

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
June 2012

Mathematics

MPC3

Unit Pure Core 3

Thursday 31 May 2012 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	



J U N 1 2 M P C 3 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

- 1** Use the mid-ordinate rule with four strips to find an estimate for $\int_{0.4}^{1.2} \cot(x^2) \, dx$,
giving your answer to three decimal places. (4 marks)

QUESTION
PART
REFERENCE

Answer space for question 1



QUESTION
PART
REFERENCE**Answer space for question 1**

2 For $0 < x \leq 2$, the curves with equations $y = 4 \ln x$ and $y = \sqrt{x}$ intersect at a single point where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.5. (2 marks)

(b) Show that the equation $4 \ln x = \sqrt{x}$ can be rearranged into the form

$$x = e^{\left(\frac{\sqrt{x}}{4}\right)} \quad (1 \text{ mark})$$

(c) Use the iterative formula

$$x_{n+1} = e^{\left(\frac{\sqrt{x_n}}{4}\right)}$$

with $x_1 = 0.5$ to find the values of x_2 and x_3 , giving your answers to three decimal places. (2 marks)

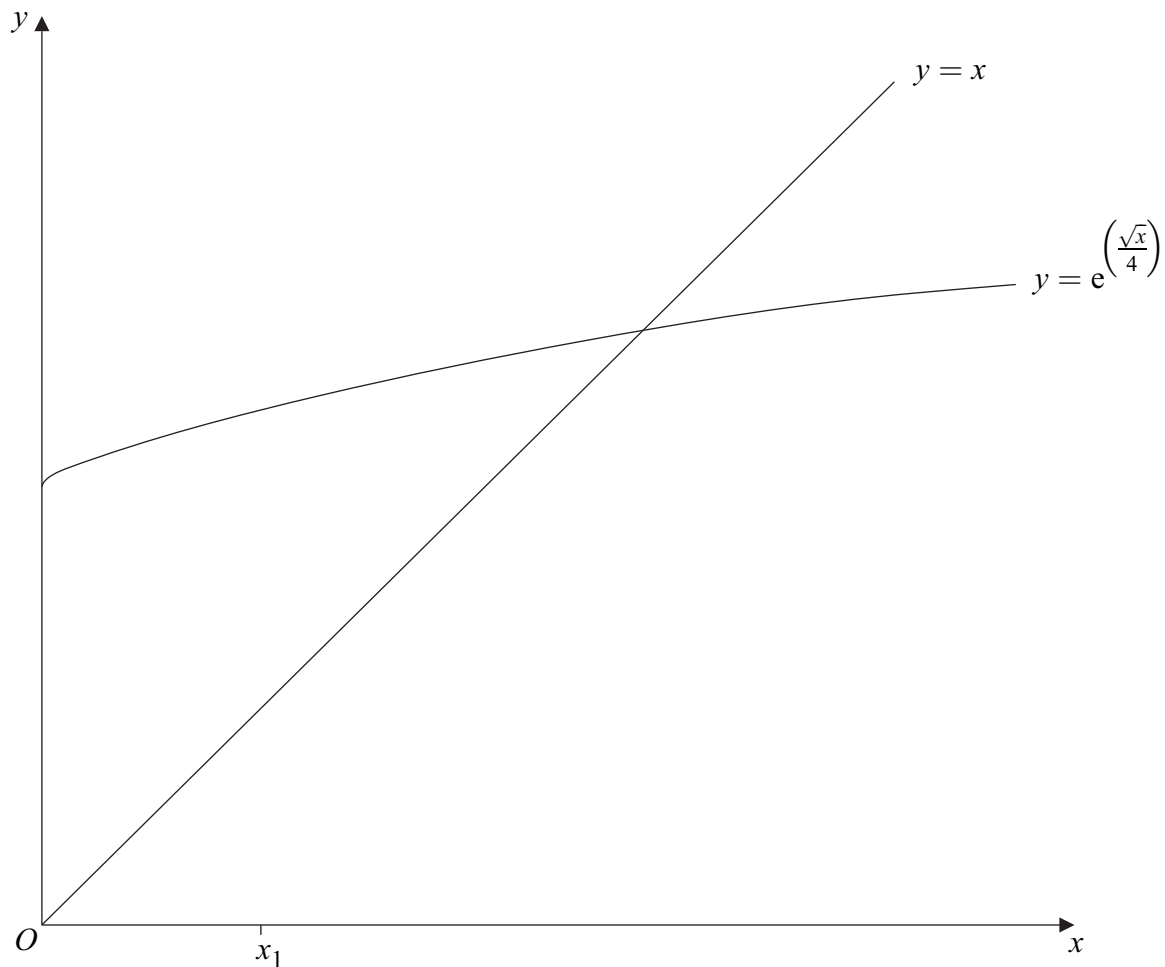
(d) **Figure 1**, on the opposite page, shows a sketch of parts of the graphs of $y = e^{\left(\frac{\sqrt{x}}{4}\right)}$ and $y = x$, and the position of x_1 .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x -axis. (2 marks)

