



General Certificate of Education (A-level)
June 2012

Mathematics

MS2B

(Specification 6360)

Statistics 2B

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)	$\bar{x} = \frac{\sum x}{n} = \frac{546}{15} = \frac{182}{5} = 36.4$	B1		oe
	$s^2 = \frac{\sum (x - \bar{x})^2}{n-1} = \frac{1407.6}{14} = 100.54$ (or $s = 10.03$)	B1		$\sigma^2 = 93.84$ or $\sigma = 9.687$ iff $\frac{\sigma}{\sqrt{14}}$ used below
	$t_{crit} = \pm 2.624$	B1		ignore signs for t_{crit} (allow $t = 2.62$) (if z used then max (B1B1B0 M0A0A0))
	98% CI for μ : $36.4 \pm 2.624 \times \frac{s}{\sqrt{15}}$	M1		their $\bar{x} \pm t_{14} \times \frac{\text{their } s}{\sqrt{15}}$ or their $\bar{x} \pm t_{14} \times \frac{\text{their } \sigma}{\sqrt{14}}$
	$(29.6, 43.2)$ 36.4 ± 6.8	A1ft		(allow any of the following for t_{14} : 1.345; 1.761; 2.145; 2.624; 2.977)
	$= 29.6, 43.2$	A1	6	cao
(b)	$40.0 \in \text{C.I.}$ \Rightarrow no change	E1ft E1ft	2	Must refer to 40 (dep M1) Dep on previous mark
			8	
2(a)	$H_0: \mu = 4.0$ $H_1: \mu > 4.0$	B1		(both)
	$z_{calc} = \frac{4.2 - 4}{1.1/\sqrt{40}}$	M1		Alternative: $P(\bar{X} > 4.2) = P(Z > 1.15)$ M1A1
	$= 1.15$	A1		awrt
	$z_{crit} = 1.6449$	B1		$= 1 - 0.87493$ $= 0.125$ B1
	Accept H_0 [or Reject H_1]	A1		$0.125 > 0.05 \Rightarrow$ accept H_0 Adep1 Dep on B1M1B1
	Insufficient evidence at 5% level to support Julian's claim	E1	6	Dep on previous mark
(b)	Type II error. Accepted H_0 when H_0 was false (oe)	B1ft E1	2	Follow through on conclusion in (a) Dep on previous mark If Reject H_0 in (a) then: No error (B1ft) Rejected H_0 when H_0 was false (oe) (E1)
	Total		8	

Q	Solution	Marks	Total	Comments
3(a)	for $-5 \leq x \leq 15$ $f(x) = \frac{d}{dx} F(x) = \frac{d}{dx} \left(\frac{x+5}{20} \right) = \frac{1}{20}$	B1	1	AG
(b)(i)	$P(X \geq 7) = 1 - F(7)$ $= 1 - \frac{12}{20}$ $= \frac{2}{5}$ or $\left[\frac{8}{20}; \frac{4}{10}; 0.4 \right]$	B1	1	Alternative: Use of $f(x) = \frac{1}{20}$ or graph \Rightarrow $P(X \geq 7) = \frac{1}{20} \times (15 - 7) = \frac{2}{5}$ (oe)
(ii)	$P(X \neq 7) = 1$	B1	1	cao
(iii)	$E(X) = \frac{1}{2}(-5 + 15) = 5$	B1	1	Alternative: $E(X) = \int_{-5}^{15} \frac{x}{20} dx = \left[\frac{x^2}{40} \right]_{-5}^{15}$ $= \frac{1}{40}(225 - 25)$ $= \frac{1}{40} \times 200$ $= 5$ B1 (cao)
(iv)	$E(3X^2) = \int_{-5}^{15} \frac{3x^2}{20} dx \left\{ \text{(ignore limits)} \right.$ $\left. \left[\frac{x^3}{20} \right]_{-5}^{15} \right\}$ $\frac{1}{20}(3375 + 125)$ $168\frac{3}{4} + 6\frac{1}{4}$ $= 175$	M1		
	$= 175$	A1		correct limits seen / used
	Alternative: $\text{Var}(X) = \frac{1}{12}(15 - -5)^2 = \frac{400}{12}$ (oe)	A1	3	(cao) (allow 174.9)
	$E(3X^2) = 3 \times \left[\frac{400}{12} + 5^2 \right]$	(B1)		$E(3X^2) = 3E(X^2)$
	$= 175$	(M1)		$= 3 \times \left[\{ \text{their Var}(X) > 0 \} + \{ \text{their } E(X) \}^2 \right]$ used
		(A1)		(\Rightarrow M1)
	Total		7	

