



General Certificate of Education (A-level)
June 2011

Mathematics

MPC3

(Specification 6360)

Pure Core 3

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

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Q	Solution	Marks	Total	Comments																	
1 (a)	$\frac{1}{6}$ or $\left(\frac{1}{6}, 0\right)$	B1	1	condone 0.167 AWRT																	
	$\left(\frac{dy}{dx}\right)=\frac{1}{x}$	M1		$\frac{k}{x}$ where $k=1, 6$ or $\frac{1}{6}$																	
		A1	2	$k=1$																	
	(c)																				
		<table><tr><td>x</td><td>y</td></tr><tr><td>1</td><td>$\ln 6=1.7918$</td></tr><tr><td>2</td><td>$\ln 12=2.4849$</td></tr><tr><td>3</td><td>$\ln 18=2.8904$</td></tr><tr><td>4</td><td>$\ln 24=3.1781$</td></tr><tr><td>5</td><td>$\ln 30=3.4012$</td></tr><tr><td>6</td><td>$\ln 36=3.5835$</td></tr><tr><td>7</td><td>$\ln 42=3.7377$</td></tr></table>	x	y	1	$\ln 6=1.7918$	2	$\ln 12=2.4849$	3	$\ln 18=2.8904$	4	$\ln 24=3.1781$	5	$\ln 30=3.4012$	6	$\ln 36=3.5835$	7	$\ln 42=3.7377$	M1		5+ y -values correct, either exact or correct to 3SF (rounded or truncated) or better
		x	y																		
		1	$\ln 6=1.7918$																		
		2	$\ln 12=2.4849$																		
		3	$\ln 18=2.8904$																		
		4	$\ln 24=3.1781$																		
5	$\ln 30=3.4012$																				
6	$\ln 36=3.5835$																				
7	$\ln 42=3.7377$																				
		A1		all 7 y -values correct (and only these 7 values), either exact or correct to 3SF (rounded or truncated) or better																	
$A=\frac{1}{3} \times 1\left[(1.7918+3.7377)\right. \\ \left.+4(2.4849+3.1781+3.5835)\right. \\ \left.+2(2.8904+3.4012)\right]$ $=18.4$	M1		correct use of Simpson's rule on their 7 y -values, condone missing square brackets																		
	A1	4	CAO this value only																		
Total			7																		

MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$y = xe^{2x}$ $\left(\frac{dy}{dx} = \right) 2xe^{2x} + e^{2x}$	M1 A1 A1 ISW	3	$kxe^{2x} + le^{2x}$ where k and l are 1s or 2s $\left. \begin{matrix} k=2 \\ l=1 \end{matrix} \right\}$ Independent of each other $(= e^{2x}(2x+1))$
(ii)	$x=1 \Rightarrow \frac{dy}{dx} = 3e^2$ tangent: $y - e^2 = 3e^2(x-1)$ OE	M1 A1	2	correct substitution of $x=1$ into their $\frac{dy}{dx}$ but must have earned M1 in part (i) CSO (no ISW), must have scored first 4 marks common correct answer: $y = 3e^2x - 2e^2$
(b)	$y = \frac{2\sin 3x}{1+\cos 3x}$ $\left(\frac{dy}{dx} = \right) \frac{(1+\cos 3x)6\cos 3x - 2\sin 3x(-3\sin 3x)}{(1+\cos 3x)^2}$ $= \frac{6\cos 3x + 6\cos^2 3x + 6\sin^2 3x}{(1+\cos 3x)^2}$ $= \frac{6\cos 3x + 6}{(1+\cos 3x)^2}$ $= \frac{6}{1+\cos 3x}$	M1 A1 m1 A1	4	$\frac{\pm p(1+\cos 3x)\cos 3x \pm q\sin 3x(\sin 3x)}{(1+\cos 3x)^2}$ where p and q are rational numbers condone poor use/omission of brackets PI by further working this line must be seen in this form (ie in terms of $\cos^2 3x$ and $\sin^2 3x$), but allow $\sin^2 3x$ replaced by $1 - \cos^2 3x$ condone denominator correctly expanded correct use of $k\sin^2 3x + k\cos^2 3x = k$ or $k\sin^2 3x = k(1 - \cos^2 3x)$ CSO
	Total		9	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	note: if degrees used then no marks in (a) and (c)			
	$f(x) = \cos^{-1}(2x-1) - e^x$			or reverse
	$f(0.4) = 0.3$ $f(0.5) = -0.1$	M1		sight of ± 0.3 (AWRT) AND ∓ 0.1 (AWRT)
	change of sign $\therefore 0.4 < \alpha < 0.5$	A1	2	CSO, note $f(x)$ must be defined, condone $0.4 \leq \alpha \leq 0.5$
		(M1)		alternative method $e^{0.4} = 1.5, \cos^{-1}(2 \times 0.4 - 1) = 1.8$ $e^{0.5} = 1.65, \cos^{-1}(2 \times 0.5 - 1) = 1.57$
		(A1)		$\left. \begin{array}{l} \text{at } 0.4 \ e^x < \cos^{-1}(2x-1) \\ \text{at } 0.5 \ e^x > \cos^{-1}(2x-1) \end{array} \right\}$ $\therefore 0.4 < \alpha < 0.5$
(b)	$\cos^{-1}(2x-1) = e^x$ $2x-1 = \cos(e^x)$ $x = \frac{1}{2}(\cos(e^x) + 1) = \frac{1}{2} + \frac{1}{2}\cos(e^x)$	B1	1	AG must see middle line, and no errors seen, but condone $\cos e^x$
(c)	$x_1 = 0.4$ $x_2 = 0.539$ $x_3 = 0.428$	B1 B1	2	CAO CAO
	Total		5	

