



**General Certificate of Education**

**Mathematics 6360**

**MS03      Statistics 3**

**Mark Scheme**

*2009 examination - June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: [www.aqa.org.uk](http://www.aqa.org.uk)

Copyright © 2009 AQA and its licensors. All rights reserved.

#### COPYRIGHT

AQA retains the copyright on all its publications. However, registered centres for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to centres to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

## Key to mark scheme and abbreviations used in marking

|         |                                                                    |  |  |
|---------|--------------------------------------------------------------------|--|--|
| M       | mark is for method                                                 |  |  |
| m or dM | mark is dependent on one or more M marks and is for method         |  |  |
| A       | mark is dependent on M or m marks and is for accuracy              |  |  |
| B       | mark is independent of M or m marks and is for method and accuracy |  |  |
| E       | mark is for explanation                                            |  |  |

  

|              |                                               |     |                            |
|--------------|-----------------------------------------------|-----|----------------------------|
| ✓ or ft or F | follow through from previous incorrect result | MC  | mis-copy                   |
| CAO          | correct answer only                           | MR  | mis-read                   |
| CSO          | correct solution only                         | RA  | required accuracy          |
| AWFW         | anything which falls within                   | FW  | further work               |
| AWRT         | anything which rounds to                      | ISW | ignore subsequent work     |
| ACF          | any correct form                              | FIW | from incorrect work        |
| AG           | answer given                                  | BOD | given benefit of doubt     |
| SC           | special case                                  | WR  | work replaced by candidate |
| OE           | or equivalent                                 | FB  | formulae book              |
| A2,1         | 2 or 1 (or 0) accuracy marks                  | NOS | not on scheme              |
| -x EE        | deduct x marks for each error                 | G   | graph                      |
| NMS          | no method shown                               | c   | candidate                  |
| PI           | possibly implied                              | sf  | significant figure(s)      |
| SCA          | substantially correct approach                | dp  | decimal place(s)           |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## MS03

| Q            | Solution                                                                                                              | Marks    | Total    | Comments                                                                                                                 |
|--------------|-----------------------------------------------------------------------------------------------------------------------|----------|----------|--------------------------------------------------------------------------------------------------------------------------|
| 1(a)         | $\hat{p}_1 = \frac{102}{150} = 0.68$ $\hat{p}_2 = \frac{36}{80} = 0.45$                                               | B1       |          | Both CAO                                                                                                                 |
|              | 99% (0.99) $\Rightarrow z = 2.57$ to 2.58                                                                             | B1       |          | AWFW (2.5758)                                                                                                            |
|              | CI for $(p_1 - p_2)$ is                                                                                               |          |          |                                                                                                                          |
|              | $(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ | M1<br>m1 |          | Use of $(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\text{attempted variance}}$<br>Use of correct expression for variance |
|              | Thus<br>$(0.68 - 0.45) \pm 2.5758 \times \sqrt{\frac{0.68 \times 0.32}{150} + \frac{0.45 \times 0.55}{80}}$           | A1F      |          | F on $\hat{p}_1$ , $\hat{p}_2$ and $z$                                                                                   |
| (b)          | Hence $0.23 \pm (0.173 \text{ to } 0.174)$<br>or<br>$(0.056 \text{ to } 0.057, 0.403 \text{ to } 0.404)$              | A1       | 6        | CAO & AFWW (accept 0.17)<br>AWFW (accept 0.06 & 0.4)<br><br><b>Note:</b><br>Pooling of variances<br>Maximum of B1 B1 M1  |
|              | Whole of confidence interval is <b>above 0 or zero</b><br>so                                                          | B1F      |          | F on (a)<br>Or equivalent                                                                                                |
|              | <b>Disagree</b> with claim / claim appears <b>doubtful</b>                                                            | B1F      | 2        | F on (a)<br>Or equivalent<br>Dependent on previous B1F                                                                   |
| <b>Total</b> |                                                                                                                       |          | <b>8</b> |                                                                                                                          |

