



General Certificate of Education

Statistics 6380

SS04 Statistics Unit 4

Mark Scheme

2007 examination - January series

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Key to mark scheme and abbreviations used in marking

| | | | |
|--------------|--|-----|----------------------------|
| M | mark is for method | | |
| m or dM | mark is dependent on one or more M marks and is for method | | |
| A | mark is dependent on M or m marks and is for accuracy | | |
| B | mark is independent of M or m marks and is for method and accuracy | | |
| E | mark is for explanation | | |
| ✓ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| -x EE | deduct x marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Jan 07

SS04

| Question | Solution | Marks | Total | Comments |
|--------------|--|---|----------|---|
| 1(a) | $\hat{p} = 27/85 = 0.31765$ 95% confidence interval for \hat{p} $0.31765 \pm 1.96 \sqrt{\frac{0.31765 \times 0.68235}{85}}$ 0.31765 ± 0.09897 $0.219 \sim 0.417$ | B1 B1 M1 M1 m1 A1 | 6 | 27/85 ACF 1.96 use of $\hat{p} \pm z \times$ their s.d. method for s.d. correct method - allow incorrect z 0.219(0.218 ~ 0.22) and 0.417(0.416 ~ 0.418) or 0.317(0.316 ~ 0.318) and 0.0990(0.0989 ~ 0.099) |
| (b) | 0.17 below confidence interval - evidence that greater proportion of Simsons matches break. Bad decision. | E1✓ E1 | 2 | below confidence interval / evidence more break bad decision |
| Total | | | 8 | |
| 2(a) | $\bar{x} = 260.25 \quad s = 41.337$ 90% confidence interval for mean $260.25 \pm 1.895 \times \frac{41.337}{\sqrt{8}}$ 260.25 ± 27.70 $232.6 \sim 287.9$ | B1 B1 B1 B1✓ M1 m1 A1 | 7 | 260.25 (260 ~ 260.3) 41.337 (41.3 ~ 41.4) 7 df 1.895 (1.89 ~ 1.9) use of $\frac{\text{their s.d.}}{\sqrt{8}}$ - generous method - allow incorrect t 232.6 (232.5 ~ 233) and 287.9 (287.5 ~ 288) or 260.25 (260 ~ 260.5) and 27.7 (27.65 ~ 27.75) |
| (b) | times were a random sample from a normal distribution | E1 E1 | 2 | random normal – allow independent |
| | | | 9 | |

SS04 (cont)

| Question | Solution | Marks | Total | Comments |
|----------|--|--|-------|---|
| 3(a) | $H_0: p = 0.4$ $H_1: p \neq 0.4$ $B(1240, 0.4) \rightarrow$ Normal, mean 496 s.d. $\sqrt{1240 \times 0.4 \times 0.6} = \sqrt{297.6}$ $= 17.25$ $z = \frac{476.5 - 496}{17.25}$ $= -1.13$ c.v. ± 1.96 accept $H_0 \rightarrow$ accept that 40% of householders in Birmingham will make a donation when approached. SC if exact probabilities used (Binomial 0.129, Poisson mean 496 0.191) allow B1 B0 M0 A0 m0 A0 B1 comparison with 0.025 A1✓ A1✓ SC Poisson approx then normal approx used - allow max B1 B1 M0 A0 M1 m0 A0 B1 A0 A1 | B1 B1 M1 A1 M1 m1 A1 B1 A1✓ A1✓ | 10 | both hypotheses - accept p as implying population attempt at normal approximation - generous e.g. allow if via Poisson method for s.d. 496 CAO and 17.25 (17.2~17.3) may be implied method for z - their mean and s.d. - allow no or incorrect c.c - ignore sign method for z - disallow incorrect cc - ignore sign -1.13 (-1.12 ~ -1.17) 1.96 ignore sign conclusion - correct tail compared correct conclusion - their figures in context - not necessarily correct tail. allow comparison of p -value 0.131 (0.12 ~ 0.132) with 0.025 |
| | (b) $H_0: p = 0.005$ $H_1: p > 0.005$ $B(440, 0.005) \rightarrow$ Poisson mean 2.2 $P(7 \text{ or more}) = 1 - 0.9925 = 0.0075$ $0.0075 < 0.05$ reject $H_0 \rightarrow$ significant evidence that more than 0.5% of Birmingham householders would agree to make a monthly donation. SC allow critical value 5 or more (closest to 5%) or 6 or more (less than 5%) SC if exact probabilities 0.0073 used allow B1 B0 B0 M1 A0 A1✓ A1✓ SC if normal approx used allow B1 B0 B1 M1 A0 A0✓ A0✓ | B1 B1 B1✓ M1 A1 A1✓ A1✓ | 7 | both hypotheses - accept p , etc as implying population attempt at appropriate Poisson approx mean 440×0.005 attempt to calculate $P(7 \text{ or more})$ - generous 0.0075 (0.0074 ~ 0.0075) conclusion - their probability compared with 0.05 correct conclusion - their figures - in context last 2 marks require use of $p = 0.005$ |

