

General Certificate of Education
January 2006
Advanced Level Examination



MATHEMATICS
Unit Statistics 2B

MS2B

Thursday 12 January 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2B.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Statistics 2B has a **written paper only**.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 A study undertaken by Goodhealth Hospital found that the number of patients each month, X , contracting a particular superbug can be modelled by a Poisson distribution with a mean of 1.5 .
- (a) (i) Calculate $P(X = 2)$. (2 marks)
- (ii) Hence determine the probability that exactly 2 patients will contract this superbug in each of three consecutive months. (2 marks)
- (b) (i) Write down the distribution of Y , the number of patients contracting this superbug in a given 6-month period. (1 mark)
- (ii) Find the probability that at least 12 patients will contract this superbug during a given 6-month period. (2 marks)
- (c) State **two** assumptions implied by the use of a Poisson model for the number of patients contracting this superbug. (2 marks)
- 2 Year 12 students at Newstatus School choose to participate in one of four sports during the Spring term.

The students' choices are summarised in the table.

	Squash	Badminton	Archery	Hockey	Total
Male	5	16	30	19	70
Female	4	20	33	53	110
Total	9	36	63	72	180

- (a) Use a χ^2 test, at the 5% level of significance, to determine whether the choice of sport is independent of gender. (10 marks)
- (b) Interpret your result in part (a) as it relates to students choosing hockey. (2 marks)

- 3 The time, T minutes, that parents have to wait before seeing a mathematics teacher at a school parents' evening can be modelled by a normal distribution with mean μ and standard deviation σ .

At a recent parents' evening, a random sample of 9 parents was asked to record the times that they waited before seeing a mathematics teacher.

The times, in minutes, are

5 12 10 8 7 6 9 7 8

- (a) Construct a 90% confidence interval for μ . (7 marks)
- (b) Comment on the headteacher's claim that the mean time that parents have to wait before seeing a mathematics teacher is 5 minutes. (2 marks)

- 4 (a) A random variable X has probability density function defined by

$$f(x) = \begin{cases} k & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

- (i) Show that $k = \frac{1}{b-a}$. (1 mark)
- (ii) Prove, using integration, that $E(X) = \frac{1}{2}(a+b)$. (4 marks)
- (b) The error, X grams, made when a shopkeeper weighs out loose sweets can be modelled by a rectangular distribution with the following probability density function:

$$f(x) = \begin{cases} k & -2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Write down the value of the mean, μ , of X . (1 mark)
- (ii) Evaluate the standard deviation, σ , of X . (2 marks)
- (iii) Hence find $P\left(X < \frac{2-\mu}{\sigma}\right)$. (3 marks)

- 5 The Globe Express agency organises trips to the theatre. The cost, £ X , of these trips can be modelled by the following probability distribution:

x	40	45	55	74
$P(X=x)$	0.30	0.24	0.36	0.10

- (a) Calculate the mean and standard deviation of X . (4 marks)
- (b) For special celebrity charity performances, Globe Express increases the cost of the trips to £ Y , where

$$Y = 10X + 250$$

Determine the mean and standard deviation of Y . (2 marks)

- 6 In previous years, the marks obtained in a French test by students attending Topnotch College have been modelled satisfactorily by a normal distribution with a mean of 65 and a standard deviation of 9.

Teachers in the French department at Topnotch College suspect that this year their students are, on average, underachieving.

In order to investigate this suspicion, the teachers selected a random sample of 35 students to take the French test and found that their mean score was 61.5.

- (a) Investigate, at the 5% level of significance, the teachers' suspicion. (6 marks)
- (b) Explain, in the context of this question, the meaning of a Type I error. (2 marks)

- 7 Engineering work on the railway network causes an increase in the journey time of commuters travelling into work each morning.

The increase in journey time, T hours, is modelled by a continuous random variable with probability density function

$$f(t) = \begin{cases} 4t(1 - t^2) & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that $E(T) = \frac{8}{15}$. (3 marks)
- (b) (i) Find the cumulative distribution function, $F(t)$, for $0 \leq t \leq 1$. (2 marks)
- (ii) Hence, or otherwise, for a commuter selected at random, find

$$P(\text{mean} < T < \text{median})$$
(5 marks)

- 8 Bottles of sherry nominally contain 1000 millilitres. After the introduction of a new method of filling the bottles, there is a suspicion that the mean volume of sherry in a bottle has changed.

In order to investigate this suspicion, a random sample of 12 bottles of sherry is taken and the volume of sherry in each bottle is measured.

The volumes, in millilitres, of sherry in these bottles are found to be

996	1006	1009	999	1007	1003
998	1010	997	996	1008	1007

Assuming that the volume of sherry in a bottle is normally distributed, investigate, at the 5% level of significance, whether the mean volume of sherry in a bottle differs from 1000 millilitres. (10 marks)

END OF QUESTIONS

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