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General Certificate of Education (A-level)
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Mathematics MPC3

(Specification 6360)

Pure Core 3

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

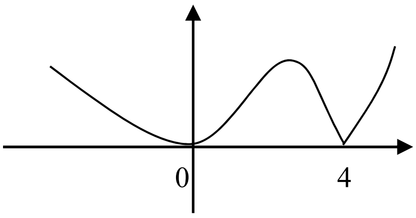
Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1(a)	$f(2) = -3$ $f(3) = 10$	M1		$f(x) = x^3 - 6x + 1$ must have both values correct allow $f(2) < 0$ and $f(3) > 0$ only if $f(x)$ is defined and no errors seen
	change of sign $\Rightarrow 2 < \alpha < 3$	A1	2	must have **both statement and interval** which may be written in words/symbols
(b)	$x^3 = 6x - 1$ or $x^2 - 6 + \frac{1}{x} = 0$ or $x^2 - 6 = -\frac{1}{x}$			must see one of these lines and no errors
	$x^2 = 6 - \frac{1}{x}$	B1	1	AG
(c)	$x_2 = \sqrt{6 - \frac{1}{2.5}} = 2.366(432)$	B1		at least 4sf needed PI by correct x_3
	$x_3 = 2.362$	B1	2	SC1 if B0B0 scored and x_3 = 2.3617
	Total		**5**	

Q	Solution	Marks	Total	Comments
2(a)	$y(0)=0$			
	$y(1)=\frac{1}{3}=0.\dot{3}$			
	$y(2)=\frac{1}{3}=0.\dot{3}$	B1		all 5 x-values PI by 5 correct y-values
	$y(3)=\frac{3}{11}=0.\dot{2}\dot{7}$			
	$y(4)=\frac{4}{18}=0.\dot{2}$	B1		at least 4 y-values exact or rounded or truncated to at least 4sf
	$\frac{1}{3}\times 1\left(0+0.\dot{2}+4\left[0.\dot{3}+0.\dot{2}\dot{7}\right]+2\left[0.\dot{3}\right]\right)$	M1		correct use of Simpson's rule using $\frac{1}{3}$ and 4 and 2 correctly with candidate's 5 y-values
	$=1.104$	A1	4	CAO (must be exactly this value)
(b)	$\int_0^4 \frac{x}{x^2+2}\,\mathrm{d}x=\frac{1}{2}\left[\ln\left(x^2+2\right)\right]$	M1 A1		for $k\ln\left(x^2+2\right)$ all correct; limits not needed
	$=\frac{1}{2}(\ln 18-\ln 2)$	A1F		For k (ln18 – ln2)
	$=\frac{1}{2}\ln 9$	A1F		combining candidate's logarithms correctly (must be seen)
	$=\ln 3$	A1	5	CAO (must be exactly this) NMS scores 0/5
	Total		**9**	

Q	Solution	Marks	Total	Comments
3(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)3\mathrm{e}^{3x}+\frac{1}{x}$	B1 B1	2	B1 for one term correct B1 all correct
(b)(i)	$\left(\frac{\mathrm{d}u}{\mathrm{d}x}=\right)\frac{\pm\cos x(1+\cos x)\pm\sin x(\sin x)}{(1+\cos x)^2}$	M1		clear attempt at quotient/product rule condone poor use of brackets
	$\frac{\cos x(1+\cos x)-\sin x(-\sin x)}{(1+\cos x)^2}$	A1		any correct form seen
	$=\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$			
	$=\frac{\cos x+1}{(1+\cos x)^2}$			
	$=\frac{1}{1+\cos x}$	A1cso	3	AG be convinced correct use of brackets and correct notation used throughout (eg A0 if $\cos x^2$ etc seen)
(ii)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{1+\cos x}{\sin x}\times\frac{1}{1+\cos x}$ OE	M1		correct use of chain rule
	$=\frac{1}{\sin x}$			
	$=\operatorname{cosec} x$	A1	2	AG, must see $=\frac{1}{\sin x}$ and no errors seen; condone incorrect use of brackets only if penalised in part (b)(i)
	Total		**7**	