**MS03 (cont)**

| <b>Q</b>       | <b>Solution</b>                                                                                                                                                                                                                                                                                                                                                                                                     | <b>Marks</b>               | <b>Total</b> | <b>Comments</b>                                                                                                               |
|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|--------------|-------------------------------------------------------------------------------------------------------------------------------|
| <b>2(a)(i)</b> | $P(B \& B) = (0.30 \times 0.80) + (0.55 \times 0.10) + (0.15 \times 0.30)$ $= 0.24 + 0.055 + 0.045 = 0.34$                                                                                                                                                                                                                                                                                                          | M1<br>A1                   | 2            | Use of <b>3 possibilities</b> each the product of <b>2 probabilities</b><br>CAO; <b>AG</b>                                    |
| <b>(ii)</b>    | $P(HB \cap \text{Coastal}) = 0.55 \times 0.65$ $= 143/400 \text{ or } 0.357 \text{ to } 0.358$                                                                                                                                                                                                                                                                                                                      | M1<br>A1                   | 2            | Can be implied by <b>correct</b> answer<br>CAO/AWFW (0.3575)                                                                  |
| <b>(iii)</b>   | $P(\text{Coastal}   HB) = \frac{P(\text{Coastal} \cap HB)}{P(HB)}$ $= \frac{0.3575}{(0.3 \times 0.15) + (0.3575) + (0.15 \times 0.5)}$ $= \frac{0.3575}{0.4775} = 143/191 \text{ or } 0.747 \text{ to } 0.75$                                                                                                                                                                                                       | M1<br>M1<br>A1F<br>A1      | 4            | $\frac{\text{answer to (ii)}}{\sum(3 \times 2) \text{ probabilities}}$<br>F on (ii)<br>CAO/AWFW (0.74869)                     |
| <b>(b)</b>     | $P(\text{City}   HB) = \frac{0.3 \times 0.15}{P(HB)} = \frac{0.045}{0.4775} = \frac{90}{955}$ $P(\text{Country}   HB) = \frac{0.15 \times 0.5}{P(HB)} = \frac{0.075}{0.4775} = \frac{30}{191}$ $\text{Thus Probability} = \frac{0.045}{P(HB)} \times \frac{0.3575}{P(HB)} \times \frac{0.075}{P(HB)}$ $\text{Multiplied by } 3! = 6$ $= 0.09424 \times 0.74869 \times 0.15707 \times 6$ $= 0.063 \text{ to } 0.068$ | M1<br>M1<br>M1<br>B1<br>A1 | 5            | Or $\left(1 - (a)(iii) - \frac{0.045}{0.4775}\right)$<br>Multiplication of 3 different probabilities<br>CAO<br>AWFW (0.06649) |
|                | <b>Total</b>                                                                                                                                                                                                                                                                                                                                                                                                        |                            | <b>13</b>    |                                                                                                                               |

**MS03 (cont)**

| <b>Q</b> | <b>Solution</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | <b>Marks</b>                                                 | <b>Total</b> | <b>Comments</b>                                                                                                                                                                                                                                                    |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>3</b> | <p>98% (0.98) CI <math>\Rightarrow z = 2.32</math> to <math>2.33</math></p> <p>CI width is <math>2 \times z \times \sqrt{\frac{p(1-p)}{n}}</math></p> <p><math>p = 0.35</math> or <math>0.50</math></p> <p>Thus <math>2 \times 2.3263 \times \sqrt{\frac{0.35 \times 0.65}{n}} = 0.1</math></p> <p>Thus <math>\sqrt{n} = \frac{2 \times 2.3263}{0.1} \times \sqrt{0.35 \times 0.65}</math></p> <p>Thus <math>n = 492.5</math> (<math>p = 0.35</math>)<br/>or <math>n = 541.2</math> (<math>p = 0.50</math>)</p> <p>Thus to nearest 10 <math>n = 500</math> or <math>490</math></p> <p><b>Notes:</b><br/>No '<math>\times 2</math>' gives <math>n = 123.1</math><br/>No '<math>\times 2</math>' and <math>p = 0.50</math> gives <math>n = 135.3</math></p> | <p>B1</p> <p>M1</p> <p>B1</p> <p>A1F</p> <p>m1</p> <p>A1</p> | <p>6</p>     | <p>AWFW (2.3263)</p> <p>Used; allow <math>z \times \sqrt{\frac{p(1-p)}{n}}</math></p> <p>Or equivalent<br/>F on <math>z</math>; allow no multiplier of 2 and/or <math>p = 0.50</math></p> <p>Solving for <math>\sqrt{n}</math> or <math>n</math></p> <p>Either</p> |
|          | <b>Total</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                              | <b>6</b>     |                                                                                                                                                                                                                                                                    |