MS2B

Q	Solution	Marks	Total	Comments												
4(a)	<table border="1"><tr><td>r</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>p</td><td>.5</td><td>.24</td><td>.144</td><td>.0864</td><td>0.0296</td></tr></table>	r	1	2	3	4	5	p	.5	.24	.144	.0864	0.0296			
	r	1	2	3	4	5										
	p	.5	.24	.144	.0864	0.0296										
	$0.4 \times 0.6 = 0.24$ $0.24 \times 0.6 = 0.144$ $0.144 \times 0.6 = 0.0864$	B2,1	2	B1 for any 1 correct (unsimplified) (B1) B2 all correct and simplified												
	(b) P(fewer than 3 bedrooms are rented) $= P(R = 1, 2) \Rightarrow$ P(fewer than 3 bedrooms not rented) $= 1 - P(R = 1, 2)$ $= 1 - P(1 \text{ or } 2 \text{ rooms are rented})$ $= 1 - (0.5 + \mathbf{0.24})$ [their $0 < p(2) \leq 0.4704$ value from table used] $= 1 - 0.74$ $= 0.26$	M1 m1 A1	 3	Alternative: P(fewer than 3 not rented) $= P(0, 1 \text{ or } 2 \text{ not rented})$ $= P(5, 4 \text{ or } 3 \text{ are rented})$ $= P(R = 3, 4, 5)$ M1 $p = 0.4 \times 0.6^2 + 0.4 \times 0.6^3 + 0.0296$ $= \mathbf{0.144} + \mathbf{0.0864} + 0.0296$ m1 [or their $p(3) + p(4) \leq 0.4704$ value from table used] $= 0.26$ (cao) A1 [SC 0.74 for B1]												
	(c)(i) $E(R) = 0.5 \times 1 + 0.4 \times 0.6 \times 2$ $+ 0.4 \times 0.6^2 \times 3 + 0.4 \times 0.6^3 \times 4$ $+ 0.0296 \times 5$ $= 0.5 \times 1 + 0.24 \times 2 + 0.144 \times 3 + 0.0864 \times 4$ $+ 0.0296 \times 5$ $= \mathbf{0.5} + \mathbf{0.48} + \mathbf{0.432} + \mathbf{0.3456} + \mathbf{0.148}$ $\left[= \frac{1}{2} + \frac{12}{25} + \frac{54}{125} + \frac{216}{625} + \frac{37}{250} \right]$ $\therefore E(R) = 1.9056$	M1 A1	 2	$\sum_1^5 r_i \times P(R = r_i)$ from their table [awfw 1.9 to 1.91] $\left[1 \frac{566}{625} \right]$												
	(ii) $E(R^2) = 0.5 \times 1^2 + 0.4 \times 0.6 \times 2^2$ $+ 0.4 \times 0.6^2 \times 3^2 + 0.4 \times 0.6^3 \times 4^2$ $+ 0.0296 \times 5^2$ $E(R^2) = 4.8784$ $\text{Var}(R) = 4.8784 - 1.9056^2$ $(= 1.24708864)$ $= 1.25$ (3sf)	B1 M1 A1	 3	AG $4.8784 - \text{their } E^2(R)$ (awfw 1.23 to 1.25)												