Q	Solution	Marks	Total	Comments
4(a)		M1		reflection in the x-axis for the negative $\mathrm{f}(x)$ and remainder as given on sketch
		A1	2	correct curvatures, correct cusp at $x = 4$ condone straight lines for $x < 0$ and $x > 4$ 4 marked on x-axis
(b)	**Either** 1. Stretch	M1		1 and either 2 or 3
	2. ‖ x-axis			
	3. by factor 0.5	A1		1, 2 and 3
	(followed by) translation	E1		
	$\begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$	B1	4	
	or			
	translation	(E1)		
	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	(B1)		
	(followed by) 1. Stretch	(M1)		1 and either 2 or 3
	2. ‖ x-axis			
	3. by factor 0.5	(A1)		1, 2 and 3
	Total		**6**	

Q	Solution	Marks	Total	Comments
5(a)		M1		$\mathrm{f}(x) > -\frac{4}{3}$, $\mathrm{f} \geq -\frac{4}{3}$, range $\geq -\frac{4}{3}$
	$\mathrm{f}(x) \geq -\frac{4}{3}$	A1	2	
(b)(i)	$x \geq -\frac{4}{3}$	B1F	1	correct or FT from **(a)**
(ii)	$x^2 = 3y + 4$			
	$x = (\pm)\sqrt{3y+4}$	M1		either order – M1 for correctly changing the subject or reversing operations; M1 for replacing y with x
	$\left(\mathrm{f}^{-1}(x) =\right)(-)\sqrt{3x+4}$	M1		
	$\left(\mathrm{f}^{-1}(x) =\right) -\sqrt{3x+4}$	A1	3	(dependent on both M1 marks) correct sign
(c)(i)	$3x - 1 = 1$	M1		Or $3x - 1 = \mathrm{e}^0$ or $3x - 1 = \pm 1$
	$\frac{2}{3}$ OE	A1	2	CAO, NMS $\frac{2}{3}$ OE scores 2/2
(ii)	g has **NO** inverse because two values of x map to one value (of y) **or** it is many-one **or** it is not one-one **or** 'it is two-one'	B1	1	must indicate no inverse with valid reason; do not accept contradictory reasons
(iii)	$\ln\left\|3 \times \frac{x^2 - 4}{3} - 1\right\|$	M1		
	$\ln\left\|x^2 - 5\right\|$	A1	2	NMS scores 0/2, condone $k = -5$ after correct expression seen
(iv)	$\ln\left\|x^2 - 5\right\| = 0$			
	$\left\|x^2 - 5\right\| = 1$			
	$x^2 - 5 = 1$ (or -1 or e^0 or $-\mathrm{e}^0$ seen)	M1		$x^2 - k = 1$ etc, for candidate's positive integer, k
	$x^2 = 6, 4$ or candidate's $k + 1$ or $k - 1$			
	$x = \sqrt{6}$, 2	A1F		exact values PI by correct answers
	$x = -\sqrt{6}, -2$	A1F		
	$(x \leq 0 \Rightarrow)$ $x = -\sqrt{6}, -2$	A1	4	CAO, rejecting the positive
	Total		**15**	