SS04 (cont)

| Question | Solution | Marks | Total | Comments |
|-------------|--|---|-----------|---|
| 3(c) | <p>part (a) suggests that 40% would make a single donation.</p> <p>monthly donations worth 80 times as much. $40/80 = 0.5$.</p> <p>hence if more than 0.5% would make a monthly donation this would be more profitable in the long run. Part (b) provides significant evidence that this is the case.</p> | <p>E1</p> <p>E1</p> <p>E1</p> | 3 | <p>40% single donation/more than 0.5% monthly donation - must be based on correct work</p> <p>$\frac{40}{80} = 0.5$</p> <p>monthly donations more profitable</p> |
| | Total | | 20 | |
| 4(a) | <p>$z = \frac{15 - 11.4}{2.4} = 1.5$</p> <p>probability > 15 minutes</p> <p>$= 1 - 0.93319$</p> <p>$= 0.0668$</p> | <p>B1</p> <p>M1</p> <p>A1</p> | 3 | <p>method for z - ignore sign</p> <p>completely correct method</p> <p>0.0668 (0.0668 ~ 0.0669)</p> |
| (b) | <p>time for 3 games → normal</p> <p>mean $3 \times 11.4 = 34.2$</p> <p>s.d. $\sqrt{3 \times 2.4^2} = 4.157$</p> <p>$z = \frac{30 - 34.2}{4.157} = -1.010$</p> <p>probability < 30 minutes</p> <p>$= 1 - 0.84375$</p> <p>$= 0.156$</p> | <p>B1</p> <p>M1</p> <p>m1</p> <p>m1</p> <p>A1</p> | 5 | <p>mean 34.2</p> <p>method for s.d (or variance) - even if not called s.d.</p> <p>method for z - ignore sign</p> <p>completely correct method</p> <p>0.156 (0.156 ~ 0.157)</p> |
| (c) | <p>Time for 3 games – time to library → normal</p> <p>mean $34.2 - 45 = -10.8$</p> <p>s.d. $\sqrt{3 \times 2.4^2 + 4.1^2} = 5.839$</p> <p>$z = \frac{-10.8}{5.839} = -1.850$</p> <p>probability Gwyneth back at hostel before 3 games completed is</p> <p>$1 - 0.96783 = 0.0322$</p> | <p>M1</p> <p>B1</p> <p>m1</p> <p>m1</p> <p>A1</p> | 5 | <p>attempt to find s.d of (3 games – time to library)</p> <p>-10.8 ignore sign - may be implied</p> <p>method for s.d. or variance - their value in (b)</p> <p>method - allow wrong tail</p> <p>0.0322 (0.032 ~ 0.0323)</p> |
| (d) | <p>very little chance of going to library and returning in time to play.</p> <p>Must either play and pay fine or go to library and miss turn</p> | <p>E1</p> <p>E1✓</p> | 2 | <p>small chance of both</p> <p>choose one or other</p> |
| | Total | | 15 | |