MS2B

Q	Solution	Marks	Total	Comments
(d)	$E(M) = 1250E(R) - 282$ $= 1250 \times 1.9056 - 282$ $= 2100$ $\text{Var}(M) = 1250^2 \times [4.8784 - 1.9056^2]$ $\text{sd}(M) = 1250 \times \sqrt{1.24708864}$ $= 1395.91$	 B1 M1 A1	 3	 cao $1250^2 \times \text{their Var}(R) > 0$ in (c)(ii) (1 948 473 to 1 953 125) $\text{sd}(M) = \sqrt{1948437} = 1395.9$ $(\sqrt{1953125} = 1397.5)$ (awfw 1395 to 1400)
	Total		13	

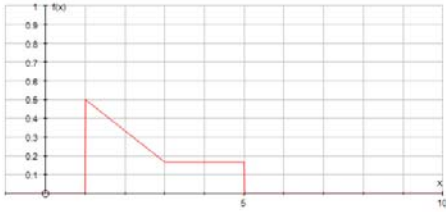
MS2B

	Solution	Marks	Total	Comments
5(a)(i)	$P(X \geq 9) = 1 - P(X \leq 8)$ $= 1 - 0.5231$ $= 0.4769$	B2,1	2	$1 - 0.6530 = 0.347$ (B1) awfw 0.476 and 0.477
(ii)	$P(5 < X < 10) = P(X \leq 9) - P(X \leq 5)$ $= 0.653 - 0.1496$ $= 0.5034$	B3,2,1	3	awfw 0.503 to 0.504 $0.7634 - 0.1496 = 0.613$ to 0.614 (B2) $0.6530 - 0.2562 = 0.397$ to 0.398 (B2) $0.7634 - 0.2562 = 0.507$ to 0.508 (B1) $\alpha - 0.1496$ or $0.653 - \alpha$ (B1) iff $0 < p < 1$
(b)	$P(Y < 2) = P(Y \leq 1) = P(Y = 0 \text{ or } Y = 1)$ $= e^{-1.5} + e^{-1.5} \times 1.5$ $[0.2231 + 0.3347]$ $= 0.5578254$ $= 0.558$	M1 A1	2	0.8 to 0.81 (B1) (both) awfw 0.557 to 0.56
(c)(i)	$\lambda = 8.5 + 1.5 = 10$	B1	1	Allow P(10) or Po(10)
(ii)	$P(T > 16) = 1 - P(T \leq 16)$ $= 1 - 0.9730$ $= 0.027$	M1 A1	2	
(iii)	$p = {}^3C_2 \cdot 0.027^2 \times 0.973$ $+ 0.027^3$ $= 0.002128 + 0.00001968$ $= 0.0021$ [4 dp]	M1 M1 A1	3	for either term correct for addition of the two correct terms 0.0021 or 0.0022 [iff M1M1 (+ 4dp)]
	Alternative: $p = 1 - P(X \leq 1)$ $P(X = 0) + P(X = 1)$ $= 0.973^3 + 3 \times 0.973^2 \times 0.027$ $= 0.921167 + 0.076685$ $p = 1 - 0.99785$ $= 0.0021$	(M1) (M1) (A1)		for either term correct for 1 - [sum of two correct terms] 0.0021 or 0.0022 [iff M1M1 (+ 4dp)]
	Total		13	

