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| For Examiner's Use | |
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| Examiner's Initials | |
| Question | Mark |
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| TOTAL | |

General Certificate of Education
Advanced Level Examination
January 2012

# Mathematics MPC3

## Unit Pure Core 3

**Friday 20 January 2012 1.30 pm to 3.00 pm**

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

**1 (a)** Use Simpson's rule with 7 ordinates (6 strips) to find an estimate for $\int_0^3 4^x \, dx$.

*(4 marks)*

**(b)** A curve is defined by the equation $y = 4^x$. The curve intersects the line $y = 8 - 2x$ at a single point where $x = \alpha$.

**(i)** Show that $\alpha$ lies between 1.2 and 1.3. *(2 marks)*

**(ii)** The equation $4^x = 8 - 2x$ can be rearranged into the form $x = \dfrac{\ln(8 - 2x)}{\ln 4}$.

Use the iterative formula $x_{n+1} = \dfrac{\ln(8 - 2x_n)}{\ln 4}$ with $x_1 = 1.2$ to find the values of $x_2$ and $x_3$, giving your answers to three decimal places. *(2 marks)*

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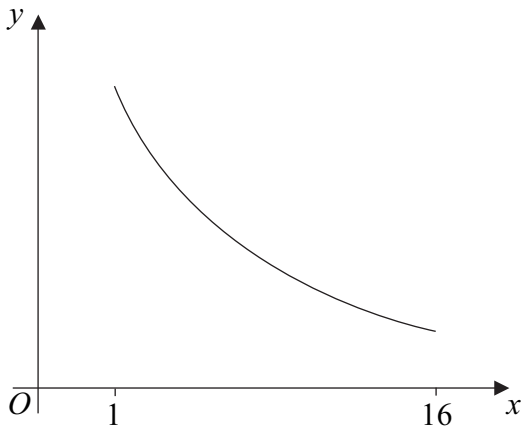
**2** The curve with equation $y = \dfrac{63}{4x - 1}$ is sketched below for $1 \leqslant x \leqslant 16$.




The function f is defined by $\mathrm{f}(x) = \dfrac{63}{4x - 1}$ for $1 \leqslant x \leqslant 16$.

**(a)** Find the range of f. *(2 marks)*

**(b)** The inverse of f is $\mathrm{f}^{-1}$.

**(i)** Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

**(ii)** Solve the equation $\mathrm{f}^{-1}(x) = 1$. *(2 marks)*

**(c)** The function g is defined by $\mathrm{g}(x) = x^2$ for $-4 \leqslant x \leqslant -1$.

**(i)** Write down an expression for $\mathrm{fg}(x)$. *(1 mark)*

**(ii)** Solve the equation $\mathrm{fg}(x) = 1$. *(3 marks)*

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**3 (a)** Given that $y = 4x^3 - 6x + 1$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(1 mark)*

**(b)** Hence find $\displaystyle\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1}\,\mathrm{d}x$, giving your answer in the form $p \ln q$, where $p$ and $q$ are rational numbers. *(5 marks)*

QUESTION PART REFERENCE

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**4 (a)** By using a suitable trigonometrical identity, solve the equation

$$\tan^2\theta = 3(3 - \sec\theta)$$

giving all solutions to the nearest 0.1° in the interval $0° < \theta < 360°$. *(6 marks)*

**(b)** Hence solve the equation

$$\tan^2(4x - 10°) = 3[3 - \sec(4x - 10°)]$$

giving all solutions to the nearest 0.1° in the interval $0° < x < 90°$. *(3 marks)*

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**5 (a)** Describe a sequence of two geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4\ln(x - \mathrm{e})$. *(4 marks)*

**(b)** Sketch, on the axes given below, the graph of $y = |4\ln(x - \mathrm{e})|$, indicating the exact value of the $x$-coordinate where the curve meets the $x$-axis. *(3 marks)*

**(c) (i)** Solve the equation $|4\ln(x - \mathrm{e})| = 4$. *(3 marks)*

**(ii)** Hence, or otherwise, solve the inequality $|4\ln(x - \mathrm{e})| \geqslant 4$. *(3 marks)*

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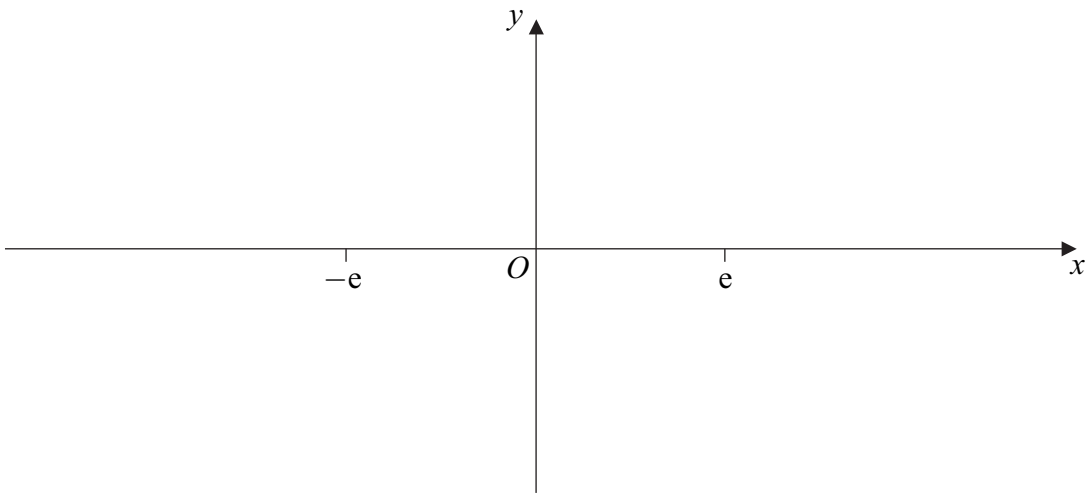
**(b)**



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**6 (a)** Given that $x = \dfrac{1}{\sin\theta}$, use the quotient rule to show that $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = -\operatorname{cosec}\theta\cot\theta$.

*(3 marks)*

**(b)** Use the substitution $x = \operatorname{cosec}\theta$ to find $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,\mathrm{d}x$, giving your answer to three significant figures.

*(9 marks)*

QUESTION PART REFERENCE

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**7 (a)** A curve has equation $y = x^2 e^{-\frac{x}{4}}$.

Show that the curve has exactly two stationary points and find the exact values of their coordinates. *(7 marks)*

**(b) (i)** Use integration by parts twice to find the exact value of $\int_0^4 x^2 e^{-\frac{x}{4}} dx$. *(7 marks)*

**(ii)** The region bounded by the curve $y = 3xe^{-\frac{x}{8}}$, the $x$-axis from 0 to 4 and the line $x = 4$ is rotated through 360° about the $x$-axis to form a solid.

Use your answer to part **(b)(i)** to find the exact value of the volume of the solid generated. *(2 marks)*

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