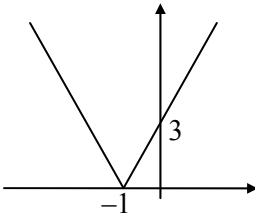
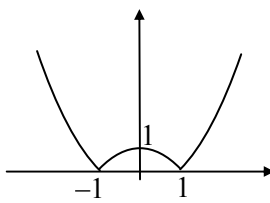
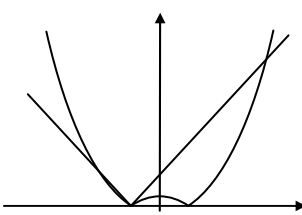
MPC3 (cont)

Q	Solution	Marks	Total	Comments		
4(a)(i)	$(\sin^{-1} \pm 0.25 =) \pm 14.5$	M1	2	PI by sight of 194.5 etc		
	$\theta = 194.5, 345.5$ (AWRT)	A1		condone ± 14.4		
	(ii)	$2 \cot^2(2x + 30) = 2 - 7 \operatorname{cosec}(2x + 30)$			no extras in interval, ignore answers outside interval	
		$2(\operatorname{cosec}^2(2x + 30) - 1) = 2 - 7 \operatorname{cosec}(2x + 30)$	M1		condone replacing $2x + 30$ by Y	
		$2 \operatorname{cosec}^2(2x + 30) + 7 \operatorname{cosec}(2x + 30) - 4 (= 0)$	A1		correct use of $\operatorname{cosec}^2 Y = 1 + \cot^2 Y$	
		$(2 \operatorname{cosec}(2x + 30) \pm 1)(\operatorname{cosec}(2x + 30) \pm 4) (= 0)$	m1		must be in this form	
		$\operatorname{cosec}(2x + 30) = \frac{1}{2}$ or -4	A1		attempt at factorisation	
		$2x + 30 = 194.5, 345.5$			must be this line using $f(2x + 30)$	
	(b)	$x = 82.2, 157.8$ (AWRT)	B1	6	one correct answer, allow 82.3, ignore extra solutions	
			B1		CAO both answers correct and no extras in interval, ignore answers outside interval	
		(b)	stretch (I)			
			scale factor $\frac{1}{2}$ (II)			
			parallel to x -axis (III)	M1		I and either II or III
				A1		I + II + III
	translate		E1			
	$\begin{pmatrix} -15 \\ 0 \end{pmatrix}$		B1	4	condone '15 to left' or '-15 in x (direction)'	
alternative method						
translate	(E1)					
$\begin{pmatrix} -30 \\ 0 \end{pmatrix}$	(B1)					
stretch	(M1)		as above			
scale factor $\frac{1}{2}$						
parallel to x -axis	(A1)		as above			
	Total		12			

