



General Certificate of Education (A-level)
June 2012

Mathematics

MS04

(Specification 6360)

Statistics 4

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS04

Q	Solution	Marks	Total	Comments
1	$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$ $\bar{x}_1 = 6.362 \quad s_1 = 0.3062 \quad (s_1^2 = 0.09377)$ $\bar{x}_2 = 6.548 \quad s_2 = 0.2926 \quad (s_2^2 = 0.08562)$ $s^2 = \frac{8 \times 0.3062^2 + 7 \times 0.2926^2}{9 + 8 - 2} = 0.08996$ $t_{calc} = \frac{0.1862 - 0}{0.2999 \sqrt{\frac{1}{9} + \frac{1}{8}}} = 1.28$ $\nu = 15 \quad t_{crit} = \pm 1.753$ $1.28 < 1.753 \Rightarrow \text{accept } H_0.$ Insufficient evidence to indicate that means are different.	B1 B1 B1 M1A1 M1A1 B1B1 A1✓	 10	Both Both (or 0.2887) Both (or 0.2737) Or: $\frac{9 \times 0.2887^2 + 8 \times 0.2737^2}{9 + 8 - 2}$ awrt Both signs not required. (Compares) – may be implied; and states conclusion in context. ✓ t (requires final M1 only).
	Total		10	
2(a)	$s = 0.0157762 \dots \text{or } s^2 = 0.0002488\dots$ $\chi_9^2(0.01) = 2.088 \quad \chi_9^2(0.99) = 21.666$ $\sqrt{\frac{9s^2}{21.666}} \quad \text{and} \quad \sqrt{\frac{9s^2}{2.088}}$ (0.0102, 0.0328)	B1 B1 M1 A1A1 A1	 6	$\sum (x - \bar{x})^2 = 0.00224$ awrt Both Or variances accept $\geq 1\text{sf}$ e.g. (0.01, 0.03)
(b)	$H_0: \sigma_A^2 = \sigma_B^2 \quad H_1: \sigma_A^2 \neq \sigma_B^2$ $s_A^2 = 0.0008571\dots \quad s_B^2 = 0.0002488\dots$ $F_{calc} = \frac{0.0008571\dots}{0.0002488\dots} = 3.44$ $\nu_A = 9 \quad \nu_B = 7 \quad , F_{7,9}(0.10) = 3.293$ $3.44 > 3.293 \Rightarrow \text{Reject } H_0$ Sufficient evidence to conclude that variances differ.	B1 B1 M1A1 B1B1 A1✓	 7	Both Both awrt (Compare) and state conclusion. ✓ on F and CV.
	Total		13	

MS04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\bar{x} = \frac{125}{100} = 1.25$	B1		cao
	$5p = 1.25 \Rightarrow p = 0.25$	B1	2	cao
	(b) Distribution B(5,0.25): 0.2373 0.3955 0.2637 0.0879 0.0146 0.0010 (May be implied by E_i s.)	M1A1 A1		A1 for 3 correct. 2 nd A1 for all correct.(≥3dp)
	Expected frequencies are: 23.73 39.55 26.37 8.79 1.46 0.1	A1✓	4	Probabilities × 100.
	(c) H_0 : B(5, p) is an appropriate model.	B1		Condone $p = 0.25$.
	O 25 41 20 14 E 23.73 39.55 26.37 10.35	M1		Combines last three classes.
	$\chi^2_{calc} = \sum \left\{ \frac{(O-E)^2}{E} \right\} = 2.947$	M1A1		awfw 2.94 to 2.95
	$\nu = 4 - 2 = 2$ $\chi^2_{crit} = 5.99$	B1B1		
	$2.947 < 5.99 \Rightarrow$ Accept H_0 B(5, p) is a suitable model.	A1✓	7	(Compare) and state conclusion in context. ✓ on χ^2
	(d) It gives some support. If, for example, the probabilities were different for a seed in the front row, say, then this would not be discernible from figures for 100 rows.	B1✓ B1dep	2	Ft on their conclusion
	Total		15	

