of the normal to C at the point P.
(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of triangle APB.
(4)

Leave blank

Question 11 continued

Leave blank

Question 11 continued

Leave blank

Question 11 continued

Q11

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

Answer **all** questions.

1 The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$.

(a) Find the coordinates of the mid-point of BC. *(2 marks)*

(b) (i) Find the gradient of AB, in its simplest form. *(2 marks)*

(ii) Hence find an equation of the line AB, giving your answer in the form $x + qy = r$, where q and r are integers. *(2 marks)*

(iii) Find an equation of the line passing through C which is parallel to AB. *(2 marks)*

(c) Prove that angle ABC is a right angle. *(3 marks)*

2 The curve with equation $y = x^4 - 32x + 5$ has a single stationary point, M.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Hence find the x-coordinate of M. *(3 marks)*

(c) (i) Find $\dfrac{d^2y}{dx^2}$. *(1 mark)*

(ii) Hence, or otherwise, determine whether M is a maximum or a minimum point. *(2 marks)*

(d) Determine whether the curve is increasing or decreasing at the point on the curve where $x = 0$. *(2 marks)*

3 (a) Express $5\sqrt{8} + \dfrac{6}{\sqrt{2}}$ in the form $n\sqrt{2}$, where n is an integer. *(3 marks)*

(b) Express $\dfrac{\sqrt{2}+2}{3\sqrt{2}-4}$ in the form $c\sqrt{2} + d$, where c and d are integers. *(4 marks)*

4 A circle with centre C has equation $x^2 + y^2 - 10y + 20 = 0$.

(a) By completing the square, express this equation in the form

$$x^2 + (y - b)^2 = k$$ *(2 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle, leaving your answer in surd form. *(1 mark)*

(c) A line has equation $y = 2x$.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation $x^2 - 4x + 4 = 0$. *(2 marks)*

(ii) Hence show that the line is a tangent to the circle and find the coordinates of the point of contact, P. *(3 marks)*

(d) Prove that the point $Q(-1, 4)$ lies inside the circle. *(2 marks)*

5 (a) Factorise $9 - 8x - x^2$. *(2 marks)*

(b) Show that $25 - (x + 4)^2$ can be written as $9 - 8x - x^2$. *(1 mark)*

(c) A curve has equation $y = 9 - 8x - x^2$.

(i) Write down the equation of its line of symmetry. *(1 mark)*

(ii) Find the coordinates of its vertex. *(2 marks)*

(iii) Sketch the curve, indicating the values of the intercepts on the x-axis and the y-axis. *(3 marks)*

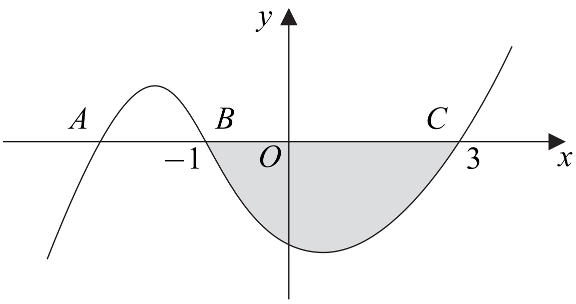
Turn over for the next question

6 (a) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^3 - 7x - 6$.

(i) Use the Factor Theorem to show that $x + 1$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

(ii) Express $\mathrm{p}(x) = x^3 - 7x - 6$ as the product of three linear factors. *(3 marks)*

(b) The curve with equation $y = x^3 - 7x - 6$ is sketched below.



The curve cuts the x-axis at the point A and the points $B(-1, 0)$ and $C(3, 0)$.

(i) State the coordinates of the point A. *(1 mark)*

(ii) Find $\displaystyle\int_{-1}^{3} (x^3 - 7x - 6)\,\mathrm{d}x$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - 7x - 6$ and the x-axis between B and C. *(1 mark)*

(iv) Find the gradient of the curve $y = x^3 - 7x - 6$ at the point B. *(3 marks)*

(v) Hence find an equation of the normal to the curve at the point B. *(3 marks)*

7 The curve C has equation $y = x^2 + 7$. The line L has equation $y = k(3x + 1)$, where k is a constant.

(a) Show that the x-coordinates of any points of intersection of the line L with the curve C satisfy the equation

$$x^2 - 3kx + 7 - k = 0$$ *(1 mark)*

(b) The curve C and the line L intersect in two distinct points. Show that

$$9k^2 + 4k - 28 > 0$$ *(3 marks)*

(c) Solve the inequality $9k^2 + 4k - 28 > 0$. *(4 marks)*

END OF QUESTIONS

Answer **all** questions.

1 The points A and B have coordinates $(1, 6)$ and $(5, -2)$ respectively. The mid-point of AB is M.

(a) Find the coordinates of M. *(2 marks)*

(b) Find the gradient of AB, giving your answer in its simplest form. *(2 marks)*

(c) A straight line passes through M and is perpendicular to AB.

(i) Show that this line has equation $x - 2y + 1 = 0$. *(3 marks)*

(ii) Given that this line passes through the point $(k, k+5)$, find the value of the constant k. *(2 marks)*

2 (a) Factorise $2x^2 - 5x + 3$. *(1 mark)*

(b) Hence, or otherwise, solve the inequality $2x^2 - 5x + 3 < 0$. *(3 marks)*

3 (a) Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers. *(4 marks)*

(b) Express $\sqrt{45} + \dfrac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. *(3 marks)*

4 (a) (i) Express $x^2 + 2x + 5$ in the form $(x+p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence show that $x^2 + 2x + 5$ is always positive. *(1 mark)*

(b) A curve has equation $y = x^2 + 2x + 5$.

(i) Write down the coordinates of the minimum point of the curve. *(2 marks)*

(ii) Sketch the curve, showing the value of the intercept on the y-axis. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 2x + 5$. *(3 marks)*

5 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 20t^2 + 66t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{dx}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2x}{dt^2}$. *(2 marks)*

(b) Verify that x has a stationary value when $t = 3$, and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of x with respect to t when $t = 1$. *(2 marks)*

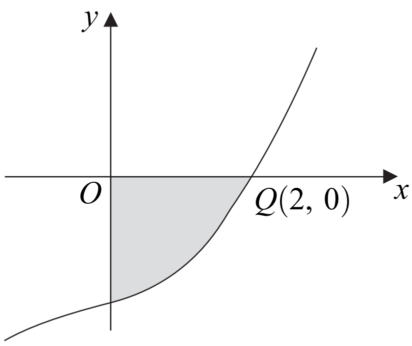
(d) Determine whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$. *(2 marks)*

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

(i) Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ in the form $(x-2)(x^2 + ax + b)$, where a and b are constants. *(2 marks)*

(b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x-axis at the point $Q(2, 0)$.



(i) Find the gradient of the curve C at the point Q. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve C at the point Q. *(2 marks)*

(iii) Find $\int (x^3 + x - 10)\, dx$. *(3 marks)*

(iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes. *(2 marks)*

Turn over for the next question

7 A circle with centre C has equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) The point D has coordinates $(7, \ -2)$.

(i) Verify that the point D lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point D, giving your answer in the form $mx + ny = p$, where m, n and p are integers. *(3 marks)*

(d) (i) A line has equation $y = kx$. Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$ *(2 marks)*

(ii) Find the values of k for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

has equal roots. *(5 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (d)(ii). *(1 mark)*

END OF QUESTIONS