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$[f(x)] \text{ not } 1-1$	E1	1	OE
(b)	$y = \frac{1}{2x+1}$ $x = \frac{1}{2y+1}$ $2y+1 = \frac{1}{x}$ $[g^{-1}(x)] = \frac{1}{2}\left(\frac{1}{x}-1\right)$ OE	M1 M1 A1	3	swap x and y a correct next line } either order $[y =] \frac{1}{2}\left(\frac{1}{x}-1\right)$
(c)	$[g^{-1}(x)] \neq -0.5$	B1	1	sight of $\neq -0.5$ OE
(d)	$\left(\frac{1}{2x+1}\right)^2 = \frac{1}{2x+1}$ $(2x+1) = (2x+1)^2$ or $2x+1 = 4x^2 + 4x + 1$ or $\frac{1}{2x+1} = 1$ or $2x+1 = 1$ $x = 0$	B1 M1 A1	3	sight of $\left(\frac{1}{2x+1}\right)^2$ or $\frac{1}{(2x+1)^2}$ one correct step, must be one of these four lines CSO
Total			8	
6(a)	$3 \ln x = 4$ $\left(\ln x = \frac{4}{3}\right)$ $x = e^{\frac{4}{3}}$	B1	1	ISW. Condone $\sqrt[3]{e^4}$
(b)	$3 \ln x + \frac{20}{\ln x} = 19$ $3(\ln x)^2 + 20 = 19 \ln x$ $3(\ln x)^2 - 19 \ln x + 20 (= 0)$ $(3 \ln x \pm 4)(\ln x \pm 5) (= 0)$ $\ln x = \frac{4}{3}, 5$ $x = e^{\frac{4}{3}}, e^5$	M1 A1 m1 A1 A1	5	correctly multiplying by $\ln x$. use of formula, or completing the square must be correct condone $\sqrt[3]{e^4}$
Total			6	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)		M1 A1	2	modulus graph, approximate V shape, touching negative x -axis and crossing y -axis -1, 3 marked, graph symmetrical, straight lines
(ii)		M1 A1 A1	3	modulus graph in 3 sections, touching x -axis and crossing positive y -axis correct curvature their $x > 1$, their $x < -1$ } independent correct curve $-1 \leq x \leq 1$ and $x = \pm 1, y = 1$ marked }
(b)(i)	$ 3x+3 = x^2-1 $ $(3x+3 = x^2-1)$ $(0 =) x^2 - 3x - 4$ —A $x = 4, -1$ $(3x+3 = 1-x^2)$ $x^2 + 3x + 2 (= 0)$ —B $x = -1, -2$	M1 A1,A1 A1,A1	5	either A or B seen, all terms on one side $\therefore x = -2, -1, 4$ SC NMS or partial method 1 correct value 1/5 2 correct values 2/5 3 correct values 5/5 } independent of more than 3 distinct values max 2/5 method mark
(ii)	 $x > 4, x < -2$	M1,A1	2	$x >$ their largest, $x <$ their smallest; CAO
Total			12	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8	$\int \frac{1}{\cos^2 x (1 + 2 \tan x)^2} dx$ $u = 1 + 2 \tan x$ $\left(\frac{du}{dx} = \right) 2 \sec^2 x \quad \text{OE}$ $\int = \int \frac{du}{2u^2}$ $= \frac{1}{2} u^{-1}$ $= - \frac{1}{2u}$ $= - \frac{1}{2(1 + 2 \tan x)} (+c)$	M1 m1 A1 A1F A1	5	condone $\left(\frac{du}{dx} = \right) a \sec^2 x$ where a is a constant $\int \frac{k}{u^2} (du)$, where k is a constant correct, or $\frac{1}{2} \int u^{-2} (du)$ correct integral of their expression but must have scored M1 m1 CSO, no ISW
	Total		5	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9 (a)	$\int x \ln x \, dx$ $\left. \begin{array}{l} u = \ln x \quad \frac{dv}{(dx)} = x \\ \frac{du}{(dx)} = \frac{1}{x} \quad v = \frac{x^2}{2} \end{array} \right\}$ $\int = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} (dx)$ $= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+c)$	M1 A1 A1	 3	correct direction and sight of $\frac{1}{x}, \frac{x^2}{2}$
(b)	$y = (\ln x)^2$ $\left(\frac{dy}{dx} \right) = 2 \ln x \times \frac{1}{x}$	M1 A1	 2	$\frac{k}{x} \ln x$ where $k = \frac{1}{2}, 1$ or 2 $k = 2$
(c)	$y = \sqrt{x} \ln x$ $(V) = \pi \int_1^e x (\ln x)^2 \, dx$ $\left. \begin{array}{l} u = (\ln x)^2 \quad \frac{dv}{(dx)} = x \\ \frac{du}{(dx)} = 2 \ln x \frac{1}{x} \quad v = \frac{x^2}{2} \end{array} \right\}$ $\int = \frac{x^2}{2} (\ln x)^2 - \int \frac{x^2}{2} \times \frac{2}{x} \ln x (dx)$ $= \frac{x^2}{2} (\ln x)^2 - \int x \ln x (dx)$ $= \frac{x^2}{2} (\ln x)^2 - \frac{1}{4} x^2 (2 \ln x - 1) \text{ OE}$ $V = (\pi) \left[\frac{x^2}{2} (\ln x)^2 - \frac{1}{4} x^2 (2 \ln x - 1) \right]_1^e$ $= (\pi) \left[\left(\frac{e^2}{2} - \frac{1}{4} e^2 \right) - \left(0 + \frac{1}{4} \right) \right]$ $= \frac{\pi}{4} [e^2 - 1] \quad \text{OE}$	B1 M1 m1 A1 m1 A1	 6	all correct, incl brackets, π , limits and dx (but dx may be seen BEFORE this line) correct direction with $\frac{du}{(dx)} = \frac{k}{x} \ln x$ where $k = \frac{1}{2}, 1$ or 2 and sight of $\frac{x^2}{2}$ correct substitution of their terms into the parts formula integral needs to be simplified to $\int x \ln x$ correct substitution of 1 and e into their expressions of the form $px^2 (\ln x)^2 + qx^2 \ln x + rx^2$ where p, q and r are non-zero rational numbers, and an intention to subtract Do not condone $F(1) - F(e)$ $\pi \left[\frac{e^2}{4} - \frac{1}{4} \right]$ etc
	Total		11	
	TOTAL		75	