Version 1.0

General Certificate of Education
June 2010

Mathematics MPC3

Pure Core 3

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

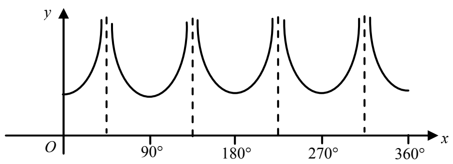
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1(a)	$\mathrm{f}(x)=3^x-10+x^3$ (or reverse) $\mathrm{f}(1)=-6$ $\mathrm{f}(2)=7$	M1		Attempt to evaluate f(1) and f(2)
	Change of sign $\therefore 1<\alpha<2$	A1	2	All working must be correct plus statement
	OR			
	LHS $(1)=3$ RHS $(1)=9$ LHS $(2)=9$ RHS $(2)=2$	(M1)		Must be these values
	At 1 LHS < RHS, At 2 LHS > RHS $\therefore 1<\alpha<2$	(A1)		
(b)(i)	$3^x=10-x^3$ $x^3=10-3^x$ $x=\sqrt[3]{10-3^x}$	B1	1	This line must be seen AG
(ii)	$(x_1=1)$ $x_2=1.913$	M1		Sight of AWRT 1.9 or AWRT 1.2
	$x_3=1.221$	A1	2	Both values correct
	Total		**5**	