7

**MS03 (cont)**

| <b>Q</b>      | <b>Solution</b>                                                                   | <b>Marks</b> | <b>Total</b> | <b>Comments</b>                                                                        |
|---------------|-----------------------------------------------------------------------------------|--------------|--------------|----------------------------------------------------------------------------------------|
| <b>5</b>      | $X \sim B(n, p)$                                                                  |              |              |                                                                                        |
| <b>(a)</b>    | $\text{Var}(X) = E(X^2) - [E(X)]^2$                                               | M1           |              | Used; may be implied                                                                   |
|               | $= E[X(X-1)] + E(X) - [E(X)]^2$                                                   | M1           |              | Rearranging & substitution                                                             |
|               | $= n(n-1)p^2 + np - n^2p^2$                                                       |              |              |                                                                                        |
|               | $= np - np^2 = np(1-p)$                                                           | A1           |              | Or equivalent                                                                          |
|               | <b>OR</b>                                                                         |              |              |                                                                                        |
|               | $E[X(X-1)] = E(X^2) - E(X)$                                                       |              |              |                                                                                        |
|               | $= n(n-1)p^2 = n^2p^2 - np^2$                                                     | (M1)         |              | Expansion & substitution                                                               |
|               | $\text{Var}(X) = E(X^2) - [E(X)]^2$                                               | (M1)         |              | Used; may be implied                                                                   |
|               | $= \{n^2p^2 - np^2 + E(X)\} - n^2p^2$                                             |              |              |                                                                                        |
|               | $= np - np^2 = np(1-p)$                                                           | (A1)         | 3            | Or equivalent                                                                          |
| <b>(b)(i)</b> | Mean $= np = 36$ SD $= \sqrt{np(1-p)} = 4.8$                                      | B1           |              | Both CAO                                                                               |
|               | Thus $36(1-p) = 4.8^2$                                                            | M1           |              | Attempt to solve for $p$ or $n$                                                        |
|               | Thus $n = 100$ & $p = 0.36$                                                       | A1           | 3            | Both CAO                                                                               |
| <b>(ii)</b>   | $P(30 < X < 40) =$                                                                |              |              |                                                                                        |
|               |                                                                                   | M1           |              | Standardising (39.5, 40 or 40.5) or (29.5, 30 or 30.5) with 36 and 4.8 and/or (36 - x) |
|               | $P\left(Z < \frac{39.5-36}{4.8}\right) - P\left(Z < \frac{30.5-36}{4.8}\right) =$ | B1           |              | Use of 39.5 & 30.5                                                                     |
|               | $P(Z < 0.73) - P(Z < -1.15) =$                                                    |              |              |                                                                                        |
|               | $P(Z < 0.73) - [1 - P(Z < 1.15)] =$                                               | m1           |              | Area change                                                                            |
|               | $0.76730 - [1 - (0.87286 \text{ to } 0.87493)] =$                                 |              |              |                                                                                        |
|               | 0.64 to 0.643                                                                     | A1           | 4            | AWFW (0.64112)                                                                         |
|               | <b>Total</b>                                                                      |              | <b>10</b>    |                                                                                        |

**MS03 (cont)**

| Q      | Solution                                                                | Marks | Total | Comments                                    |
|--------|-------------------------------------------------------------------------|-------|-------|---------------------------------------------|
| 6(a)   | $E(X) = \underline{2.2}$                                                | B1    | 3     | CAO                                         |
|        | $\text{Var}(X) = E(X^2) - 2.2^2 =$                                      | M1    |       | Used; or equivalent                         |
|        | $6.8 - 4.84 = 1.96$                                                     | A1    |       | CAO                                         |
| (b)(i) | $E(S) = E(X) + 2.0 = 4.2$                                               | B1F   | 5     | F on (a)                                    |
|        | $\text{Var}(S) = \text{Var}(X) + 1.5 + 2 \times (-0.43)$                | M1    |       | Used for $S$ or $D$                         |
|        | $= 2.6$                                                                 | A1F   |       | F on (a)                                    |
| (ii)   | $E(D) = E(X) - 2.0 = 0.2$                                               | B1F   | 5     | F on (a)                                    |
|        | $\text{Var}(D) = \text{Var}(X) + 1.5 - 2 \times (-0.43)$                | A1F   |       | F on (a)                                    |
|        | $= 4.32$                                                                |       |       |                                             |
| (c)    | <u><math>L \sim N(2.31, 0.89^2) \quad M \sim N(2.04, 0.43^2)</math></u> | B1 B1 | 5     | Both CAO; SD = 0.98843                      |
|        | $T = L + M \sim N(\underline{4.35}, \underline{0.977})$                 |       |       |                                             |
|        | $P(T > 5) = P\left(Z > \frac{5 - 4.35}{\sqrt{0.977}}\right)$            | M1    |       | Standardising 5 or 5.01 using C's mean & SD |
|        | $= P(Z > 0.66) = 1 - P(Z < 0.66)$                                       | m1    |       | Area change                                 |
|        | $0.25 \text{ to } 0.26$                                                 | A1    |       | AWFW (0.25540)                              |
|        | Total                                                                   |       | 13    |                                             |