MS2B

Q	Solution	Marks	Total	Comments																										
6(a)	H_0 : No association between A level grade and class of degree	B1		At least H_0 correct																										
	H_1 : Association between A level grade and class of degree																													
	<table><tr><td>O_i</td><td>E_i</td></tr><tr><td>20</td><td>11.6</td></tr><tr><td>9</td><td>17.4</td></tr><tr><td>36</td><td>36.4</td></tr><tr><td>55</td><td>54.6</td></tr><tr><td>22</td><td>28</td></tr><tr><td>48</td><td>42</td></tr><tr><td>2</td><td>4</td></tr><tr><td>8</td><td>6</td></tr><tr><td>200</td><td>200</td></tr></table>				O_i	E_i	20	11.6	9	17.4	36	36.4	55	54.6	22	28	48	42	2	4	8	6	200	200						
	O_i				E_i																									
	20				11.6																									
	9				17.4																									
	36				36.4																									
	55				54.6																									
	22				28																									
	48				42																									
2	4																													
8	6																													
200	200																													
Combine Class 2(ii) and 3	M1	For E_i 's attempted																												
<table><tr><td>20</td><td>11.6</td><td>8.4</td><td>6.0827</td></tr><tr><td>9</td><td>17.4</td><td>−8.4</td><td>4.0552</td></tr><tr><td>36</td><td>36.4</td><td>−0.4</td><td>0.0044</td></tr><tr><td>55</td><td>54.6</td><td>0.4</td><td>0.0029</td></tr><tr><td>24</td><td>32</td><td>−8</td><td>2.0</td></tr><tr><td>56</td><td>48</td><td>8</td><td>1.3333</td></tr><tr><td>200</td><td>200</td><td>0</td><td>13.47</td></tr></table>	20	11.6	8.4	6.0827	9	17.4	−8.4	4.0552	36	36.4	−0.4	0.0044	55	54.6	0.4	0.0029	24	32	−8	2.0	56	48	8	1.3333	200	200	0	13.47	M1	For combining attempted
20	11.6	8.4	6.0827																											
9	17.4	−8.4	4.0552																											
36	36.4	−0.4	0.0044																											
55	54.6	0.4	0.0029																											
24	32	−8	2.0																											
56	48	8	1.3333																											
200	200	0	13.47																											
$\nu = 2$	M1	For final column attempted																												
$\chi^2_{1\%}(2) = 9.210$	A1	(awrt 13.5)																												
Reject H_0	B1	[$\nu = 3$ with $\chi^2 = 11.345$ (B0B1ft)]																												
Fiona's belief justified	B1																													
	A1	Dep on B1 M1M1M1 B1B1, not A1																												
(b)	Fewer than expected gained a Class 1 degree having gained grade B in A-level Mathematics.	E1	9	Dep on B1 M1M1M1 B1B1, not A1																										
	More than expected gained a Class 2(ii) degree having gained grade B in A-level Mathematics.	E1	2	correct comments (see below)																										
	<table><tr><td>1</td><td>2(i)</td><td>2(ii)</td><td>3</td><td>comb</td></tr><tr><td>9</td><td>55</td><td>48</td><td>8</td><td>56</td></tr><tr><td>17.6</td><td>54.6</td><td>42</td><td>6</td><td>48</td></tr><tr><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr></table>	1	2(i)	2(ii)	3	comb	9	55	48	8	56	17.6	54.6	42	6	48	A	B	C	D	E			A: fewer than expected B: as expected C: more than expected D: more or similar than expected E: more than expected						
	1	2(i)	2(ii)	3	comb																									
	9	55	48	8	56																									
17.6	54.6	42	6	48																										
A	B	C	D	E																										
Total			11																											