QUESTION
PART
REFERENCE

Answer space for question 2



QUESTION
PART
REFERENCE**Answer space for question 2****(d)****Figure 1****Turn over ►**

3 A curve has equation $y = x^3 \ln x$.

(a) Find $\frac{dy}{dx}$. (2 marks)

(b) (i) Find an equation of the tangent to the curve $y = x^3 \ln x$ at the point on the curve where $x = e$. (3 marks)

(ii) This tangent intersects the x -axis at the point A . Find the exact value of the x -coordinate of the point A . (2 marks)

QUESTION
PART
REFERENCE

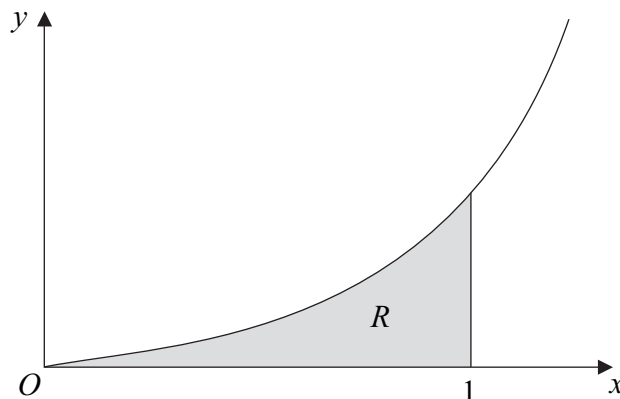
Answer space for question 3



QUESTION
PART
REFERENCE**Answer space for question 3****Turn over ►**

4 (a) By using integration by parts, find $\int x e^{6x} dx$. (4 marks)

(b) The diagram shows part of the curve with equation $y = \sqrt{x} e^{3x}$.



The shaded region R is bounded by the curve $y = \sqrt{x} e^{3x}$, the line $x = 1$ and the x -axis from $x = 0$ to $x = 1$.

Find the volume of the solid generated when the region R is rotated through 360° about the x -axis, giving your answer in the form $\pi(pe^6 + q)$, where p and q are rational numbers. (3 marks)

QUESTION
PART
REFERENCE

Answer space for question 4



QUESTION
PART
REFERENCE

Answer space for question 4

Handwriting practice area with horizontal dotted lines.

Turn over ►



5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{2x - 5}, \quad \text{for } x \geq 2.5$$

$$g(x) = \frac{10}{x}, \quad \text{for real values of } x, \quad x \neq 0$$

(a) State the range of f . (2 marks)

(b) (i) Find $fg(x)$. (1 mark)

(ii) Solve the equation $fg(x) = 5$. (2 marks)

(c) The inverse of f is f^{-1} .

(i) Find $f^{-1}(x)$. (3 marks)

(ii) Solve the equation $f^{-1}(x) = 7$. (2 marks)

QUESTION
PART
REFERENCE

Answer space for question 5



QUESTION
PART
REFERENCE**Answer space for question 5****Turn over ►**

6

Use the substitution $u = x^4 + 2$ to find the value of $\int_0^1 \frac{x^7}{(x^4 + 2)^2} dx$, giving your answer in the form $p \ln q + r$, where p , q and r are rational numbers. (6 marks)

QUESTION
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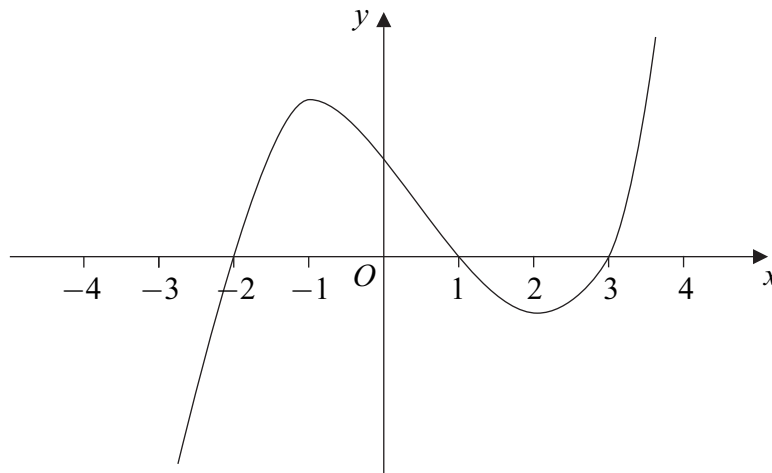
Answer space for question 6



Answer space for question 6

[illegible]

- 7 The sketch shows part of the curve with equation $y = f(x)$.



