



**General Certificate of Education (A-level)**  
**January 2012**

**Mathematics**

**MS2B**

**(Specification 6360)**

**Statistics 2B**

**Final**

***Mark Scheme***

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## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## MS2B

Question	Solution	Marks	Total	Comments
1(a)	21.05 and 21.15	B1	1	both (allow 21.049 and 21.149)
(b)	$E(X) = 0$ (symmetry)	B1		For $R[-a, a]: E(X) = 0$ iff $a = 0.05, 0.1, 0.5$ then:
	$\text{Var}(X) = \frac{1}{12}(0.05 - -0.05)^2 = \frac{1}{12} \times \frac{1}{100}$ $\Rightarrow sd(X) = \sqrt{\frac{1}{12} \times \frac{1}{100}} = \frac{1}{20\sqrt{3}}$	M1		$\text{Var}(X) = \frac{1}{12}(a - -a)^2 \quad \text{or}$ their $a = 0.049$ to $0.05$ used for <b>M1</b>
(c)	$P(-0.01 \leq X \leq 0.03) = 0.04 \times 10$ $= 0.4$	A1	3	or $\frac{\sqrt{3}}{60}$ or $\sqrt{\frac{1}{1200}}$ 0.0289 (3sf) A0
		B1	1	cao from correct value used $\int_{-0.01}^{0.03} 10dx = [10x]_{-0.01}^{0.03} = 0.4$ oe
	<b>Total</b>		<b>5</b>	

## MS2B (cont)

Question	Solution	Marks	Total	Comments
2(a)(i)	$H_0: \mu = 61.4$	B1		(both)
	$H_1: \mu \neq 61.4$			
	$z_{calc} = \frac{65.0 - 61.4}{7.5 / \sqrt{16}}$	M1		<b>Alternative:</b>
	$= 1.92$	A1		$P(\bar{X} > 65.0) = P(Z > 1.92)$
	$z_{crit} = \pm 1.96$			$= 1 - 0.97257$
	<b>or</b> (shown in / implied by diagram)	B1		$= 0.02743$
				$\geq 0.025 \quad \therefore \text{Accept } H_0$
	Accept $H_0$	Adep1		Use of $t \Rightarrow \max(B1M1A1)$
	Insufficient / No evidence (at 5% level) to suggest / show <b>mean</b> (age has) <b>changed</b> (from 61.4 years.)			dep(B1M1) but not A1B1
	<b>Mean</b> (age) has <b>not changed at 1% level</b> (of significance)	Edep1	6	If incorrect <b>or</b> no hypothesis then <b>B0</b> $\Rightarrow \max(M1A1B1)$ i.e. final Adep1Edep1 <b>not</b> available
(ii)	$61.4 - 3 \times 7.5 = 38.9 > 25$ $\Rightarrow$ none under the age of 25 years. Very unlikely any members < 25 yrs.	B1	1	dep(Adep1) $z = \frac{25 - 61.4}{7.5} = -4.85$ $\Rightarrow P(Z < -4.85) \approx 0$ $\Rightarrow$ none aged under 25 included
(b)(i)	$\bar{y} = \frac{\sum y}{n} = \frac{702}{12} = 58.5$	B1		( $s = 2.83$ )
	$s^2 = \frac{\sum (y - \bar{y})^2}{n - 1} = \frac{88.25}{11} = 8.02$	B1		$\left( \sigma^2 = 7.35 \text{ or } \sigma = 2.71 \right)$
	$t_{crit} = \pm 1.796$	B1		iff $\sigma / \sqrt{11}$ used below
	90% CI for $\mu$ :			Ignore signs for $t_{crit}$
	$58.5 \pm 1.796 \times \frac{s}{\sqrt{12}}$			If $z$ used then $\max(B1B1B0M0A0)$
	$58.5 \pm 1.4685$	M1		$(\text{their } \bar{y}) \pm t_{11} \times \frac{(\text{their } s)}{\sqrt{12}} \quad \text{OR}$
	$= 57.03, 59.97$			$(\text{their } \bar{y}) \pm t_{11} \times \frac{(\text{their } \sigma)}{\sqrt{11}}$
	$= (57.0, 60.0)$	A1	5	
	(ii) upper limit < 61.4			
	$\Rightarrow$ recruitment drive lowered the average age of the club membership	B1ft	1	Must refer to 61.4 (on their CI)
<b>Total</b>			<b>13</b>	

