



General Certificate of Education (A-level)
June 2011

Mathematics

MS2B

(Specification 6360)

Statistics 2B

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)(i)	$X \sim \text{Po}(13)$	B1	1	Both Poisson and $\lambda = 13$
(ii)	$P(X = 20) = P(X \leq 20) - P(X \leq 19)$ $= 0.975(0) - 0.957(3)$ [allow 0.975 – 0.957] $= 0.0177$ (3sf)	M1 A1	2	Must use $\lambda = 13$ otherwise M0A0 AWFW 0.0176 to 0.018 or $P(X = 20) = \frac{e^{-13} \times 13^{20}}{20!}$ M1 $= 0.0177$ A1
(iii)	$P(6 \leq X \leq 18) = P(X \leq 18) - P(X \leq 5)$ $= 0.930(2)$ – (0.0107 or 0.0259) $= 0.920$ (3sf)	M1 M1 A1	3	AWFW 0.919 to 0.92
(b)	Cars not random Cars not independent Mean and Variance of cars different / not equal Mean / Average / λ / 2.6 greater / less / smaller / different / variable / not constant / too small / too large Any contextual reason that suggests a change in traffic flow, eg due to: rush hour / congestion / traffic jams / accidents / work traffic / school traffic / peak time	B1 B1		Allow (number of) cars not random / not independent B1 for any one of these 3 statements Must indicate a reference to cars Correct comment about value of $\lambda \neq 2.6$ Any combination (one from each group): eg mean greater <i>due to</i> rush hour, or λ smaller <i>due to</i> congestion, or 2.6 too small <i>due to</i> school traffic
(c)	$Y \sim \text{Bin}(20, 0.2)$ $P(Y \geq 5) = 1 - P(Y \leq 4)$ $= 1 - 0.6296$ $= 0.37(0)$ (3sf)	M1 A1	2	or: $1 - \left(0.01153 + 0.05765 + 0.13691 + 0.20536 + 0.21820 \right)$ $1 - 0.6296$ (Allow $1 - 0.8042$ seen for M1)
(d)	X and Y independent $p = 0.0177 \times 0.3704$ $= 0.00656$ (3sf)	B1 M1 A1	3	Any statement which indicates two / both events are independent [their (c)] $\times$ [their (a)(ii)] AWFW 0.0065 and 0.0067
			13	

MS2B (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	Area / $F(x) = 10u \times 0.01\pi$ (OE)	B1	2	Shown by any correct method
	$= 1 \Rightarrow u = \frac{10}{\pi}$	Bdep1		Alternatives: $f = \frac{1}{10u}$ B1
	or $u = \frac{10}{\pi} \Rightarrow F(x) = 1$	(Bdep1)		Show $u = \frac{10}{\pi}$ or show $\frac{1}{10u} = 0.01\pi$ Bdep1
(ii)	$E(X) = \frac{1}{2}(11u + u) = 6u = 6 \times \frac{10}{\pi} = \frac{60}{\pi}$	B1	1	Must be in terms of π (eg $60\pi^{-1}$)
	$\text{Var}(X) = \frac{1}{12}(b-a)^2$ $\text{Var}(X) = \frac{1}{12}(11u - u)^2$ $= \frac{1}{12} \times 100 \times \frac{100}{\pi^2} = \frac{100^2}{12\pi^2}$	B1	1	Alternatives: $\frac{10000}{12\pi^2} = \frac{5000}{6\pi^2} = \frac{2500}{3\pi^2} = \left(\frac{50}{\pi\sqrt{3}}\right)^2 = \frac{(AWRT\ 833)}{\pi^2}$ Must be in terms of π
(iii)	$C = \pi \left(X + \frac{10}{\pi} \right)$			
	$E(C) = \left. \begin{array}{l} \pi \times [\text{their } E(X)] + 10 \\ \pi \times \frac{60}{\pi} + 10 \end{array} \right\}$	M1		Their numerical value of $E(X)$ used correctly Must have a multiplier of π or 2π
	$= 70$	A1		CAO
	$\text{Var}(C) = \pi^2 \times \frac{100^2}{12\pi^2} = \frac{100^2}{12}$ $= 833\frac{1}{3} \text{ (833.}\dot{3}\text{)}$	M1 A1	4	$\pi^2 \times [\text{their } \text{Var}(X) > 0]$ Must have a multiplier of π^2 or $4\pi^2$ Alternatives: $\frac{10000}{12} = \frac{5000}{6} = \frac{2500}{3}$ Must be exact: 833.3 gets A0
(b)	$n = 100$ and $\bar{a} = 40.5$			
	$95\% \text{ CI for } \mu = \left. \begin{array}{l} 40.5 \pm z \times \frac{\sqrt{25}}{\sqrt{100}} \\ 40.5 \pm 1.0 \end{array} \right\}$	B1 M1		For $z = 1.96$ $z = 1.96$ or 1.64 to 1.65 only
	$= (39.5, 41.5)$	A1	3	AWRT
Total			11	