MPC3

Q	Solution	Marks	Total	Comments
2(a)(i)	$(y=)\ 1$	B1	1	Condone 1 marked at A, $A=1$ etc but not $\frac{1}{\cos 0}$, sec 0
(ii)		M1		Modulus graph $y>0$
		A1		$3+2\times\frac{1}{2}$ sections roughly as shown, condone sections touching, variable minimum heights
		A1	3	Correct graph with correct behaviour at 4 asymptotes but need not show broken lines; and roughly same minima
(b)	$\cos x=\frac{1}{2}$ or $\cos^{-1}\frac{1}{2}$ seen	M1		or sight of $\pm 60^\circ$ or $\pm\frac{\pi}{3}$, ± 1.05 (AWRT)
	$x=60^\circ, 300^\circ$	A1	2	Condone extra values outside $0^\circ < x < 360^\circ$, but no extras in interval
(c)	$\sec(2x-10^\circ)=2$, $\sec(2x-10^\circ)=-2$ $\cos(2x-10^\circ)=\frac{1}{2}$ or $\cos(2x-10^\circ)=-\frac{1}{2}$	M1		Either of these, PI by further working
	$2x-10^\circ=60^\circ, 300^\circ$ or $2x-10^\circ=120^\circ, 240^\circ$ (ignore values outside $0^\circ < x < 360^\circ$)	A1		Both correct values from one equation or 2 correct values and no wrong values from both equations, but must have "$2x-10^\circ=$" PI by $2x=70^\circ, 130^\circ, 250^\circ, 310^\circ$
	$x=35^\circ, 65^\circ, 125^\circ, 155^\circ$	B1		3 correct (and not more than 1 extra value in $0^\circ < x < 180^\circ$)
		B1	4	All 4 correct (and no extras in interval)
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$y=\ln(5x-2)$			
		M1		$\frac{k}{5x-2}$
	$\left(\frac{dy}{dx}=\right)\ \frac{5}{5x-2}$	A1	2	No ISW, eg $\frac{5}{5x-2}=\frac{1}{x-2}$ (M1A0)
(ii)	$y=\sin 2x$			
		M1		$k\cos 2x$
	$\left(\frac{dy}{dx}=\right)\ 2\cos 2x$	A1	2	
(b)(i)		M1		
	$f(x)\geqslant \ln 0.5$ or $f(x)\geqslant -\ln 2$	A1	2	
(ii)	$(gf(x)=)\quad \sin[2\ln(5x-2)]$ or $(gf(x)=)\quad \sin\ln(5x-2)^2$	B1	1	Condone $\sin 2\ln(5x-2)$ or $\sin 2(\ln(5x-2))$ but **not** $\sin 2(\ln 5x-2)$ or $\sin 2\ln 5x-2$
(iii)	$gf(x)=0$ $\sin[2\ln(5x-2)]=0$			
	$2\ln(5x-2)=0$	M1		Correct first step from their (b)(ii)
	$5x-2=1$	m1		Their $f(x)=1$ from $k\ln(f(x))=0$
	$x=\frac{3}{5}$	A1	3	Withhold if clear error seen other than omission of brackets
(iv)	$x=\sin 2y$			
	$\sin^{-1}x=2y\quad(\text{or } \sin^{-1}y=2x)$	M1		Correct equation involving $\sin^{-1}$
	$(g^{-1}(x)=)\ \frac{1}{2}\sin^{-1}x$	A1	2	
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	<table><tr><th>x</th><th>y</th></tr><tr><td>0.5</td><td>$\frac{4}{9}=0.\dot{4}$</td></tr><tr><td>0.75</td><td>$\frac{48}{91}=0.5275$</td></tr><tr><td>1</td><td>$\frac{1}{2}=0.5$</td></tr><tr><td>1.25</td><td>$\frac{80}{189}=0.4233$</td></tr><tr><td>1.5</td><td>$\frac{12}{35}=0.3429$</td></tr><tr><td>1.75</td><td>$\frac{112}{407}=0.2752$</td></tr><tr><td>2</td><td>$\frac{2}{9}=0.\dot{2}$</td></tr></table>	B1 B1		x values correct PI At least 5 y values that would be correct to 2sf or better, or exact values. May be seen within working.
	$\left[\left(\frac{4}{9}+\frac{2}{9}\right)+4\left(\frac{48}{91}+\frac{80}{189}+\frac{112}{407}\right)+2\left(\frac{1}{2}+\frac{12}{35}\right)\right]$	M1		Clear attempt to use 'their' y values within Simpson's rule
	$\int=\frac{1}{3}\times 0.25[\ \ ]$			
	$=0.605$	A1	4	Answer must be 0.605 with no extra sf (Note 0.605 with no evidence of Simpson's rule scores 0/4)
(b)	$\int_0^1 \frac{x^2}{1+x^3}\,\mathrm{d}x$			
	$=\frac{1}{3}\ln\left(1+x^3\right)$	M1 A1		$k\ln\left(1+x^3\right)$ condone missing brackets Correct. A1 may be recovered for missing brackets if implied later
	$=\frac{1}{3}\ln(1+1)\ \left(-\frac{1}{3}\ln 1\right)$	m1		F(1) (– F(0))
