

General Certificate of Education
June 2006
Advanced Level Examination



MATHEMATICS
Unit Statistics 2B

MS2B

Wednesday 24 May 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Statistics 2B has a **written paper only**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The number of A-grades, X , achieved in total by students at Lowkey School in their Mathematics examinations each year can be modelled by a Poisson distribution with a mean of 3.
- (a) Determine the probability that, during a 5-year period, students at Lowkey School achieve a total of more than 18 A-grades in their Mathematics examinations. *(3 marks)*
- (b) The number of A-grades, Y , achieved in total by students at Lowkey School in their English examinations each year can be modelled by a Poisson distribution with a mean of 7.
- (i) Determine the probability that, during a year, students at Lowkey School achieve a total of fewer than 15 A-grades in their Mathematics and English examinations. *(3 marks)*
- (ii) What assumption did you make in answering part (b)(i)? *(1 mark)*

- 2 The weights of lions kept in captivity at Wildcat Safari Park are normally distributed.

The weights, in kilograms, of a random sample of five lions were recorded as

46 48 57 49 54

- (a) Construct a 95% confidence interval for the mean weight of lions kept in captivity at Wildcat Safari Park. *(6 marks)*
- (b) State the probability that this confidence interval does **not** contain the mean weight of lions kept in captivity at Wildcat Safari Park. *(1 mark)*

- 3 Morecrest football team always scores at least one goal but never scores more than four goals in each game. The number of goals, R , scored in each game by the team can be modelled by the following probability distribution.

r	1	2	3	4
$P(R=r)$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

- (a) Calculate exact values for the mean and variance of R . (4 marks)
- (b) Next season the team will play 32 games. They expect to win 90% of the games in which they score at least three goals, half of the games in which they score exactly two goals and 20% of the games in which they score exactly one goal.

Find, for next season:

- (i) the number of games in which they expect to score at least three goals; (1 mark)
- (ii) the number of games that they expect to win. (2 marks)

- 4 It is claimed that the area within which a school is situated affects the age profile of the staff employed at that school. In order to investigate this claim, the age profiles of staff employed at two schools with similar academic achievements are compared.

Academia High School, situated in a rural community, employs 120 staff whilst Best Manor Grammar School, situated in an inner-city community, employs 80 staff.

The **percentage** of staff within each age group, for each school, is given in the table.

Age	Academia High School	Best Manor Grammar School
22–34	17.5	40.0
35–39	60.0	45.0
40–59	22.5	15.0

- (a) (i) Form the data into a contingency table suitable for analysis using a χ^2 distribution. (2 marks)
- (ii) Use a χ^2 test, at the 1% level of significance, to determine whether there is an association between the age profile of the staff employed and the area within which the school is situated. (9 marks)
- (b) Interpret your result in part (a)(ii) as it relates to the 22–34 age group. (2 marks)

Turn over ►

- 5 (a) The continuous random variable X follows a rectangular distribution with probability density function defined by

$$f(x) = \begin{cases} \frac{1}{b} & 0 \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

- (i) Write down $E(X)$. (1 mark)
- (ii) Prove, using integration, that

$$\text{Var}(X) = \frac{1}{12} b^2 \quad (5 \text{ marks})$$

- (b) At an athletics meeting, the error, in seconds, made in recording the time taken to complete the 10 000 metres race may be modelled by the random variable T , having the probability density function

$$f(t) = \begin{cases} 5 & -0.1 \leq t \leq 0.1 \\ 0 & \text{otherwise} \end{cases}$$

Calculate $P(|T| > 0.02)$. (3 marks)

- 6 The lifetime, X hours, of Everwhite camera batteries is normally distributed. The manufacturer claims that the mean lifetime of these batteries is 100 hours.

- (a) The members of a photography club suspect that the batteries do not last as long as is claimed by the manufacturer. In order to investigate their suspicion, the members test a random sample of five of these batteries and find the lifetimes, in hours, to be as follows:

85 92 100 95 99

Test the members' suspicion at the 5% level of significance. (9 marks)

- (b) The manufacturer, believing that the mean lifetime of these batteries has not changed from 100 hours, decides to determine the lifetime, x hours, of each of a random sample of 80 Everwhite camera batteries. The manufacturer obtains the following results, where $\bar{x}$ denotes the sample mean:

$$\sum x = 8080 \quad \text{and} \quad \sum (x - \bar{x})^2 = 6399$$

Test the manufacturer's belief at the 5% level of significance. (8 marks)

7 The continuous random variable X has probability density function defined by

$$f(x) = \begin{cases} \frac{1}{5}(2x + 1) & 0 \leq x \leq 1 \\ \frac{1}{15}(4 - x)^2 & 1 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f . (2 marks)

(b) (i) Show that the cumulative distribution function, $F(x)$, for $0 \leq x \leq 1$ is

$$F(x) = \frac{1}{5}x(x + 1) \quad (3 \text{ marks})$$

(ii) Hence write down the value of $P(X \leq 1)$. (1 mark)

(iii) Find the value of x for which $P(X \geq x) = \frac{17}{20}$. (5 marks)

(iv) Find the lower quartile of the distribution. (4 marks)

END OF QUESTIONS

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