## MS2B (cont)

Question	Solution	Marks	Total	Comments									
3(a)(i)	$E_i: \frac{mp}{N} ; \frac{mq}{N} ; \frac{np}{N} ; \frac{nq}{N}$	B2,1	2	B1 any one correct B2 all correct (simplified)									
(ii)	$\left. \begin{aligned} \sum_i E_i &= \frac{mp + mq + np + nq}{N} \\ &= \frac{m(p + q)}{N} + \frac{n(p + q)}{N} \text{ (oe)} \end{aligned} \right\}$ $= \frac{mN}{N} + \frac{nN}{N}$ $= m + n$ $= N$ (since $p + q = m + n = N$ )	M1  Mdep1  Adep1	3	$\sum_i E_i = \frac{mp + mq + np + nq}{N}$ $= \frac{m(p + q) + n(p + q)}{N}$ (or use of unsimplified forms) $= \frac{(p + q)(m + n)}{N} = \frac{N \times N}{N} = N$ (AG)									
(b)	$H_0$ : No association between Andy's results and wind conditions  $E_i$ : <table border="1"><tr><td>17.82</td><td>15.18</td><td>33</td></tr><tr><td>9.18</td><td>7.82</td><td>17</td></tr><tr><td>27</td><td>23</td><td>50</td></tr></table> $\Rightarrow  0_i - E_i  - 0.5 = 2.32$ $X^2 = 0.3020 + 0.3546 + 0.5863 + 0.6883 = 1.93$ $\chi^2_{10\%}(1) = 2.706$ $\Rightarrow \text{Accept } H_0$  <b>No association</b> (between Andy's results and wind conditions)	17.82	15.18	33	9.18	7.82	17	27	23	50	B1  M1  M1 A1  B1  Adep1  Edep1	8	Attempt E's   Yates' correction attempted  Final column attempted awrt  correct value of $\chi^2$ only (allow 2.71)  dep (B1 for $H_0$ )  Appropriate conclusion dep(B1 for $H_0$ ; M1final column; $\chi^2_{10\%} = 2.706$ )
17.82	15.18	33											
9.18	7.82	17											
27	23	50											
Total			13										
(a)(ii)	An example of unsimplified values derived from $a = \frac{mp}{N}$ : $\Rightarrow b = m - \frac{mp}{N} ; c = p - \frac{mp}{N} ;$ $d = n - \frac{mp}{n} \text{ (oe)}$												

## MS2B (cont)

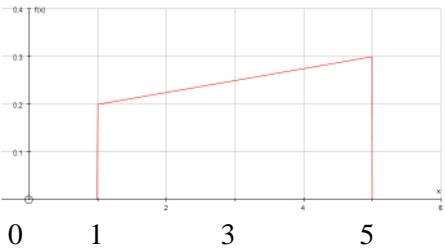
Question	Solution	Marks	Total	Comments
4(a)(i)	Poisson	B1	1	
(ii)	$E(3X - 1) = 3\lambda - 1$ $\text{Var}(3X - 1) = 9\lambda$	B1 B1	2	oe (allow $3^2\lambda$ )
(iii)	$P(X = x+1) = \frac{e^{-\lambda} \times \lambda^{x+1}}{(x+1)!}$  $P(X = x+1) = \frac{e^{-\lambda} \times \lambda^{x+1}}{(x+1)!}$  $\left. \begin{aligned} &= \frac{e^{-\lambda} \times \lambda^x \times \lambda}{(x+1)x!} \\ &= \frac{\lambda}{x+1} \times \frac{e^{-\lambda} \times \lambda^x}{x!} \end{aligned} \right\}$ $= \frac{\lambda}{x+1} P(X = x)$	B1   Mdep1  Adep1	3	dep(B1)  AG
(b)(i)	$\lambda_{\text{car}} = 500/\text{hour}$ $\lambda_{\text{coach}} = 10/\text{hour}$  $\Rightarrow \lambda_{\text{vehicle}} = 510/\text{hour} = 8.5/\text{min}$  $P(V \geq 10) = 1 - 0.6530$  $= 0.347$	B1 M1 A1	3	for 8.5 stated / used special case: $\lambda = 10 \Rightarrow$ B1M0A0 B1 $\Rightarrow 1 - 0.458$ or 0.542
(ii)	$\mu_{\text{car}} = 836/\text{hour}$ $\mu_{\text{coach}} = 22/\text{hour}$  $\Rightarrow \mu_{\text{vehicle}} = 858/\text{hour} = 14.3/\text{min}$  $P(V \leq 3) = P(V = 0, 1, 2, 3)$  $= \begin{cases} e^{-14.3} \left[ 1 + \frac{14.3}{1} + \frac{14.3^2}{2} + \frac{14.3^3}{6} \right] \\ e^{-14.3} \times 604.91283 \\ 0.0003726 \text{ to } 0.000373 \end{cases}$  $= 0.00037 \text{ (2sf)}$	B1  M1  Adep1	3	for 14.3 stated /used  All 4 terms required for <b>any</b> $\lambda > 0$ M0 for use of normal approximation  dep M1
	<b>Total</b>		<b>12</b>	