MS2B

Q	Solution	Marks	Total	Comments
7(a)		B2,1	2	Straight line from (1, 0.5) to (3, $\frac{1}{6}$). Horizontal straight line from (3, $\frac{1}{6}$) to (5, $\frac{1}{6}$).
(b)	$E(X) = \frac{1}{6} \int_1^3 x(4-x) dx + \frac{1}{6} \int_3^5 x dx$ $= \frac{1}{6} \left[2x^2 - \frac{x^3}{3} \right]_1^3 + \frac{1}{6} \left[\frac{x^2}{2} \right]_3^5$ $= \frac{1}{6} \left[(18-9) - \left(2 - \frac{1}{3} \right) \right] + \frac{1}{6} \left[\frac{25}{2} - \frac{9}{2} \right]$ $= \frac{1}{6} \left[7\frac{1}{3} + 8 \right]$ $= 2\frac{5}{9}$	M1 A1 m1 A1	4	ignore limits (both parts attempted) ignore limits (both correct) use of correct limits. dep on M1A1 (AG)
(c)(i)	$P(X > 2.5) = \frac{1}{3} + \frac{1}{2} \times \left(0.25 + \frac{1}{6} \right) \times \frac{1}{2}$ $= \frac{7}{16}$	M1 A1	2	Or $1 - \int_1^{2.5} \frac{1}{6}(4-x) dx = 1 - \left[\frac{1}{6} \left(4x - \frac{x^2}{2} \right) \right]_1^{2.5}$ cao (0.4375)
(ii)	$P(1.5 < X < 4.5) = \frac{1}{2} \times \left(\frac{5}{12} + \frac{1}{6} \right) \times 1.5 \left\{ \right.$ $\left. + (4.5 - 3) \times \frac{1}{6} \right\}$ $= \frac{7}{16} + \frac{1}{4}$ $= \frac{11}{16}$	M1 A1 A1	3	Or $\int_{1.5}^3 \frac{1}{6}(4-x) dx + \int_3^{4.5} \frac{1}{6} dx$ cao ($= \frac{11}{16}$ or 0.6875)
(iii)	$P(X > 2.5 \text{ and } 1.5 < X < 4.5)$ $= P(2.5 < X < 4.5)$ $= \frac{1}{2} \times \left(0.25 + \frac{1}{6} \right) \times 0.5 + \frac{1}{4}$ $= \frac{5}{48} + \frac{1}{4}$ $= \frac{17}{48}$	M1 A1	2	$\int_{2.5}^3 \frac{1}{6}(4-x) dx = \left[\frac{1}{6} \left(4x - \frac{x^2}{2} \right) \right]_{2.5}^3 = \frac{5}{48}$ cao (0.35416)
(iv)	$P(X > 2.5 1.5 < X < 4.5) = \frac{17/48}{11/16}$ $= \frac{17}{33}$	M1 A1	2	their $\frac{(iii)}{(ii)}$ iff $0 < p's < 1$ cao (allow 0.51)

MS2B

Q	Solution	Marks	Total	Comments
7(c)	Alternative Solution			
	$F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{12}(x-1)(7-x) & 1 \leq x < 3 \\ \frac{1}{6}(x+1) & 3 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$			
(i)	$P(X > 2.5) = 1 - F(2.5)$ $= 1 - \frac{1}{12}(2.5-1)(7-2.5)$ $= 1 - \frac{1}{12} \times 1.5 \times 4.5$ $= 1 - 0.5625$ $= 0.4375$ or $\frac{7}{16}$	(M1) (A1)		cao
(ii)	$P(1.5 < X < 4.5) = F(4.5) - F(1.5)$ $= \frac{1}{6}(4.5+1) - \frac{1}{12}(1.5-1)(7-1.5)$ $= \frac{11}{12} - \frac{11}{48}$ $= \frac{11}{16}$ or 0.6875	(M1) (A1) (A1)		cao
(iii)	$P(X > 2.5 \text{ and } 1.5 < X < 4.5)$ $= P(2.5 < X < 4.5)$ $= F(4.5) - F(2.5)$ $= \frac{11}{12} - \frac{9}{16}$ $= \frac{17}{48}$	(M1) (A1)		cao
(iv)	$P(X > 2.5 1.5 < X < 4.5)$ $= \frac{F(4.5) - F(2.5)}{F(4.5) - F(1.5)}$ or $\frac{\text{their (iii)}}{\text{their (ii)}}$ $= \frac{17/48}{11/16}$ $= \frac{17}{33}$ or (allow 0.51)	(M1) (A1)		cao
	Total		15	
	TOTAL		75	