

General Certificate of Education

Mathematics 6360

MS03 Statistics 3

Mark Scheme

2008 examination - June series

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Key to mark scheme and abbreviations used in marking

M	mark is for method				
m or dM	mark is dependent on one or more M marks and is for method				
A	mark is dependent on M or m marks and is for accuracy				
В	mark is independent of M or m marks and is for method and accuracy				
Е	mark is for explanation				
√or ft or F	follow through from previous				
v 01 10 01 1	incorrect result	MC	mis-copy		
CAO	correct answer only	MR	mis-read		
CSO	correct solution only	RA	required accuracy		
AWFW	anything which falls within	FW	further work		
AWRT	anything which rounds to	ISW	ignore subsequent work		
ACF	any correct form	FIW	from incorrect work		
AG	answer given	BOD	given benefit of doubt		
SC	special case	WR	work replaced by candidate		
OE	or equivalent	FB	formulae book		
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme		
−x EE	deduct x marks for each error	G	graph		
NMS	no method shown	c	candidate		
PI	possibly implied	sf	significant figure(s)		
SCA	substantially correct approach	dp	decimal place(s)		

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments	
1 (a)	$r = \frac{1.3781}{\sqrt{7.0036 \times 0.8464}} =$	M1		Used	
	0.56 to 0.57	A1	2	AWFW (0.56	6602)
(b)	$H_0: \rho = 0$ $H_1: \rho > 0$	B1		Both	
	SL $\alpha = 0.01 (1\%)$ CV $r = 0.515$ to 0.516	B1		AWFW (0	5155)
	Calculated $r > $ Tabulated r	M1		Comparison	
	Evidence, at 1% level, of a positive correlation between x and y	A1√	4	ft on r and CV	
	Special Case for part (b)				
	CV t _{n-2} (0.99) 2.552	(B1)			
	$r\sqrt{\frac{n-2}{1-r^2}} = 2.913$	(B1)			
(c)	(Strong) evidence of a positive correlation between best performances of junior athletes in the long jump and in the high jump	B1√	1	ft on (b) ; or equivalent	
	Total		7		

MS03 (cont)				
Q	Solution	Marks	Total	Comments
(a)	$\hat{p} = \frac{132}{200} = 0.66$	B1		CAO; or equivalent
	$98\% \implies z = 2.32 to 2.33$	B1		AWFW (2.3263)
	CI for p : $\hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1		Variance term
	ie $0.66 \pm 2.3263 \times \sqrt{\frac{0.66 \times 0.34}{200}}$	M1 A1√		CI expression used ft on \hat{p} and z
	ie 0.66 ± 0.08 or $(0.58, 0.74)$	A1	6	AWRT; or equivalent
(b)	Value of 0.6 (60%) is within CI	B1√		ft on (a)
	Reason to doubt claim of more than 60%	B1√	2	dependent on previous B1 ft on (a); or equivalent
	Total		8	
3	$H_0: \mu_X = \mu_Y$ $H_1: \mu_X \neq \mu_Y$	B1		Both
	SL $\alpha = 0.01(1\%)$ CV $z = (\pm)$ 2.57 to 2.58	B1		AWFW (2.5758)
	$z = \frac{ 157 - 162 }{\sqrt{\frac{4.5^2}{10} + \frac{5.7^2}{15}}} =$	M1		Numerator
	$\sqrt{\frac{4.5^2}{10} + \frac{5.7^2}{15}}$	M1		Denominator
	(\pm) 2.44 to 2.445	A1		AWFW (2.4424)
	No evidence, at 1% level, to reject hypothesis that $\mu_X = \mu_Y$	A1√	6	ft on z, CV and signs; or equivalent
	Total		6	

Q (cont)	Solution	Marks	Total	Comments
4 (a)		B1		B, S & D with 3 probabilities
		B2	3	3 × (E, M & W) each with 3 probabilities
	E(0.55) 0.0825 D(0.15)——M(0.35) 0.0525 W(0.10) 0.0150	(B1)		≥ 1 × (E, M & W) (each) with 3 probabilities
(b)(i)	$P(E) = (0.25 \times 0.3) + (0.6 \times 0.4) + (0.15 \times 0.55)$ $= 0.075 + 0.24 + 0.0825 =$	M1		≥ 1 term correct
	0.397 to 0.398 or 159/400	A 1	2	AWFW/CAO (0.3975)
(ii)	$P(D \mid E) = \frac{0.0825}{(b)(i)} =$	M1		Or equivalent
	0.207 to 0.208 or 11/53	A1	2	AWFW/CAO (0.2075)
(c)	$X \sim B(10, (b)(ii))$	M1		Used
	$P(X=4) = {10 \choose 4} (0.2075)^4 (0.7925)^6 =$	A1√		ft on (b)(ii)
	0.0955 to 0.0975	A1	3	AWFW (0.09645)
	Total		10	

