



General Certificate of Education  
Advanced Level Examination  
January 2010

# Mathematics

# MPC3

## Unit Pure Core 3

Friday 15 January 2010 1.30 pm to 3.00 pm

### For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 2 (enclosed).

You may use a graphics calculator.

### Time allowed

- 1 hour 30 minutes

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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**1** A curve has equation  $y = e^{-4x}(x^2 + 2x - 2)$ .

(a) Show that  $\frac{dy}{dx} = 2e^{-4x}(5 - 3x - 2x^2)$ . (3 marks)

(b) Find the exact values of the coordinates of the stationary points of the curve. (5 marks)

**2** [**Figure 1**, printed on the insert, is provided for use in this question.]

(a) (i) Sketch the graph of  $y = \sin^{-1} x$ , where  $y$  is in radians. State the coordinates of the end points of the graph. (3 marks)

(ii) By drawing a suitable straight line on your sketch, show that the equation

$$\sin^{-1} x = \frac{1}{4}x + 1$$

has only one solution. (2 marks)

(b) The root of the equation  $\sin^{-1} x = \frac{1}{4}x + 1$  is  $\alpha$ . Show that  $0.5 < \alpha < 1$ . (2 marks)

(c) The equation  $\sin^{-1} x = \frac{1}{4}x + 1$  can be rewritten as  $x = \sin\left(\frac{1}{4}x + 1\right)$ .

(i) Use the iteration  $x_{n+1} = \sin\left(\frac{1}{4}x_n + 1\right)$  with  $x_1 = 0.5$  to find the values of  $x_2$  and  $x_3$ , giving your answers to three decimal places. (2 marks)

(ii) The sketch on **Figure 1** shows parts of the graphs of  $y = \sin\left(\frac{1}{4}x + 1\right)$  and  $y = x$ , and the position of  $x_1$ .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of  $x_2$  and  $x_3$  on the  $x$ -axis. (2 marks)

- 3 (a) Solve the equation

$$\operatorname{cosec} x = 3$$

giving all values of  $x$  in radians to two decimal places, in the interval  $0 \leq x \leq 2\pi$ .  
(2 marks)

- (b) By using a suitable trigonometric identity, solve the equation

$$\cot^2 x = 11 - \operatorname{cosec} x$$

giving all values of  $x$  in radians to two decimal places, in the interval  $0 \leq x \leq 2\pi$ .  
(6 marks)

- 4 (a) Sketch the graph of  $y = |8 - 2x|$ . (2 marks)

- (b) Solve the equation  $|8 - 2x| = 4$ . (2 marks)

- (c) Solve the inequality  $|8 - 2x| > 4$ . (2 marks)

- 5 (a) Use the mid-ordinate rule with four strips to find an estimate for  $\int_0^{12} \ln(x^2 + 5) \, dx$ ,  
giving your answer to three significant figures. (4 marks)

- (b) A curve has equation  $y = \ln(x^2 + 5)$ .

- (i) Show that this equation can be rewritten as  $x^2 = e^y - 5$ . (1 mark)

- (ii) The region bounded by the curve, the lines  $y = 5$  and  $y = 10$  and the  $y$ -axis is rotated through  $360^\circ$  about the  $y$ -axis. Find the exact value of the volume of the solid generated. (4 marks)

- (c) The graph with equation  $y = \ln(x^2 + 5)$  is stretched with scale factor 4 parallel to the  $x$ -axis, and then translated through  $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$  to give the graph with equation  $y = f(x)$ .  
Write down an expression for  $f(x)$ . (3 marks)

**Turn over for the next question**

**Turn over ►**

6 The functions  $f$  and  $g$  are defined with their respective domains by

$$f(x) = e^{2x} - 3, \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{3x+4}, \quad \text{for real values of } x, \quad x \neq -\frac{4}{3}$$

(a) Find the range of  $f$ . (2 marks)

(b) The inverse of  $f$  is  $f^{-1}$ .

(i) Find  $f^{-1}(x)$ . (3 marks)

(ii) Solve the equation  $f^{-1}(x) = 0$ . (2 marks)

(c) (i) Find an expression for  $gf(x)$ . (1 mark)

(ii) Solve the equation  $gf(x) = 1$ , giving your answer in an exact form. (3 marks)

7 It is given that  $y = \tan 4x$ .

(a) By writing  $\tan 4x$  as  $\frac{\sin 4x}{\cos 4x}$ , use the quotient rule to show that  $\frac{dy}{dx} = p(1 + \tan^2 4x)$ , where  $p$  is a number to be determined. (3 marks)

(b) Show that  $\frac{d^2y}{dx^2} = qy(1 + y^2)$ , where  $q$  is a number to be determined. (5 marks)

8 (a) Using integration by parts, find  $\int x \sin(2x - 1) dx$ . (5 marks)

(b) Use the substitution  $u = 2x - 1$  to find  $\int \frac{x^2}{2x - 1} dx$ , giving your answer in terms of  $x$ . (6 marks)

**END OF QUESTIONS**