## MS2B (cont)

Question	Solution	Marks	Total	Comments																		
5(a)	<table><tr><th><math>n</math></th><th>Outcome</th><th><math>P(N = n)</math></th></tr><tr><td>1</td><td>H</td><td><b>0.5</b> (<math>\frac{1}{2}</math>)</td></tr><tr><td>2</td><td>TH</td><td><b>0.25</b> (<math>\frac{1}{4}</math>)</td></tr><tr><td>3</td><td>TTH</td><td>0.125 (<math>\frac{1}{8}</math>)</td></tr><tr><td>4</td><td>TTTH</td><td><b>0.0625</b> (<math>\frac{1}{16}</math>)</td></tr><tr><td>5</td><td>TTTTA</td><td>0.0625 (<math>\frac{1}{16}</math>)</td></tr></table> $E(N) = \left(1 \times \frac{1}{2}\right) + \left(2 \times \frac{1}{4}\right) + \left(3 \times \frac{1}{8}\right) + \left(4 \times \frac{1}{16}\right) + \left(5 \times \frac{1}{16}\right)$ $= \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{16} = \frac{31}{16}$ $= 1\frac{15}{16} \text{ (1.9375)}$	$n$	Outcome	$P(N = n)$	1	H	<b>0.5</b> ( $\frac{1}{2}$ )	2	TH	<b>0.25</b> ( $\frac{1}{4}$ )	3	TTH	0.125 ( $\frac{1}{8}$ )	4	TTTH	<b>0.0625</b> ( $\frac{1}{16}$ )	5	TTTTA	0.0625 ( $\frac{1}{16}$ )	B2,1		B1 for <b>one</b> correct entry for $n = 1, 2, 4$ B2 for all 3 correct  Can be implied by correct $E(N)$
$n$	Outcome	$P(N = n)$																				
1	H	<b>0.5</b> ( $\frac{1}{2}$ )																				
2	TH	<b>0.25</b> ( $\frac{1}{4}$ )																				
3	TTH	0.125 ( $\frac{1}{8}$ )																				
4	TTTH	<b>0.0625</b> ( $\frac{1}{16}$ )																				
5	TTTTA	0.0625 ( $\frac{1}{16}$ )																				
(b)	<table><tr><th>m</th><th>Outcome</th><th><math>P(M = m)</math></th></tr><tr><td>1</td><td>H</td><td><math>\frac{1}{4}</math></td></tr><tr><td>2</td><td>TH</td><td><math>\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}</math></td></tr><tr><td>3</td><td>TTH</td><td><math>\left(\frac{3}{4}\right)^2 \times \frac{1}{4} = \frac{9}{64}</math></td></tr><tr><td>4</td><td>TTTH</td><td><math>\left(\frac{3}{4}\right)^3 \times \frac{1}{4} = \frac{27}{256}</math></td></tr><tr><td>5</td><td>TTTTA</td><td><math>\left(\frac{3}{4}\right)^4 \times 1 = \frac{81}{256}</math></td></tr></table>	m	Outcome	$P(M = m)$	1	H	$\frac{1}{4}$	2	TH	$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$	3	TTH	$\left(\frac{3}{4}\right)^2 \times \frac{1}{4} = \frac{9}{64}$	4	TTTH	$\left(\frac{3}{4}\right)^3 \times \frac{1}{4} = \frac{27}{256}$	5	TTTTA	$\left(\frac{3}{4}\right)^4 \times 1 = \frac{81}{256}$	A1	4	(awfw 1.93 to 1.94)
m	Outcome	$P(M = m)$																				
1	H	$\frac{1}{4}$																				
2	TH	$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$																				
3	TTH	$\left(\frac{3}{4}\right)^2 \times \frac{1}{4} = \frac{9}{64}$																				
4	TTTH	$\left(\frac{3}{4}\right)^3 \times \frac{1}{4} = \frac{27}{256}$																				
5	TTTTA	$\left(\frac{3}{4}\right)^4 \times 1 = \frac{81}{256}$																				
(c)(i)	$P(J, R)$ : $P(1,1) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ (oe) $P(2,2) = \frac{1}{4} \times \frac{3}{16} = \frac{3}{64}$ (oe) $P(3,3) = \frac{1}{8} \times \frac{9}{64} = \frac{9}{512}$ (oe) $P(4,4) = \frac{1}{16} \times \frac{27}{256} = \frac{27}{4096}$ (oe) $P(5,5) = \frac{1}{16} \times \frac{81}{256} = \frac{81}{4096}$ (oe)  $p = \sum_{n=1}^{n=5} P(n,n)$ $\Rightarrow p = \frac{221}{1024} \text{ (0.2158)}$	M1		e.g 0.125 attempt at any $P(n,n)$																		
(ii)	$= 3 \times \left(\frac{221}{1024}\right)^2 \times \left(\frac{803}{1024}\right)$ $+ \left(\frac{221}{1024}\right)^3$ $P(X \geq 2) = P(X = 2) + P(X = 3)$ $= 0.120 \text{ (3dp)}$	A1	5	(B1 any one correct) (B2 any 2 correct) (B3 all 3 correct)																		
		B3,2,1	3																			
		M1																				
		A1																				
		A1																				
		m1		$\sum_{n=1}^{n=5} P(n,n)$ with all 5 values attempted																		
		A1		(awfw 0.215 to 0.217)																		
		M1		(either term with <b>their p</b> used) ( $0 < p < 1$ )																		
		M1		(second term with <b>their p</b> used) ( $0 < p < 1$ )																		
		Mdep1		dep (M1M1)																		
		A1	4	(allow 0.119; 0.12; 0.121)																		
	<b>Total</b>		<b>16</b>																			