SS04 (cont)

| Question | Solution | Marks | Total | Comments |
|----------|---|-------|-------|--|
| 5(a) | $H_0: \mu = 5.00$ | B1 | 9 | both hypotheses - μ or population needed |
| | $H_1: \mu \neq 5.00$ | B1 | | 5.132 (5.13 ~ 5.135) and 0.8611 (0.861 ~ 0.8615) |
| | $\bar{x} = 5.132 \quad s = 0.8611$ | M1 | | use of their $\frac{s}{\sqrt{11}}$ |
| | $t = \frac{5.132 - 5.00}{\frac{0.8611}{\sqrt{11}}}$ | m1 | | method for t - ignore sign |
| | $= 0.508$ | A1 | | 0.508 (0.507 ~ 0.508) |
| | c.v. t_{10} are ± 2.228 | B1 | | 10 df |
| | accept H_0 : i.e. accept mean weight of potatoes in bags is 5kg | B1 | | 2.228 - ignore sign |
| | | A1✓ | | correct conclusion their figures - AG |
| | | A1✓ | | correct conclusion their figures in context |
| | SC critical values $\frac{5.00 \pm 2.228 \times 0.8611}{\sqrt{11}}$ 5.00 ± 0.578 $4.42 \sim 5.58$ confidence interval $\frac{5.132 \pm 2.228 \times 0.8611}{\sqrt{11}}$ 5.132 ± 0.578 $4.55 \sim 5.71$ | | | |
| (b) | contents much less than 5kg will lead to customer complaints, contents much greater than 5kg will use more potatoes than necessary. | E1 | 1 | reason – either |
| (c) | $H_0: \mu = 0.7$ | B1 | 4 | both hypotheses - don't penalise for same reason as (a) |
| | $H_1: \mu > 0.7$ | B1 | | 2.68 (2.65 ~ 2.69) - allow use of $\frac{0.52 \times \sqrt{60}}{59}$ |
| | $z = \frac{0.88 - 0.7}{\frac{0.52}{\sqrt{60}}} = 2.68$ | B1 | | 2.3263 (2.326 ~ 2.33) or 2.39 (2.39 ~ 2.392) |
| | c.v. 2.3263 | B1 | | conclusion based on correct method of calculation and c.v. from z or t -tables |
| | reject H_0 : Evidence mean magnitude of differences greater than 0.7 | B1✓ | | |
| | SC critical value $0.7 + 2.3263 \times \frac{0.52}{\sqrt{60}} = 0.856$ confidence interval $0.88 \pm 2.3263 \times \frac{0.52}{\sqrt{60}}$ $0.724 \sim 1.036$ | | | |

SS04 (cont)

| Question | Solution | Marks | Total | Comments |
|----------------|---|-------|-----------|---|
| 5(d)(i) | $H_0: \mu = 5.00$ | B1 | 4 | both hypotheses - don't penalise for same reason as (a) |
| | $H_1: \mu \neq 5.00$ | B1 | | 4.71 (4.66~4.72) allow use of $0.12 \times \frac{\sqrt{50}}{49}$ |
| | $z = \frac{5.08 - 5.00}{\frac{0.12}{\sqrt{50}}} = 4.71$ | B1 | | 1.96 ignore sign - or 2.01 (2.009 ~ 2.01) |
| | c.v. ± 1.96 reject H_0 : significant evidence mean weight of potatoes in bags packed by Sybil \neq (greater than) 5kg | B1✓ | | conclusion based on correct method of calculation and c.v. from z or t tables |
| | SC critical values $\frac{5.00 \pm 1.96 \times 0.12}{\sqrt{50}}$ 4.967 ~ 5.033 confidence interval $\frac{5.08 \pm 1.96 \times 0.12}{\sqrt{50}}$ 5.047 ~ 5.113 | | | |
| (d)(ii) | to test $H_0: \mu = 0.7$; $H_1: \mu > 0.7$ the critical value would be positive but the test statistic would be negative hence H_0 must be accepted. | B2, 1 | 2 | both marks for a clear explanation |
| (e) | there is evidence that Sybil's mean is a little over 5kg while Maxwell's may equal 5kg | E1 | 3 | comparison of means |
| | but on average Sybil's bags are much closer to 5kg. | E1 | | comparison of variability |
| | Maxwell's bags are erratic and therefore unsatisfactory | E1 | | Maxwell unsatisfactory |
| | Total | | 23 | |
| | TOTAL | | 75 | |