MS2B (cont)

Q	Solution	Marks	Total	Comments
3(a)	H ₀ : no association (between type of school and performance of 16 year olds in their GCSEs)	B1	1	H ₀ : type of school and performance of 16 year olds in their GCSEs independent
(b)	$\frac{(O - E)^2}{E}$ 0.195819311 0.482160711 0.003569447 1.080536181 0.062507172 1.269422099 0.785491128 0.183802623 0.541856652 0.044011976 3.274102564 4.096492891 $X^2 = \sum \frac{(O - E)^2}{E}$ = 12.01977275 = 12.0 (1dp)	M1 m1		Attempt at $\frac{(O - E)^2}{E}$ (≥ 4 correct values seen to 2dp) Attempt to add ≥ 8 terms Allow $11.9 \leq X^2 \leq 12.1 \Rightarrow M1m1$ CAO
(c)	$\nu = 6 \Rightarrow \chi^2_{1\%} = 16.8(12)$ No (significant evidence to suggest an association between (type of) school and (GCSE) performance (of 16 year olds))	B1,B1 Adep1	3 3	$\nu = 6$ can be implied by $\chi^2_{1\%} = 16.8(12)$ Insufficient/no evidence to support Emily's belief. School and performance are independent. Correct conclusion in context Dep on B1M1m1B1B1 given in (a), (b), (c) and $11.9 \leq X^2 \leq 12.1$
(d)	More than expected gained at least / more than 5 GCSEs Fewer than expected gained at least / more than 1 GCSE but less than 5 GCSEs Fewer than expected gained no GCSEs	B1	1	Since conclusion of no association between school and GCSE performance, it may be misleading to look at individual differences in any great detail Any one of these 4 comments seen
(e)	$\chi^2_{10\%} = 10.6(45)$ Reject H ₀ at 10% level of significance. (Evidence to suggest) an association between (type of) school and (GCSE) performance	B1 Bdep1	 2	Correct value of χ^2 only Evidence to support Emily's (Joanne's) belief. (Type of) school + (GCSE) performance dependent. Dep on B1M1m1 and $11.9 \leq X^2 \leq 12.1$ and B1 in (e)
Total			10	

MS2B (cont)

Q	Solution	Marks	Total	Comments																		
4(a)	$E(X) = \sum xp$ $= \frac{3}{40} + \left(2 \times \frac{6}{40}\right) + \left(3 \times \frac{9}{40}\right) + \left(4 \times \frac{12}{40}\right) + \left(5 \times \frac{5}{20}\right) = 3.5$	B2,1	2																			
(b)(i)	$E\left(\frac{1}{X}\right) = \sum \frac{1}{x} \times p$ $= \left(1 \times \frac{3}{40}\right) + \left(\frac{1}{2} \times \frac{6}{40}\right) + \left(\frac{1}{3} \times \frac{9}{40}\right) + \left(\frac{1}{4} \times \frac{12}{40}\right) + \left(\frac{1}{5} \times \frac{5}{20}\right)$ $= \frac{7}{20}$	M1 A1	2	At least 4 of these terms added (accept decimal equivalents) AG (allow 0.35 seen)																		
(ii)	$E\left(\frac{1}{X^2}\right) = \sum \frac{1}{x^2} \times p$ $= \left(1 \times \frac{3}{40}\right) + \left(\frac{1}{4} \times \frac{6}{40}\right) + \left(\frac{1}{9} \times \frac{9}{40}\right) + \left(\frac{1}{16} \times \frac{12}{40}\right) + \left(\frac{1}{25} \times \frac{5}{20}\right)$ $= \frac{133}{800} \quad (0.16625)$ $\text{Var}\left(\frac{1}{X}\right) = \frac{133}{800} - \frac{49}{400}$ $= \frac{7}{160}$	M1 A1 m1 Adep1	4	At least 4 of these terms added (accept decimal equivalents) (can be implied by $\frac{133}{800}$ seen with no other working shown) $\left[\text{their } E\left(\frac{1}{X^2}\right) \right] - \left(\frac{7}{20}\right)^2$ AG (allow 0.04375 seen)																		
(c)(i)	<table border="1"><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>40</td><td>20</td><td>13⅓</td><td>10</td><td>8</td></tr><tr><td>p</td><td>$\frac{3}{40}$</td><td>$\frac{6}{40}$</td><td>$\frac{9}{40}$</td><td>$\frac{12}{40}$</td><td>$\frac{10}{40}$</td></tr></table> Identifying $X = (2), 3, 4, 5$ or $Y = (20), 13\frac{1}{3}, 10, 8$ $P(X > 2) = \frac{9}{40} + \frac{12}{40} + \frac{5}{20}$ $= P(Y < 20)$ $= \frac{31}{40} \quad (0.775)$	x	1	2	3	4	5	y	40	20	13⅓	10	8	p	$\frac{3}{40}$	$\frac{6}{40}$	$\frac{9}{40}$	$\frac{12}{40}$	$\frac{10}{40}$	M1 A1 A1	3	Alternative: $Y < 20 \Rightarrow \frac{40}{X} < 20 \Rightarrow 40 < 20X \Rightarrow X > 2$ M1 (allow < or $\leq$ and > or $\geq$ in above) $P(Y < 20) = P(X > 2)$ $= 1 - \left(\frac{3}{40} + \frac{6}{40}\right) \quad \text{A1}$ $= \frac{31}{40} \quad (0.775) \quad \text{A1}$
x	1	2	3	4	5																	
y	40	20	13⅓	10	8																	
p	$\frac{3}{40}$	$\frac{6}{40}$	$\frac{9}{40}$	$\frac{12}{40}$	$\frac{10}{40}$																	
(ii)	$\frac{9}{40}$ seen irrespective of labelling $P(X < 4 \mid Y < 20) = \frac{\frac{9}{40}}{\frac{31}{40}} = \frac{0.225}{0.775}$ $= \frac{9}{31} \quad (0.290)$	B1 M1 A1	3	As numerator or final answer (0.225) $= \frac{\frac{9}{40}}{\left(\text{their (c)(i)}\right)} \quad (\text{or correct use of table})$ AWFW 0.29 to 0.2904																		
	Total		14																			