	$=\frac{1}{3}\ln 2$	A1	4	ln 1 must not be left in final answer
	Alternative $u=1+x^3 \quad \mathrm{d}u=3x^2\mathrm{d}x$			
	$\int=\int\frac{\mathrm{d}u}{3u}$	(M1)		$\frac{\mathrm{d}u}{\mathrm{d}x}$ correct and integral of form $k\int\frac{\mathrm{d}u}{u}$
	$=\frac{1}{3}[\ln u]$	(A1)		
	$=\frac{1}{3}\ln 2\ \left(-\frac{1}{3}\ln 1\right)$	(m1)		Correct substitution of correct u limits or conversion back to x and F(1) (– F(0))
	$=\frac{1}{3}\ln 2$	(A1)		ln 1 must not be left in final answer
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$10\operatorname{cosec}^2 x = 16 - 11\cot x$ $10(1+\cot^2 x) = 16 - 11\cot x$ $10\cot^2 x + 11\cot x - 6 = 0$	B1	1	**AG** Must see evidence of correct identity and no errors.
(b)	Attempt at factors, giving $\pm 10\cot^2 x \pm 6$ when expanded.	M1		Use of formula: condone one error
	$(5\cot x - 2)(2\cot x + 3) \quad (=0)$	A1		Correct factors
	$\left(\cot x = \frac{2}{5}, -\frac{3}{2}\right)$ $\tan x = \frac{5}{2}, -\frac{2}{3}$	A1,A1	4	1st A1 must be earned Condone AWRT –0.67 ISW if x values attempted
	Alternative 1 $10\cot^2 x + 11\cot x - 6 = 0$ $10\frac{\cos^2 x}{\sin^2 x} + 11\frac{\cos x}{\sin x} - 6 = 0$ $10\cos^2 x + 11\cos x\sin x - 6\sin^2 x = 0$ $(5\cos x - 2\sin x)(2\cos x + 3\sin x) \quad (=0)$	(M1) (A1)		Attempt at factors, gives $\pm 10\cos^2 x \pm 6\sin^2 x$ when explained As above
	$(5\cos x = 2\sin x \qquad 2\cos x = -3\sin x)$ $\frac{5}{2} = \tan x \qquad -\frac{2}{3} = \tan x$	(A1), (A1)		1st A1 must be earned Condone AWRT –0.67 ISW if x values attempted
	Alternative 2 $10 + 11\tan x - 6\tan^2 x = 0$ $(5 - 2\tan x)(2 + 3\tan x) \quad (=0)$	(M1) (A1)		Attempt at factors gives $\pm 10 \pm 6\tan^2 x$
	$\tan x = \frac{5}{2}, -\frac{2}{3}$	(A1), (A1)		1st A1 must be earned Condone AWRT –0.67 ISW if x values attempted
	Total		**5**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$y=\dfrac{\ln x}{x}$			
	(when) $y=0 \quad x=1 \quad$ or $\quad (1, 0)$	B1	1	Both coordinates must be stated, not 1 simply shown on diagram
(b)	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}=\right) \dfrac{x\times\frac{1}{x}-\ln x}{x^2}$	M1		Quotient/product rule $\dfrac{\pm\frac{x}{x}\pm\ln x}{x^2}$
	$=\dfrac{1-\ln x}{x^2}$ or $x^{-2}-x^{-2}\ln x$	A1		OE must simplify $\dfrac{x}{x}$
	At B, $\dfrac{1-\ln x}{x^2}=0$	m1		Putting their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ or numerator = 0
	$x=\mathrm{e}$	A1		CSO condone $x=\mathrm{e}^1$
	$y=\dfrac{1}{\mathrm{e}}$ or e^{-1}	A1	5	CSO must simplify ln e
(c)	Gradient at $x=\mathrm{e}^3$			
	$=\dfrac{1-\ln\mathrm{e}^3}{(\mathrm{e}^3)^2}$	M1		Substituting $x=\mathrm{e}^3$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ (condone 1 slip) but must have scored M1 in (b)
	$=\dfrac{-2}{\mathrm{e}^6}$ or $-2\mathrm{e}^{-6}$	A1		PI
	Gradient of normal $=\dfrac{1}{2}\mathrm{e}^6$	A1	3	CSO simplified to this
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$\int x\cos 4x \, \mathrm{d}x \quad u = x \quad \frac{\mathrm{d}v}{\mathrm{d}x} = \cos 4x$	M1		$\int \cos 4x, \ \frac{\mathrm{d}}{\mathrm{d}x}(x)$ attempted
	$\frac{\mathrm{d}u}{\mathrm{d}x} = 1 \quad v = \frac{\sin 4x}{4}$	A1		All correct
	$\int = x\frac{\sin 4x}{4} - \int \frac{\sin 4x}{4} \, \mathrm{d}x$	m1		Correct substitution of their terms into parts formula
	$= \frac{x\sin 4x}{4} + \frac{\cos 4x}{16} \ (+c)$	A1	4	OE with fractions unsimplified
