

General Certificate of Education  
June 2007  
Advanced Level Examination



**MATHEMATICS**  
**Unit Statistics 3**

**MS03**

Monday 11 June 2007 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 As part of an investigation into the starting salaries of graduates in a European country, the following information was collected.

|                   |             | Starting salary (€) |                           |
|-------------------|-------------|---------------------|---------------------------|
|                   | Sample size | Sample mean         | Sample standard deviation |
| Science graduates | 175         | 19 268              | 7321                      |
| Arts graduates    | 225         | 17 896              | 8205                      |

- (a) Stating a necessary assumption about the samples, construct a 98% confidence interval for the difference between the mean starting salary of science graduates and that of arts graduates. *(6 marks)*
- (b) What can be concluded from your confidence interval? *(2 marks)*
- 2 A hill-top monument can be visited by one of three routes: road, funicular railway or cable car. The percentages of visitors using these routes are 25, 35 and 40 respectively.

The age distribution, in percentages, of visitors using **each** route is shown in the table. For example, 15 per cent of visitors using the road were under 18.

|             |          | Percentage of visitors using |                   |           |
|-------------|----------|------------------------------|-------------------|-----------|
|             |          | Road                         | Funicular railway | Cable car |
| Age (years) | Under 18 | 15                           | 25                | 10        |
|             | 18 to 64 | 80                           | 60                | 55        |
|             | Over 64  | 5                            | 15                | 35        |

Calculate the probability that a randomly selected visitor:

- (a) who used the road is aged 18 or over; *(1 mark)*
- (b) is aged between 18 and 64; *(3 marks)*
- (c) used the funicular railway and is aged over 64; *(2 marks)*
- (d) used the funicular railway, given that the visitor is aged over 64. *(5 marks)*

- 3 Kutz and Styler are two unisex hair salons. An analysis of a random sample of 150 customers at Kutz shows that 28 per cent are male. An analysis of an independent random sample of 250 customers at Styler shows that 34 per cent are male.
- (a) Test, at the 5% level of significance, the hypothesis that there is no difference between the proportion of male customers at Kutz and that at Styler. (9 marks)
- (b) State, with a reason, the probability of making a Type I error in the test in part (a) if, in fact, the actual difference between the two proportions is 0.05. (2 marks)
- 4 A machine is used to fill 5-litre plastic containers with vinegar. The volume, in litres, of vinegar in a container filled by the machine may be assumed to be normally distributed with mean  $\mu$  and standard deviation 0.08.
- A quality control inspector requires a 99% confidence interval for  $\mu$  to be constructed such that it has a width of at most 0.05 litres.
- Calculate, to the nearest 5, the sample size necessary in order to achieve the inspector's requirement. (6 marks)
- 5 The duration,  $X$  minutes, of a timetabled 1-hour lesson may be assumed to be normally distributed with mean 54 and standard deviation 2.
- The duration,  $Y$  minutes, of a timetabled  $1\frac{1}{2}$ -hour lesson may be assumed to be normally distributed with mean 83 and standard deviation 3.
- Assuming the durations of lessons to be independent, determine the probability that the total duration of a random sample of three 1-hour lessons is less than the total duration of a random sample of two  $1\frac{1}{2}$ -hour lessons. (7 marks)

**Turn over for the next question**

**Turn over ►**

- 6 (a) The random variable  $X$  has a binomial distribution with parameters  $n$  and  $p$ .
- (i) Prove that  $E(X) = np$ . (4 marks)
  - (ii) Given that  $E(X^2) - E(X) = n(n-1)p^2$ , show that  $\text{Var}(X) = np(1-p)$ . (3 marks)
  - (iii) Given that  $X$  is found to have a mean of 3 and a variance of 2.97, find values for  $n$  and  $p$ . (3 marks)
  - (iv) Hence use a distributional approximation to estimate  $P(X > 2)$ . (3 marks)
- (b) Dressher is a nationwide chain of stores selling women's clothes. It claims that the probability that a customer who buys clothes from its stores uses a Dressher store card is 0.45.
- Assuming this claim to be correct, use a distributional approximation to estimate the probability that, in a random sample of 500 customers who buy clothes from Dressher stores, at least half of them use a Dressher store card. (7 marks)
- 7 In a town, the total number,  $R$ , of houses sold during a week by estate agents may be modelled by a Poisson distribution with a mean of 13.
- A new housing development is completed in the town. During the first week in which houses on this development are offered for sale by the developer, the estate agents sell a total of 10 houses.
- (a) Using the 10% level of significance, investigate whether the offer for sale of houses by the developer has resulted in a reduction in the mean value of  $R$ . (6 marks)
  - (b) Determine, for your test in part (a), the critical region for  $R$ . (2 marks)
  - (c) Assuming that the offer for sale of houses on the new housing development has reduced the mean value of  $R$  to 6.5, determine, for a test at the 10% level of significance, the probability of a Type II error. (4 marks)

**END OF QUESTIONS**