Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					



General Certificate of Education Advanced Level Examination June 2011

Mathematics

MPC4

Unit Pure Core 4

Thursday 16 June 2011 1.30 pm to 3.00 pm

For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed

• 1 hour 30 minutes

Instructions

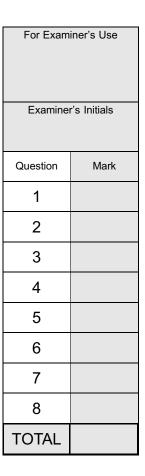
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

 Unless stated otherwise, you may quote formulae, without proof, from the booklet.





Answer all	questions	in	the	spaces	provided.
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1	1	The no	olvnomial	f(x)	1S	defined	bv	f(x)	$= 4x^{3} -$	-13x +	· 6.

- (a) Find f(-2). (1 mark)
- (b) Use the Factor Theorem to show that 2x 3 is a factor of f(x). (2 marks)
- (c) Simplify $\frac{2x^2 + x 6}{f(x)}$. (4 marks)

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2		The average weekly pay of a footballer at a certain club was £80 on 1 August 1960. By 1 August 1985, this had risen to £2000.					
		The average weekly pay of a footballer at this club can be modelled by the	he equation				
		$P=Ak^t$					
		where $\pounds P$ is the average weekly pay t years after 1 August 1960, and A a constants.	and k are				
(a) (i)	Write down the value of A .	(1 mark)				
	(ii)	Show that the value of k is 1.137411, correct to six decimal places.	(2 marks)				
(b)	Use this model to predict the year in which, on 1 August, the average we a footballer at this club will first exceed £100000.	eekly pay of (3 marks)				
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- 3 (a) (i) Find the binomial expansion of $(1-x)^{\frac{1}{3}}$ up to and including the term in x^2 .
 - (ii) Hence, or otherwise, show that

$$(125 - 27x)^{\frac{1}{3}} \approx 5 + \frac{m}{25}x + \frac{n}{3125}x^2$$

for small values of x, stating the values of the integers m and n. (3 marks)

(b) Use your result from part (a)(ii) to find an approximate value of $\sqrt[3]{119}$, giving your answer to five decimal places. (2 marks)

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- **4 (a)** A curve is defined by the parametric equations $x = 3\cos 2\theta$, $y = 2\cos \theta$.
 - (i) Show that $\frac{dy}{dx} = \frac{1}{k \cos \theta}$, where k is an integer. (4 marks)
 - (ii) Find an equation of the normal to the curve at the point where $\theta = \frac{\pi}{3}$. (4 marks)
 - **(b)** Find the exact value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx$. (5 marks)

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The points A and B have coordinates (5, 1, -2) and (4, -1, 3) respectively.

The line
$$l$$
 has equation $\mathbf{r} = \begin{bmatrix} -8 \\ 5 \\ -6 \end{bmatrix} + \mu \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$.

- (a) Find a vector equation of the line that passes through A and B. (3 marks)
- (b) (i) Show that the line that passes through A and B intersects the line I, and find the coordinates of the point of intersection, P. (4 marks)
 - (ii) The point C lies on l such that triangle PBC has a right angle at B. Find the coordinates of C. (5 marks)

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6	A curve is defined by the equation $2y + e^{2x}y^2 = x^2 + C$, where C	is a constant.
	The point $P\left(1, \frac{1}{e}\right)$ lies on the curve.	
(a) Find the exact value of C .	(1 mark)
(b	Find an expression for $\frac{dy}{dx}$ in terms of x and y.	(7 marks)
(с	Verify that $P\left(1, \frac{1}{e}\right)$ is a stationary point on the curve.	(2 marks)
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7		A giant snowball is melting. The snowball can be modelled as a sphere whose surface area is decreasing at a constant rate with respect to time. The surface area of the sphere is $A \mathrm{cm}^2$ at time t days after it begins to melt.
(a)		Write down a differential equation in terms of the variables A and t and a constant k , where $k>0$, to model the melting snowball. (2 marks)
(b) (i)	Initially, the radius of the snowball is 60 cm, and 9 days later, the radius has halved.
		Show that $A = 1200\pi(12 - t)$.
		(You may assume that the surface area of a sphere is given by $A=4\pi r^2$, where r is the radius.) (4 marks)
	(ii)	Use this model to find the number of days that it takes the snowball to melt completely. (1 mark)
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- 8 (a) Express $\frac{1}{(3-2x)(1-x)^2}$ in the form $\frac{A}{3-2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$. (4 marks)
 - **(b)** Solve the differential equation

$$\frac{dy}{dx} = \frac{2\sqrt{y}}{(3 - 2x)(1 - x)^2}$$

where y = 0 when x = 0, expressing your answer in the form

$$y^p = q \ln[f(x)] + \frac{x}{1 - x}$$

where p and q are constants.

(9 marks)

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