



General Certificate of Education (A-level)
June 2011

Mathematics

MS04

(Specification 6360)

Statistics 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS04

Q	Solution	Marks	Total	Comments
1	$H_0: \sigma = 0.7 \quad H_1: \sigma \neq 0.7$ $s = 0.3147$ $\nu = 11$ $\chi^2_{\text{crit}} = 2.60, 26.76$ Under H_0 $\frac{11 \times 0.3147^2}{0.49} = 2.22$ $2.22 < 2.60 \Rightarrow \text{Reject } H_0$ Evidence to suggest that $\sigma \neq 0.7$ at 1% level	B1 B1 B1 B1 M1A1 E1ft	 7	Both $\sum (x - \bar{x})^2 = 1.08916$ or $s^2 = 0.0990$ Accept 2.60 only
		Total	7	
2 (a)	$d: 2 \quad 5 \quad 1 \quad 0 \quad 4 \quad -6 \quad 3 \quad 4$ $\bar{d} = 1.625$ $s = 3.503$ $\nu = 7$ $t = 3.499$ Confidence Limits are $1.625 \pm \frac{3.499 \times 3.503}{\sqrt{8}}$ Confidence interval is $(-2.71, 5.96)$	M1 B1 B1 B1 B1 M1A1 ft A1	 8	At most two errors ft on $\bar{d}$, s , and t
(b)	Normally distributed	E1	1	
(c)	$5 \in \text{CI}$ $\Rightarrow$ Not unreasonable that $\mu_D = 5$.	E1ft E1ft	 2	ft on (a) Dependent on previous E1
		Total	11	

MS04 (cont)

Q	Solution	Marks	Total	Comments
3 (a)	$s_X^2 = 15300 \quad s_Y^2 = 4760$	M1A1	2	M1 One A1 both (15334) AWRT (4760) S.C. B1 ≥ 1 S.D.
(b)(i)	$\nu_1 = 9 \quad \nu_2 = 7$ $F_{9,7} = 6.719 \quad F_{7,9} = 5.613$ $F_{\text{calc}} = \frac{15334}{4764} = 3.219$ $\frac{1}{6.719} \leq \frac{\text{VR}}{3.219} \leq 5.613$ $\Rightarrow 0.479 \leq \text{VR} \leq 18.1$	B1 B1B1 M1 M1A1 ft A1	7	Both ft on variances and F values
(ii)	$1 \in \text{CI}$ $\Rightarrow$ No significant evidence of a difference between 'Killrust' and 'Stoprust'	E1ft E1ft	2	ft on (b)(i) Dependent on previous E1
		Total	11	
4(a)(i)	$F(t) : 0.0272 \quad 0.1792 \quad 0.4752 \quad 0.8192 \quad 1$	M1A1		Table function on GDC allowed
(ii)	$p_i : 0.0272 \quad 0.1520 \quad 0.2960 \quad 0.3440 \quad 0.1808$	M1A1	4	Differences $\Rightarrow$ M1
(b)	$H_0 : \text{Claim is correct}$ $O_i : \quad 2 \quad 9 \quad 12 \quad 22 \quad 5$ $E_i : \quad 1.36 \quad 7.60 \quad 14.80 \quad 17.20 \quad 9.04$ Combine first two classes: $\begin{pmatrix} 11 \\ 8.96 \end{pmatrix}$ $\chi_{\text{calc}}^2 = \sum \left\{ \frac{(O - E)^2}{E} \right\} = 4.139$ $\nu = 4 - 1 = 3$ $\chi_{\text{crit}}^2 = 7.815$ $4.139 < 7.815 \Rightarrow$ Accept H_0 at 5% level Evidence to suggest claim is correct	B1 M1A1 M1 M1A1 B1ft B1ft E1ft	9	Accept 4.234 based on $\nu = 5 - 1 = 4$ ft on $\nu = 4$ ft $\chi_{\text{crit}}^2 = 9.488$
		Total	13	