Q	Solution	Marks	Total	Comments
5(a)	$Y \sim N(\mu_y, 640^2)$ $n = 25$ and $\bar{y} = 19700$ $H_0: \mu_y = 20000$ $H_1: \mu_y \neq 20000$ (both) $\bar{Y} \sim N\left(20000, \frac{640^2}{25}\right)$ $z = \frac{19700 - 20000}{640/\sqrt{25}}$ $= -2.34375$ $z_{\text{crit}} = \pm 2.5758$ Accept H_0 Insufficient / no evidence (to suggest) that the mean (lifetime) has changed (from 20000 hours) Mean (lifetime) has not changed at 1% level (of significance)	 B1 M1 A1 B1 Adep1 Edep1	 6	 Alternative: $P(\bar{Y} < 19700) = P(Z < -2.34375)$ $= 1 - 0.99036$ $= 0.00964 \geq 0.005$ Accept H_0 (−2.35 to −2.34) (±2.57 to ±2.58) Use of $t \Rightarrow$ max B1M1A1 dep on B1M1B1 dep on Adep1 If incorrect hypotheses then B0 $\Rightarrow$ max M1A1B1 ie final Adep1Edep1 not available
(b)(i)	$\mu < 10000$	B1	1	
(ii)	$n = 16$ and $s = 500$; $t_{\text{crit}} = 1.753$ $\text{sd}(\bar{X}) = \frac{500}{\sqrt{16}}$ (125) Critical value is one of: $10000 \pm 1.753 \times \frac{500}{\sqrt{16}}$ (considered) Choose 9780 (3sf) ($\Rightarrow$ critical region: $\bar{x} < 9780$) $\therefore$ Range of values for $\bar{x}$ which leads Christine not to reject $H_0: \mu = 10000$ is: $\bar{x} > 9780$	 B1 B1 M1 A1 A1	 1 5	 For t_{crit} (ignore signs) Ignore notation M0 if only considered upper value No ft on incorrect t value AWFW 9780 to 9781 (ignore inequality) If z used then max B0B1M0A0A0 Allow $\bar{x} \geq 9780$ to 9781
(iii)	No error	B1	1	Ignore any subsequent statements
	Total		13	