**MS03 (cont)**

| <b>Q</b>      | <b>Solution</b>                                                                                                                                  | <b>Marks</b> | <b>Total</b> | <b>Comments</b>                                                                |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------|--------------|--------------|--------------------------------------------------------------------------------|
| <b>7</b>      | $X_D \sim \text{Po}(24)$                                                                                                                         |              |              |                                                                                |
| <b>(a)</b>    | $T = X_{\Sigma D} \sim \text{Po}(144)$                                                                                                           | B1           |              | CAO                                                                            |
|               | Thus $T \sim \text{approx } N(144, 144)$                                                                                                         | M1           |              | Normal with $\mu = \sigma^2$                                                   |
|               | $P(T_{\text{Po}} \leq 150) \approx P(T_N < \mathbf{150.5})$                                                                                      | B1           |              | CAO                                                                            |
|               | $= P\left(Z < \frac{150.5 - 144}{12}\right)$                                                                                                     | M1           |              | Standardising (149.5, 150 or 150.5) with $\mu > 24$ and $\sqrt{\mu}$           |
|               | $= P(Z < 0.54) = 0.705 \text{ to } 0.71$                                                                                                         | A1           | 5            | AWFW (0.70598)                                                                 |
| <b>(b)(i)</b> | $H_0: \lambda \text{ (or mean)} = 2 \text{ (or 10)}$<br>$H_1: \lambda \text{ (or mean)} > 2 \text{ (or 10)}$                                     | B1           |              | Both; or equivalent                                                            |
|               | $P(Y \geq 17) = 1 - P(Y \leq 16)$                                                                                                                | M1           |              | Accept $1 - P(Y \leq 17)$                                                      |
|               | $= 1 - 0.09730 = 0.027$                                                                                                                          | A1           |              | AWRT                                                                           |
|               | $< 0.10 \text{ (10\%)}$                                                                                                                          | M1           |              | Comparison of probability with 0.1                                             |
|               | $[z = 2.05 \text{ to } 2.38 > 1.2816]$                                                                                                           |              |              | Comparison of $z$ with 1.2816 or 1.6449                                        |
|               | Thus evidence, at 10% level, of increase in mean daily number of requests                                                                        | A1F          | 5            | F on probability or on $z$                                                     |
| <b>(ii)</b>   | CV of $Y$ is such that $P(Y \geq \text{CV}) \leq 0.10$ (10%)                                                                                     | M1           |              | Can be implied by 13, 14 or 15<br>Accept $P(Y = \text{CV}) = 0.10$             |
|               | Thus $P(Y \leq \text{CV} - 1) \geq 0.90$                                                                                                         | M1           |              | Can be implied by 13, 14 or 15<br>Accept $P(Y = \text{CV}) = 0.90$             |
|               | Thus $\text{CV} = 15$                                                                                                                            | A1           | 3            | CAO                                                                            |
| <b>(iii)</b>  | Power $= 1 - P(\text{Type II error})$<br>$= 1 - P(\text{accept } H_0 \mid H_0 \text{ false})$<br>$= P(\text{accept } H_1 \mid H_1 \text{ true})$ | B1           |              | Or equivalent<br>Stated or implied use                                         |
|               | $\lambda = 5 \times 3 = 15$                                                                                                                      | B1           |              | Stated or implied use of Po(15)                                                |
|               | Thus power $= P(Y \geq \text{CV})$<br>$= P(Y \geq 15) = 1 - P(Y \leq 14)$<br>$= 1 - 0.4657 = 0.53 \text{ to } 0.54$                              | M1<br>A1     | 4            | Attempt at a probability based on C's CV from (ii) and Po(15)<br>AWFW (0.5343) |
|               | <b>Total</b>                                                                                                                                     |              | <b>17</b>    |                                                                                |
|               | <b>TOTAL</b>                                                                                                                                     |              | <b>75</b>    |                                                                                |