General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Friday 5 June 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) The curve with equation

$$y = \frac{\cos x}{2x + 1}, \qquad x > -\frac{1}{2}$$

intersects the line $y = \frac{1}{2}$ at the point where $x = \alpha$.

(i) Show that α lies between 0 and $\frac{\pi}{2}$. *(2 marks)*

(ii) Show that the equation $\frac{\cos x}{2x + 1} = \frac{1}{2}$ can be rearranged into the form

$$x = \cos x - \frac{1}{2}$$ *(1 mark)*

(iii) Use the iteration $x_{n+1} = \cos x_n - \frac{1}{2}$ with $x_1 = 0$ to find x_3, giving your answer to three decimal places. *(2 marks)*

(b) (i) Given that $y = \frac{\cos x}{2x + 1}$, use the quotient rule to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(ii) Hence find the gradient of the normal to the curve $y = \frac{\cos x}{2x + 1}$ at the point on the curve where $x = 0$. *(2 marks)*

2 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \sqrt{2x+5}, \qquad \text{for real values of } x,\ \ x \geqslant -2.5$$
$$\mathrm{g}(x) = \frac{1}{4x+1}, \qquad \text{for real values of } x,\ \ x \neq -0.25$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) State the domain of f^{-1}. *(1 mark)*

(c) The composite function fg is denoted by h.

(i) Find an expression for $\mathrm{h}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{h}(x) = 3$. *(3 marks)*

3 (a) Solve the equation $\tan x = -\frac{1}{3}$, giving all the values of x in the interval $0 < x < 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation

$$3\sec^2 x = 5(\tan x + 1)$$

can be written in the form $3\tan^2 x - 5\tan x - 2 = 0$. *(1 mark)*

(c) Hence, or otherwise, solve the equation

$$3\sec^2 x = 5(\tan x + 1)$$

giving all the values of x in the interval $0 < x < 2\pi$ in radians to two decimal places. *(4 marks)*

4 (a) Sketch the graph of $y = |50 - x^2|$, indicating the coordinates of the point where the graph crosses the y-axis. *(3 marks)*

(b) Solve the equation $|50 - x^2| = 14$. *(3 marks)*

(c) Hence, or otherwise, solve the inequality $|50 - x^2| > 14$. *(2 marks)*

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 50 - x^2$. *(4 marks)*

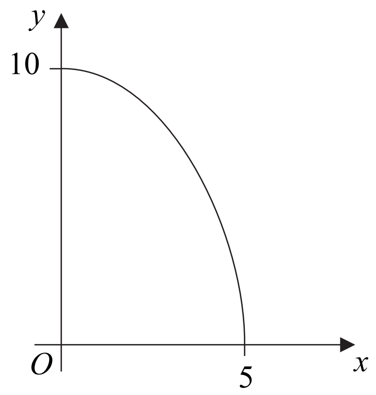
5 (a) Given that $2\ln x = 5$, find the exact value of x. *(1 mark)*

(b) Solve the equation

$$2\ln x + \frac{15}{\ln x} = 11$$

giving your answers as exact values of x. *(5 marks)*

6 The diagram shows the curve with equation $y = \sqrt{100 - 4x^2}$, where $x \geqslant 0$.



(a) Calculate the volume of the solid generated when the region bounded by the curve shown above and the coordinate axes is rotated through 360° about the ***y*-axis**, giving your answer in terms of π. *(5 marks)*

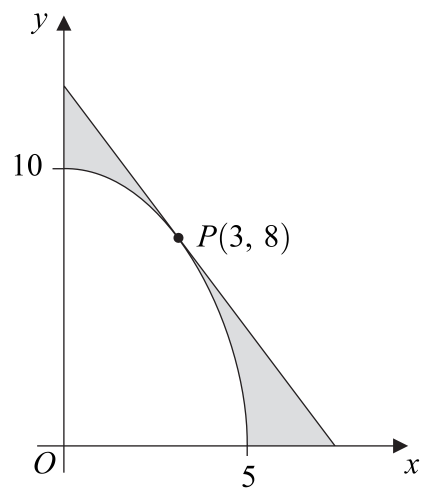
(b) Use the mid-ordinate rule with five strips of equal width to find an estimate for $\int_0^5 \sqrt{100 - 4x^2}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(c) The point P on the curve has coordinates $(3, 8)$.

(i) Find the gradient of the curve $y = \sqrt{100 - 4x^2}$ at the point P. *(3 marks)*

(ii) Hence show that the equation of the tangent to the curve at the point P can be written as $2y + 3x = 25$. *(2 marks)*

(d) The shaded regions on the diagram below are bounded by the curve, the tangent at P and the coordinate axes.



Use your answers to part (b) and part (c)(ii) to find an approximate value for the **total** area of the shaded regions. Give your answer to three significant figures. *(5 marks)*

7 (a) Use integration by parts to find $\int (t-1)\ln t \, \mathrm{d}t$. *(4 marks)*

(b) Use the substitution $t = 2x + 1$ to show that $\int 4x \ln(2x+1)\, \mathrm{d}x$ can be written as $\int (t-1)\ln t \, \mathrm{d}t$. *(3 marks)*

(c) Hence find the exact value of $\int_0^1 4x \ln(2x+1)\, \mathrm{d}x$. *(3 marks)*

END OF QUESTIONS

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