MS03 (cont)		N/ 1	TF ()	C .
Q	Solution	Marks	Total	Comments
5 (a)	$\hat{\lambda}_{A} = \frac{3312}{184} = 18$ $\hat{\lambda}_{B} = \frac{2760}{184} = 15$	B1		CAO both
	$95\% \Rightarrow z = 1.96$	B1		CAO
	CI for $(\lambda_{A} - \lambda_{B})$:	M1		Variance term
	$\left(\hat{\lambda}_{_{\mathrm{A}}}-\hat{\lambda}_{_{\mathrm{B}}} ight)\pm z\sqrt{rac{\hat{\lambda}_{_{\mathrm{A}}}}{n_{_{\mathrm{A}}}}+rac{\hat{\lambda}_{_{\mathrm{B}}}}{n_{_{\mathrm{B}}}}}$	M1		CI expression used
	ie $(18-15) \pm 1.96 \times \sqrt{\frac{18}{184} + \frac{15}{184}}$	A1√		ft on $\hat{\lambda}_{A}$, $\hat{\lambda}_{B}$ and z
	ie 3 ± 0.83 or $(2.17, 3.83)$	A1	6	AWRT
(b)	Calls from A and B are independent	B1	1	Or equivalent
(a)	Alternative Solution			
	$(3312-2760) \pm 1.96 \times \sqrt{3312+2760} =$ ie 552 ± 152.73	(M2) (B1) (A1)		1.96
	Dividing by 184	(M1)		
	ie 3 ± 0.83 or $(2.17, 3.83)$	(A1)		AWRT
	Total		7	

MS03 (cont)				T ~
Q	Solution	Marks	Total	Comments
6 (a)(i)	E(F) = 128 + 112 = 240	B1		CAO
(ii)	$Cov(X, Y) = -0.4 \times \sqrt{50 \times 50} = -20$	M1		Used; or equivalent
	$Var(F) = 50 + 50 + (2 \times -20) = 60$	M1 A1	4	V(X) + V(Y) + 2Cov(X,Y) used CAO; AG
(b)(i)	E(T) = 240 + 75 = 315	B1√		ft on (a)(i)
	Var(T) = 60 + 36 = 96	В1	2	CAO
(ii)	$E(M) = 240 - (3 \times 75) = 15$	B1√		ft on (a)(i)
	$Var(M) = 60 + \{(-3^2) \times 36\}$ = 60 + 324 = 384	M1 A1	3	$V(F) + 3^2V(S)$ used CAO
(c)(i)	$P(T > 300) = P\left(Z > \frac{300 - 315}{\sqrt{96}}\right)$	M1		Standardising 300 or 300.5 using (b)(i)
	= P(Z > -1.53) = P(Z < 1.53)	m1		Area change
	= 0.936 to 0.938	A1	3	AWFW
(ii)	$P\left(S > \frac{X+Y}{3}\right) =$	M1		Used; or equivalent
	P(3S > X + Y) = P(3S > F) =	M1		Attempt to change to M
	P(F - 3S < 0) = P(M < 0)	A1		Or equivalent
	$= P\left(Z < \frac{0-15}{\sqrt{384}}\right)$	M1		Standardising 0 using (b)(ii)
	= P(Z < -0.765) = 1 - P(Z < 0.765)	m1		Area change
	= 0.22(0) to 0.225	A1	6	
	Total		18	

MS03 (cont)		35 3		
Q	Solution	Marks	Total	Comments
7(a)(i)	$E(X(X-1)) = \sum_{x=0}^{\infty} x(x-1) \times \frac{e^{-\lambda} \lambda^{x}}{x!} =$	M1		$\sum x(x-1) \times P(X=x) \text{used}$ Ignore limits until A1
	$\sum_{x=2}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-2)!} =$	M1		$\frac{x(x-1)}{x!} = \frac{1}{(x-2)!}$ used
	$\lambda^2 e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} =$	M1		Factor of $\lambda^2 e^{-\lambda}$ used
	$(\lambda^2 e^{-\lambda}) \times (e^{\lambda}) = \lambda^2$	A1	4	Fully correct derivation; AG
(ii)	$Var(X) = E(X(X-1)) + E(X) - (E(X))^{2}$	M1		Used
	$= \lambda^2 + \lambda - \lambda^2 = \lambda$	A1	2	Fully correct derivation; AG
(b)(i)	E(D) = 5 - 2 = 3 Var(D) = 5 + 2 = 7	B1 B1	2	CAO CAO
(ii)	$E(F) = (2 \times 5) + 10 = 20$ $Var(F) = 2^2 \times 5$ = 20	B1 M1 A1	3	$CAO 2^2V(X_1) + 0 CAO$
(iii)	D: Mean ≠ Variance	B1		Negative values possible
	F: Values < 10 impossible Odd values impossible	B1	2	$2X_1 = X_1 + X_1$ is not sum of independent Po variables
(c)	$B \sim \text{Po}(5) C \sim \text{Po}(2)$			
	$T = 24 \times (5 + 2) \sim \text{Po}(168)$	B1		CAO
	$T \sim \text{approx N}(168, 168)$	M1		Normal with $\mu = \sigma^2$
	$P(T_{Po} > 175) \approx P(T_{N} > 175.5)$	В1		175.5
	$= P\left(Z > \frac{175.5 - 168}{\sqrt{168}}\right) = P(Z > 0.58) =$	M1		Standardising 174.5, 175 or 175.5 with $\mu = \sigma^2$
	1 - P(Z < 0.58) = 0.28(0) to 0.283	m1 A1	6	Area change AWFW
	Total		19	
	TOTAL		75	