## MS2B (cont)

Question	Solution	Marks	Total	Comments
6(a)		B2,1	2	<p>B2 for st. line from (1,0.2) to (5,0.3)</p> <p>B1 st. line (<math>m &gt; 0</math>) from <math>x = 1</math> to <math>x = 5</math>.</p>
(b)	$E(X) = \frac{1}{40} \int_1^5 x(x+7) dx$ $= \frac{1}{40} \left( \frac{x^3}{3} + \frac{7x^2}{2} \right)_1^5$ $= \frac{1}{40} \left( \frac{125}{3} + \frac{175}{2} - \frac{1}{3} - \frac{7}{2} \right)$ $= 3\frac{2}{15}$	M1  A1	3	<p>Ignore limits</p> <p>Ignore limits</p> <p>cao (accept 3.133 or <math>\frac{47}{15}</math> oe <i>exact</i>)</p>
(c)	$F(x) = \int_1^x \frac{1}{40}(x+7) dx$ $= \frac{1}{40} \left[ \frac{x^2}{2} + 7x \right]_1^x$ $= \frac{1}{80} (x^2 + 14x - 1 - 14)$ $= \frac{1}{80} (x^2 + 14x - 15)$ $= \frac{1}{80} (x+15)(x-1)$	M1  A1  Adep1  Adep1	4	$F(x) = \int \left( \frac{x}{40} + \frac{7}{40} \right) dx$ $= \frac{x^2}{80} + \frac{7x}{40} + c \Rightarrow (M1A1)$ $F(1) = 0 \Rightarrow c = -\frac{1}{80} - \frac{7}{40} = -\frac{15}{80}$ <p>or [use of <math>F(5) = 1</math>]</p> $\Rightarrow F(x) = \frac{1}{80} (x^2 + 14x - 15)$ $F(x) = \frac{1}{80} (x+15)(x-1) \text{ (AG)}$
(d)(i)	$P(2.5 \leq X \leq 4.5) = F(4.5) - F(2.5)$ $= \frac{1}{80} (19.5 \times 3.5 - 17.5 \times 1.5)$ $= \frac{42}{80} = \frac{21}{40} \text{ (0.525)}$	M1  A1	2	<p>Trapezium Rule</p> $\frac{1}{2} \left( \frac{23}{80} + \frac{19}{80} \right) \times 2$ $= \frac{42}{80} = \frac{21}{40}$
(ii)	$F(m) = \frac{1}{2}$ $\Rightarrow \frac{1}{80} (m^2 + 14m - 15) = \frac{1}{2}$ $(\times 80) \Rightarrow m^2 + 14m - 15 = 40$ $m^2 + 14m - 55 = 0$	B1  M1  Adep1	3	$\int_1^m \frac{1}{40} (x+7) dx = 0.5 \text{ (B1)}$ <p>Correct equation formed</p> <p>AG</p>
(e)	$m = \frac{-14 \pm \sqrt{196 + 220}}{2} = \frac{-14 \pm 20.396}{2}$ $m = \frac{-14 + 20.396}{2} \text{ (since } m > 1)$ $m = 3.198 \text{ (3dp)}$	M1  A1	2	<p>Correct attempt at solving quadratic (by formula, oe).</p> <p>cao</p>
	<b>Total</b>		<b>16</b>	
	<b>TOTAL</b>		<b>75</b>	