Q	Solution	Marks	Total	Comments
6(a)	$\dfrac{\sec^2 x}{(\sec x+1)(\sec x-1)} = \dfrac{\sec^2 x}{\sec^2 x-1}$			
	$\sec^2 x = 1+\tan^2 x$ used	M1		M1 for correct use of $\sec^2 x = 1+\tan^2 x$ at least once or $\left(\text{cosec}^2 x = 1+\cot^2 x\right)$
	$= \dfrac{\sec^2 x}{\tan^2 x}$ or $\dfrac{1+\tan^2 x}{\tan^2 x}$			$\left(= \dfrac{1}{\cos^2 x \tan^2 x}\right)$
	$= \dfrac{1}{\sin^2 x}$ or $\cot^2 x+1$	A1		Shown, with no errors
	$= \text{cosec}^2 x$	A1	3	AG (No errors, omissions or poor notations seen)
(b)	$\text{cosec}^2 x = \text{cosec}\, x + 3$ $\text{cosec}^2 x - \text{cosec}\, x - 3 = 0$	B1		must have = 0
	$\text{cosec}\, x = \dfrac{1\pm\sqrt{13}}{2}$ or (2.3... **and** – 1.3...)	M1		correct solution of the quadratic, or by completing the square $\left(\text{cosec}\, x = \pm\sqrt{\dfrac{13}{4}}+\dfrac{1}{2}\right)$ PI by values for sin x
	$\sin x = \dfrac{2}{1\pm\sqrt{13}}$	B1F		B1F for $\text{cosec}\, x = \dfrac{1}{\sin x}$ seen or implied
	$= 0.434$ **and** -0.768 $\left(\text{or } -0.767\right)$	A1		PI
	$x = 26°, 154°, -50°, -130°$	B1 B1	6	B1 for any three values correct AWRT B1 for all four values correct AWRT and no extras in the interval $-180° < x < 180°$
(c)	$2\theta - 60° = x$	M1		where x is a written value from candidate's **(b)** in degrees PI by their answer
	$\theta = 43°, 5°$	A1	2	CSO Ignore solutions outside interval $0° < \theta < 90°$
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	$y = 4x\cos 2x$			
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 4\cos 2x - 4x(2)\sin 2x$	M1		anything reducible to $A\cos 2x + Bx\sin 2x$ where A and B are non-zero integers
		A1		OE, all correct
	gradient of the tangent $A\cos\frac{2\pi}{4} + B\times\frac{\pi}{4}\sin\frac{2\pi}{4}$	m1		substituting $\frac{\pi}{4}$ into candidate's derived function
	$= -2\pi$	A1		must have -2π using correct $\frac{\mathrm{d}y}{\mathrm{d}x}$
	an equation of the tangent is $y = -2\pi\left(x - \frac{\pi}{4}\right)$	A1	5	OE, dependent on previous A1
(b)				$\left(\int_0^{\frac{\pi}{4}} 4x\cos 2x\,\mathrm{d}x\right)$
	$u = Ax \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \cos 2x$	M1		all 4 terms in this form seen or used
	$\frac{\mathrm{d}u}{\mathrm{d}x} = A \qquad v = B\sin 2x$	A1		$A = 4$ and $B = \frac{1}{2}$ **or** $A = 1$ and $B = 2$, etc
	$= \left[4x\frac{1}{2}\sin 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)} - \int_{(0)}^{\left(\frac{\pi}{4}\right)} 4\times\frac{1}{2}\sin 2x(\mathrm{d}x)$	m1		correct substitution of candidate's terms into integration by parts formula condone missing limits
	$= \left[4x\frac{1}{2}\sin 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)} - \left[-\cos 2x\right]_{(0)}^{\left(\frac{\pi}{4}\right)}$	A1F		candidate's second integration completed correctly FT on one error including coefficient condone missing limits
	$= \frac{\pi}{2} - 1$	A1	5	OE, exact value
	Total		**10**	

Q	Solution	Marks	Total	Comments
8(a)	$\int e^{1-2x}dx = ke^{1-2x}$ or $e\left(ke^{-2x}\right)$	M1		where k is a rational number
	$\int_0^{\ln 2} e^{1-2x}dx = -\frac{1}{2}e^{1-2x}\Big\|_0^{\ln 2}$ or $e\left[-\frac{1}{2}e^{-2x}\right]_0^{\ln 2}$	A1		correct integration condone missing limits
	$= -\frac{1}{2}e^{1-2\ln 2} - -\frac{1}{2}e^{1-2(0)}$	A1		correct (no decimals)
	$= -\frac{1}{2}\left(\frac{1}{4}e\right) + \frac{1}{2}e$			eliminating ln
	$= \frac{3}{8}e$	A1	4	AG, be convinced
(b)	$u = \tan x$			
	$\frac{du}{dx} = \sec^2 x$	M1		PI below, condone $du = \sec^2 x\, dx$
	Replacing dx by $\frac{1}{\sec^2 x}(du)$ in integral	A1		or $\frac{1}{1+u^2}(du)$
	$\sec^2 x = 1 + u^2$	B1		PI below
	$x = 0 \Rightarrow u = 0$ $x = \frac{\pi}{4} \Rightarrow u = 1$	B1		this could be gained by changing u to $\tan x$ after the integration and using $x = 0$ and $x = \frac{\pi}{4}$
	$\int_0^{\frac{\pi}{4}} \sec^4 x\sqrt{\tan x}\, dx$			
	$= \int\left(1+u^2\right)\sqrt{u}\,(du)$ or $\int\left(1+u^2\right)^2\sqrt{u}\,\frac{(du)}{1+u^2}$	M1		all in terms of u including replacing dx all correct, condone omission of du
	$= \int\left(u^{\frac{5}{2}} + u^{\frac{1}{2}}\right)(du)$	A1		must be in this form
	$= \frac{2}{7}u^{\frac{7}{2}} + \frac{2}{3}u^{\frac{3}{2}}$	A1		accept correct unsimplified form
	$= \frac{20}{21}$	A1	8	CAO
	Total		**12**	
	TOTAL		**75**	