(ii)	$\int x^2 \sin 4x \, \mathrm{d}x \quad u = x^2 \quad \frac{\mathrm{d}v}{\mathrm{d}x} = \sin 4x$ $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x \quad v = -\frac{\cos 4x}{4}$	M1		$\int \sin 4x, \ \frac{\mathrm{d}}{\mathrm{d}x}(x^2)$ attempted
	$\int = \frac{-x^2 \cos 4x}{4} - \int \frac{-2x\cos 4x}{4} \, \mathrm{d}x$	A1		
	$= \frac{-x^2 \cos 4x}{4} + \frac{1}{2}\int x\cos 4x \, \mathrm{d}x$			
	$= \frac{-x^2 \cos 4x}{4} + \frac{1}{2}[\ ]$ $\left[\frac{x\sin 4x}{4} + \frac{\cos 4x}{16}\right]$	m1		Clear attempt to replace integral using their answer from part (a)(i)
	$= \frac{-x^2 \cos 4x}{4} + \frac{x\sin 4x}{8} + \frac{\cos 4x}{32} \ (+c)$	A1	4	OE with fractions unsimplified
(b)	$V = (\pi)\int_{(0)}^{(0.2)} (64) x^2 \sin 4x \, (\mathrm{d}x)$	M1		
	$= (\pi \times 64)\left[\frac{-x^2 \cos 4x}{4} + \frac{x\sin 4x}{8} + \frac{\cos 4x}{32}\right]$			Must see evidence of their (a)(ii) result or starting again obtaining 3 terms of the form $\pm Ax^2 \cos 4x \pm Bx\sin 4x \pm C\cos 4x$
	$= \pi[2.09529 - 2]$	m1		**AND** F(0.2) – F(0) attempted
	$= 0.299$ AWRT	A1	3	Accept AWRT 0.0953π
	Total		**11**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$y = e^x \to e^{2x} - 1$			
	Stretch (I)			
	scale factor $\frac{1}{2}$ (II)	M1		I + (II or III)
	in x-direction (III)	A1		I + II + III
	Translation	E1		Allow "translate"
	$\begin{bmatrix} 0 \\ -1 \end{bmatrix}$	B1	4	OE "1 unit down" etc
(b)	$x = 0 \quad y = 6$ or $(0, 6)$	B1	1	Both coordinates must be stated, not simply 6 marked on diagram
(c)(i)	$e^{2x} - 1 = 4e^{-2x} + 2$			
	$e^{4x} - e^{2x} = 4 + 2e^{2x}$ or $\left(e^{2x}\right)^2 - e^{2x} = 4 + 2e^{2x}$	M1		Multiplying both sides by e^{2x}
	$(e^{2x})^2 - 3e^{2x} - 4 = 0$	A1	2	AG With no errors seen
(ii)	$\left(e^{2x} - 4\right)\left(e^{2x} + 1\right)$	M1		$\left(e^{2x} \pm 4\right)\left(e^{2x} \pm 1\right)$
	$x = \ln 2$ or $\frac{1}{2}\ln 4$	A1		
	Reject $e^{2x} = -1$ OE	A1	3	eg $e^{2x} > 0$, $e^{2x} \neq -1$, impossible etc
(d)	$\int\left(4e^{-2x} + 2\right)\, dx$ (I)			
	$= \left[\frac{4e^{-2x}}{-2} + 2x\right]_0^{\ln 2}$	M1		I or II attempted and e^{-2x} or e^{2x} integrated correctly
	$= \left(\frac{4e^{-2\ln 2}}{-2} + 2\ln 2\right) - \left(\frac{4}{-2} + 0\right)$	m1		F['their $\ln 2$' from (c)(ii)] $-$ F[0]
	$= -\frac{1}{2} + 2\ln 2 + +2 = \frac{3}{2} + 2\ln 2$			
	$\int\left(e^{2x} - 1\right)\, dx$ (II)			
	$= \left[\frac{e^{2x}}{2} - x\right]_0^{\ln 2}$	A1		Both I and II correctly integrated
	$= \left(\frac{e^{2\ln 2}}{2} - \ln 2\right) - \left(\frac{1}{2} - 0\right)$			
	$= 2 - \ln 2 - \frac{1}{2} = \frac{3}{2} - \ln 2$			
	$A = \left(\frac{3}{2} + 2\ln 2\right) - \left(\frac{3}{2} - \ln 2\right)$	B1√		Attempt to find difference of 'their I – their II'
	$= 3\ln 2$ or $\ln 8$ or $\frac{3}{2}\ln 4$ OE	A1	5	CSO must be exact

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(d)	**Alternative**			
	$A=\int\left(4\mathrm{e}^{-2x}+2\right)\,\mathrm{d}x-\int\left(\mathrm{e}^{2x}-1\right)\,\mathrm{d}x$	(B1)		Condone functions reversed
	$=\int_{(0)}^{(\ln 2)}\left(4\mathrm{e}^{-2x}-\mathrm{e}^{2x}+3\right)\,\mathrm{d}x$			
	$=\left[\dfrac{4\mathrm{e}^{-2x}}{-2}-\dfrac{\mathrm{e}^{2x}}{2}+3x\right]_0^{\ln 2}$	(M1) (A1)		e^{2x} or e^{-2x} correctly integrated
	$=\left(-2\mathrm{e}^{-2\ln 2}-\dfrac{1}{2}\mathrm{e}^{2\ln 2}+3\ln 2\right)-\left(-2-\dfrac{1}{2}\right)$	(m1)		Correct substitution of their $\ln 2$ from (c)(ii) into their integrated expression
	$=3\ln 2$ or $\ln 8$ or $\dfrac{3}{2}\ln 4$ OE	(A1)		CSO must be exact
	Total		**15**	
	TOTAL		**75**	