MS04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Unbiased estimator with the smaller Variance would, more often than not, yield an estimate closer to the parameter.	E2	2	SC ‘implies more efficient’ E1
(b)(i)	$E(\bar{X}_1 - \bar{X}_2) = E(\bar{X}_1) - E(\bar{X}_2) = \mu_1 - \mu_2$ $\text{Var}(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$	M1A1 B1	3	
(ii)	$V = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n - n_1}$ $\frac{dV}{dn_1} = -\frac{\sigma_1^2}{n_1^2} + \frac{\sigma_2^2}{(n - n_1)^2} = 0$ $\Rightarrow \frac{\sigma_1^2}{\sigma_2^2} = \frac{n_1^2}{(n - n_1)^2} = \frac{n_1^2}{n_2^2} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{n_1}{n_2}$	M1 m1A1 A1	4	Or in terms of n_2 etc AG
(iii)	$n_1 : n_2 = \sigma_1 : \sigma_2 = 5 : 9 \quad (\Rightarrow 14 \text{ parts})$ $n_1 = 5 \times 20 = 100 \quad n_2 = 9 \times 20 = 180$	M1 A1	2	Both cao
Total			11	

MS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$E(X^2) = \int_0^{\infty} kx^2 e^{-kx} dx$ $= \left[-x^2 e^{-kx} \right]_0^{\infty} + \int_0^{\infty} 2xe^{-kx} dx$ $= 0 + \left[-\frac{2x}{k} e^{-kx} \right]_0^{\infty} + \int_0^{\infty} \frac{2}{k} e^{-kx} dx$ $= 0 + \left[-\frac{2}{k^2} e^{-kx} \right]_0^{\infty}$ $= \frac{2}{k^2}$ $\text{Var}(X) = \frac{2}{k^2} - \left(\frac{1}{k} \right)^2 = \frac{1}{k^2}$	M1 M1 A1 A1 A1 A1✓	6	0 may be omitted, or limits inserted at end of process. (E(X) integral can be quoted.) Ditto. Their E(X ²) minus mean ² , provided positive . SC Allow B1 for those who write correct working and result, having failed to integrate correctly.
(b)(i)	$F(x) = \int_0^x ke^{-ku} du$ $= \left[-e^{-ku} \right]_0^x = 1 - e^{-kx}$	M1 A1A1	3	
(ii)	$\left[1 - e^{-kx} \right]_0^N = 0.9 \Rightarrow e^{-kN} = 0.1$ $\Rightarrow N = \frac{1}{k} \ln 10$	M1 M1A1	3	M1 for taking logs. cao, acf
(c)	$\text{Mean} = a = \frac{1}{k} \Rightarrow k = \frac{1}{a},$ $\text{Mean} = 3a = \frac{1}{k} \Rightarrow k = \frac{1}{3a}$ $e^{-\frac{1}{a}a} \cdot e^{-\frac{1}{3a}a} = e^{-1} \cdot e^{-\frac{1}{3}} = e^{-\frac{4}{3}}$	M1A1 M1A1	4	cwo
Total			16	

MS04 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$E(X) = p + 2qp + 3q^2p + 4q^3p + \dots$	M1	3	$q = 1 - p$
	$= p(1 + 2q + 3q^2 + 4q^3 + \dots)$	M1		
	$= p(1 - q)^{-2}$	A1		
	$= \frac{p}{p^2} = p^{-1}$			
(ii)	$\text{Var}(X) = \frac{2-p}{p^2} - \left(\frac{1}{p}\right)^2$	M1	2	$\left(\text{or } \frac{q}{p^2}\right)$
	$= \frac{2-p-1}{p^2} = \frac{1-p}{p^2}$	A1		
(b)(i)	$P(> 2 \text{ throws required})$ $= 1 - \left(\frac{1}{4} + \frac{3}{4} \times \frac{1}{4}\right) = \frac{9}{16}$	M1A1	2	
(ii)	$E(Y) = E(1 + X) = 1 + E(X) = 1 + \frac{4}{3} = \frac{7}{3}$	B1		(OE)
	$\text{Var}(Y) = \text{Var}(1 + X)$ $= 0 + \text{Var}(X) = \frac{\left(\frac{1}{4}\right)}{\left(\frac{3}{4}\right)^2} = \frac{4}{9}$	M1A1	3	(OE)
	Total		10	
	TOTAL		75	