MS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$X \sim \text{Geo}(p)$ $E(X) = \sum xP(X = x)$ $= p + 2qp + 3q^2p + \dots$ $= p(1 + 2q + 3q^2 + \dots)$ $= p(1 - q)^{-2}$ $= \frac{p}{p^2}$ $= \frac{1}{p}$	M1 A1	3	Where $p + q = 1$
		A1		CSO AG
(b)	By part (a) expect 6 throws	B1	4	Where x is the price per throw Accept £1.83
	$6x - 10 = 1$ $\Rightarrow x = £1.84$ for profit	M1A1 A1		
		Total	7	

MS04 (cont)

Q	Solution	Marks	Total	Comments
6 (a)(i)	$\mu = E(X) = \int_0^{\infty} kxe^{-kx} dx$	M1	3	AG
	$= \left[-xe^{-kx} \right]_0^{\infty} + \int_0^{\infty} e^{-kx} dx$	M1		
	$= 0 + \left[\frac{e^{-kx}}{-k} \right]_0^{\infty}$	A1		
	$= \frac{1}{k}$			
	(ii) $\int_0^m ke^{-kx} dx = \frac{1}{2}$	M1	5	Or $F(x) = 1 - e^{-kx}$ $\frac{1}{2} = 1 - e^{-km}$
	$\left[-e^{-kx} \right]_0^m = \frac{1}{2}$			
	$-e^{-km} + 1 = \frac{1}{2}$	A1		
	$e^{-km} = \frac{1}{2} \Rightarrow km = \ln 2 \Rightarrow m = \frac{\ln 2}{k}$	M1A1		
	$\ln 2 < 1 \Rightarrow m < \mu$	A1	1	AG
	(b)(i) $P(T = 0) = e^{-\frac{t}{\lambda}}$	B1		
	(ii)(A) $P(T > t) = 1 - F(t) = e^{-\frac{t}{\lambda}}$	M1		
	$P(T < t) = F(t) = 1 - e^{-\frac{t}{\lambda}} \quad t \geq 0$	A1		
(B)	$f(t) = F'(t) = \frac{1}{\lambda} e^{-\frac{t}{\lambda}} \quad t \geq 0$	M1	2	
	$\Rightarrow$ Exponential distribution	A1		
		Total	13	

MS04 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$E(T) = \theta$	B1	1	
(b)(i)	$E(T_1) = \mu$ $\Rightarrow E(4\bar{X} - a\bar{Y}) = 4E(\bar{X}) - aE(\bar{Y}) = \mu$ $\Rightarrow 4\mu - a\frac{\mu}{3} = \mu$ $\Rightarrow 3 = \frac{a}{3} \Rightarrow a = 9$	M1 A1 A1	3	
(ii)	$\text{Var}(T_1) = 16\text{Var}(\bar{X}) + a^2\text{Var}(\bar{Y})$ $= 16\frac{\sigma^2}{n} + 81b\frac{\sigma^2}{n}$ $= \frac{\sigma^2}{n}(16 + 81b)$	M1 M1 A1		AG
	$\text{Var}(T_2) = \frac{64}{81} \times \frac{\sigma^2}{n} + \frac{9}{81} \times \frac{b\sigma^2}{n}$ $= \frac{\sigma^2}{81n}(64 + 9b)$	M1 A1	5	
(iii)	$\text{RE}(T_2 \text{ wrt } T_1) = \frac{\{\text{Var}(T_2)\}^{-1}}{\{\text{Var}(T_1)\}^{-1}} = \frac{\text{Var}(T_1)}{\text{Var}(T_2)}$ $= 81 \times \frac{(16 + 81b)}{(64 + 9b)}$	M1 A1ft		ft on $\text{Var}(T_2)$
	$81 \times 16 + 81^2 b > 64 + 9b$ $(\because \{9, 16, 64, 81\} \subset \mathbb{Z}^+ \text{ and } b > 0 [\text{given}])$ $\Rightarrow \text{RE}(T_2 \text{ wrt } T_1) > 1$ $\Rightarrow T_2 \text{ more efficient than } T_1.$	E1ft E1ft	4	Dependent on previous E1
		Total	13	
	TOTAL		75	