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	$F(x) = \int \frac{3}{8}(x^2 + 1) dx$	M1		Ignore limits
	$= \frac{3}{8} \left[\frac{x^3}{3} + x \right] \text{ or } = \frac{1}{8}x^3 + \frac{3}{8}x$	A1		Either
	$= \frac{1}{8}x(x^2 + 3)$	A1	3	(including use of correct limits 0 and x or +c used and evaluated) (AG)
(b)	$F(m) = \frac{1}{2}$	B1		
	$F(1) = \frac{1}{8} \times 1 \times 4 = \frac{1}{2}$	B1	2	AG
(c)	Upper quartile lies in range $1 < x < 2$ such that $F(q) = \frac{3}{4}$			$\frac{1}{2} + \int_1^q \frac{1}{4}(5 - 2x) dx = \frac{3}{4}$
	$\int_1^q \frac{1}{4}(5 - 2x) dx = \frac{1}{4}$	M1		Alternative: $\int_q^2 \frac{1}{4}(5 - 2x) dx = \frac{1}{4}$
	$[5x - x^2]_1^q = 1$			$[5x - x^2]_q^2 = 1$
	$5q - q^2 - 4 = 1$			$(10 - 4) - (5q - q^2) = 1$
	$q^2 - 5q + 5 = 0$	A1		$6 - 5q + q^2 = 1$ $q^2 - 5q + 5 = 0$
	$q = \frac{5 \pm \sqrt{25 - 20}}{2} \text{ or } \frac{1}{2}(5 \pm \sqrt{5})$	M1		Correct use of formula (OE) to give the two surd answers to given quadratic equation
	but $1 < q < 2$ [or $(q < 2)$]	m1		
	$\therefore q = \frac{1}{2}(5 - \sqrt{5})$	A1	5	Must qualify with a numerical comparison, not just quote the given answer; dep on M1; AG
	$P(X > 1.5) = \frac{1}{2} \left[\frac{1}{2} + \frac{1}{4} \right] \times \frac{1}{2}$	M1		$P(X < 1.5) = 0.5 + \frac{1}{2} \left[\frac{3}{4} + \frac{1}{2} \right] \times \frac{1}{2} \text{ (M1)}$
	$= \frac{3}{16} \text{ (0.1875)}$	A1		$= \frac{1}{2} + \frac{1}{2} \times \frac{5}{4} \times \frac{1}{2}$ $= \frac{1}{2} + \frac{5}{16} = \frac{13}{16} \text{ (A1)}$
(d)	$P(X > q) = \frac{1}{4} \text{ (0.25)}$	B1		$P(X < q) = \frac{3}{4} \text{ (0.75)} \text{ (B1)}$
	$P(q < X < 1.5) = \frac{1}{4} - \frac{3}{16}$			$P(q < X < 1.5) = \frac{13}{16} - \frac{3}{4} = \frac{1}{16} \text{ (A1)}$
	$= \frac{1}{16} \text{ (0.0625)}$	A1	4	(0.0625)

MS2B (cont)

Q	Solution	Marks	Total	Comments
6(d) cont	OR $\int_{1\frac{1}{2}}^2 \frac{1}{4}(5-2x) \, dx = \frac{3}{16} \text{ etc (M1A1)}$ NB statement $F(1.5) - \frac{3}{4} = \frac{1}{16}$ (OE) scores 4 marks Alternative: $\int_q^{1.5} \frac{1}{4}(5-2x) \, dx = \left[-\frac{1}{16}(5-2x)^2 \right]_{\frac{5-\sqrt{5}}{2}}^{1.5} \quad (\text{M1})$ $= -\frac{1}{16}(4) - \left[-\frac{1}{16}(\sqrt{5})^2 \right] \quad (\text{sub}) \quad (\text{A1})$ $= -\frac{4}{16} + \frac{5}{16} \quad (\text{A1})$ $= \frac{1}{16} \quad (\text{A1})$			OR $\int_q^{1.5} \frac{1}{4}(5-2x) \, dx = \frac{1}{4} \left[5x - x^2 \right]_q^{1.5} \quad (\text{M1})$ (correct integration and limits) Allow use of $q = 1.38$ to $q = 1.382$ in limits for M1 Whatever follows must be exact $= \frac{1}{4} \left[(7.5 - 2.25) - (5q - q^2) \right] \quad (\text{A1})$ for use of $5q - q^2 = 5$ or showing $5q - q^2 = 5$ by substituting $q = \frac{1}{2}(5 - \sqrt{5})$ (A1) $= \frac{1}{4} [5.25 - 5] = \frac{1}{16} \quad (\text{A1})$ Alternative using F(x): for $1 \leq x \leq 2$ $F(x) = \frac{1}{2} + \int_1^x \frac{1}{4}(5-2x) \, dx$ $= \frac{1}{2} + \frac{1}{4} \left[5x - x^2 \right]_1^x$ $= \frac{1}{2} + \frac{1}{4} \left[(5x - x^2) - (5 - 1) \right]$ $= \frac{1}{4} (2 + 5x - x^2 - 4)$ $= \frac{1}{4} (5x - x^2 - 2) \quad (\text{seen or used}) \quad (\text{M1})$ $F(1.5) = \frac{1}{4} (7.5 - 2.25 - 2) = \frac{3.25}{4}$ $= 0.8125 = \frac{13}{16} \quad (\text{A1})$ $F(q) = \frac{1}{16} (50 - 10\sqrt{5} - (25 - 10\sqrt{5} + 5) - 8)$ $= \frac{12}{16} \quad \text{OE} \quad (\text{B1})$ $P(q < X < 1.5) = \frac{13}{16} - \frac{12}{16} = \frac{1}{16} \quad (\text{A1})$
	Total		14	
	TOTAL		75	