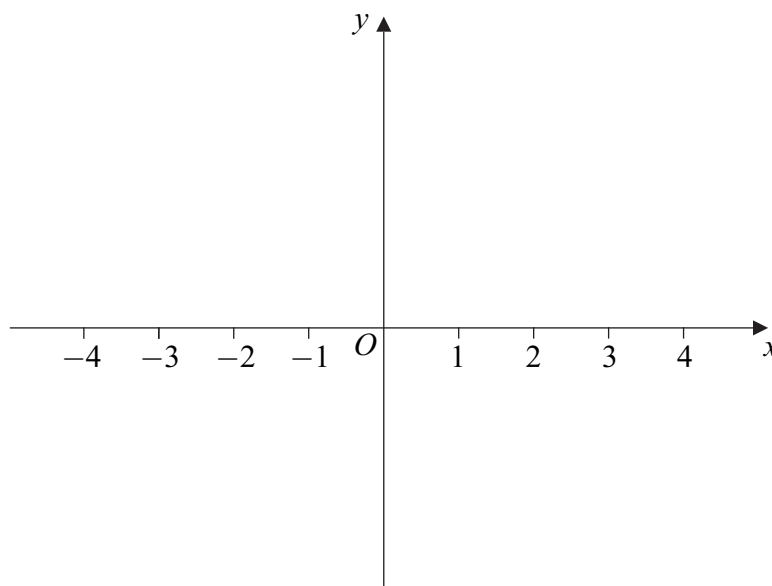
- (a) On **Figure 2** below, sketch the curve with equation $y = |f(x)|$. (3 marks)
- (b) On **Figure 3** opposite, sketch the curve with equation $y = f(|x|)$. (2 marks)
- (c) Describe a sequence of two geometrical transformations that maps the graph of $y = f(x)$ onto the graph of $y = \frac{1}{2}f(x+1)$. (4 marks)
- (d) The maximum point of the curve with equation $y = f(x)$ has coordinates $(-1, 10)$. Find the coordinates of the maximum point of the curve with equation $y = \frac{1}{2}f(x+1)$. (2 marks)

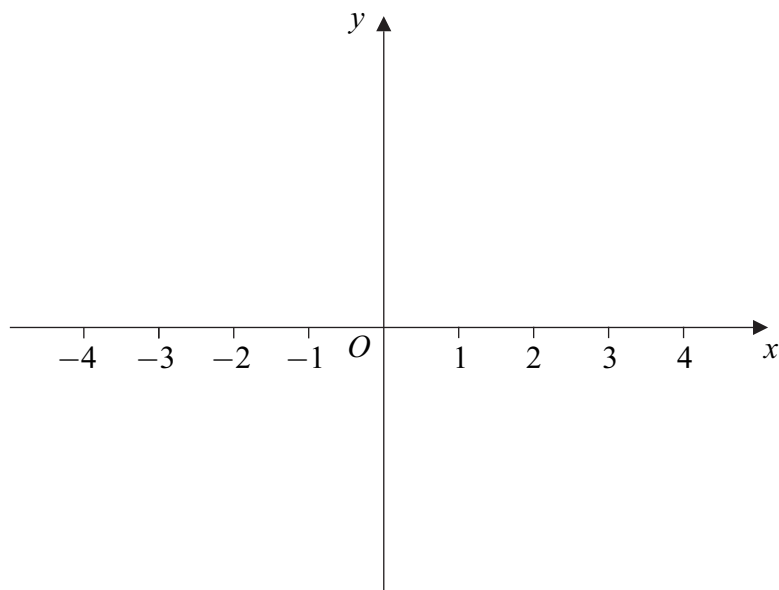
QUESTION
PART
REFERENCE

Answer space for question 7

(a)

Figure 2



QUESTION
PART
REFERENCE**Answer space for question 7****(b)****Figure 3****Turn over ►**

8 (a) Show that the equation

$$\frac{1}{1 + \cos \theta} + \frac{1}{1 - \cos \theta} = 32$$

can be written in the form

$$\operatorname{cosec}^2 \theta = 16 \quad (4 \text{ marks})$$

(b) Hence, or otherwise, solve the equation

$$\frac{1}{1 + \cos(2x - 0.6)} + \frac{1}{1 - \cos(2x - 0.6)} = 32$$

giving all values of x in radians to two decimal places in the interval $0 < x < \pi$.
(5 marks)

QUESTION
PART
REFERENCE

Answer space for question 8



Answer space for question 8

[illegible]

- 9 (a)** Given that $x = \frac{\sin y}{\cos y}$, use the quotient rule to show that

$$\frac{dx}{dy} = \sec^2 y \quad (3 \text{ marks})$$

- (b)** Given that $\tan y = x - 1$, use a trigonometrical identity to show that

$$\sec^2 y = x^2 - 2x + 2 \quad (2 \text{ marks})$$

- (c)** Show that, if $y = \tan^{-1}(x - 1)$, then

$$\frac{dy}{dx} = \frac{1}{x^2 - 2x + 2} \quad (1 \text{ mark})$$

- (d)** A curve has equation $y = \tan^{-1}(x - 1) - \ln x$.

- (i)** Find the value of the x -coordinate of each of the stationary points of the curve. (4 marks)

- (ii)** Find $\frac{d^2y}{dx^2}$. (2 marks)

- (iii)** Hence show that the curve has a minimum point which lies on the x -axis. (2 marks)

QUESTION
PART
REFERENCE

Answer space for question 9



Answer space for question 9

[illegible]

Answer space for question 9

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