

General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS
Unit Pure Core 2

MPC2



Monday 21 May 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}};$ (1 mark)

(ii) $x^{\frac{3}{2}} \div x;$ (1 mark)

(iii) $\left(x^{\frac{3}{2}}\right)^2.$ (1 mark)

(b) (i) Find $\int 3x^{\frac{1}{2}} dx.$ (3 marks)

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}} dx.$ (2 marks)

2 The n th term of a geometric sequence is u_n , where

$$u_n = 3 \times 4^n$$

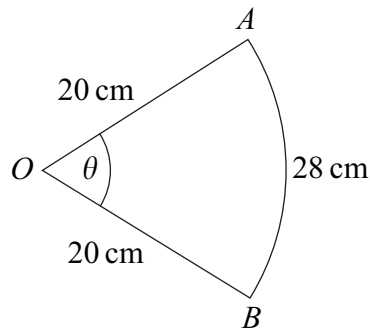
(a) Find the value of u_1 and show that $u_2 = 48.$ (2 marks)

(b) Write down the common ratio of the geometric sequence. (1 mark)

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. (3 marks)

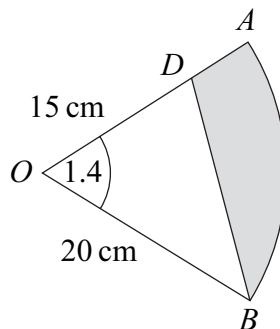
(ii) Hence find the value of $\sum_{n=2}^{12} u_n.$ (1 mark)

- 3 The diagram shows a sector OAB of a circle with centre O and radius 20 cm. The angle between the radii OA and OB is θ radians.



The length of the arc AB is 28 cm.

- Show that $\theta = 1.4$. (2 marks)
- Find the area of the sector OAB . (2 marks)
- The point D lies on OA . The region bounded by the line BD , the line DA and the arc AB is shaded.



The length of OD is 15 cm.

- Find the area of the shaded region, giving your answer to three significant figures. (3 marks)
- Use the cosine rule to calculate the length of BD , giving your answer to three significant figures. (3 marks)

- 4 An arithmetic series has first term a and common difference d .

The sum of the first 29 terms is 1102.

- (a) Show that $a + 14d = 38$. (3 marks)

- (b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d . (4 marks)

- 5 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where $x = 2$.

- (a) Find the y -coordinate of P . (1 mark)

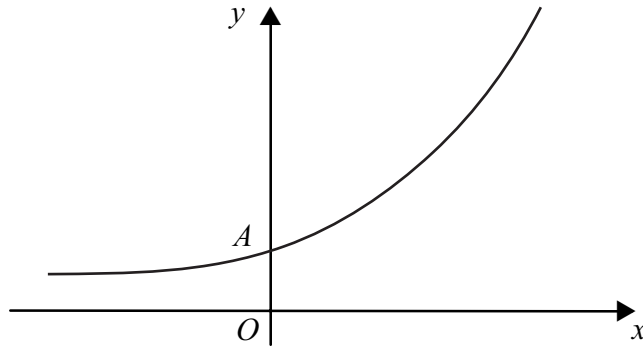
- (b) Expand $\left(1 + \frac{2}{x}\right)^2$. (2 marks)

- (c) Find $\frac{dy}{dx}$. (3 marks)

- (d) Hence show that the gradient of the curve at P is -2 . (2 marks)

- (e) Find the equation of the normal to the curve at P , giving your answer in the form $x + by + c = 0$, where b and c are integers. (4 marks)

- 6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y -axis at the point A .

- (a) Find the y -coordinate of the point A . (2 marks)
- (b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1) dx$. (4 marks)
- (c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P .
- (i) Show that the x -coordinate of P satisfies the equation $2^x = 6$ (1 mark)
- (ii) Use logarithms to find the x -coordinate of P , giving your answer to three significant figures. (3 marks)

Turn over for the next question

Turn over ►

- 7 (a) Sketch the graph of $y = \tan x$ for $0^\circ \leq x \leq 360^\circ$. (3 marks)
- (b) Write down the **two** solutions of the equation $\tan x = \tan 61^\circ$ in the interval $0^\circ \leq x \leq 360^\circ$. (2 marks)
- (c) (i) Given that $\sin \theta + \cos \theta = 0$, show that $\tan \theta = -1$. (1 mark)
- (ii) Hence solve the equation $\sin(x - 20^\circ) + \cos(x - 20^\circ) = 0$ in the interval $0^\circ \leq x \leq 360^\circ$. (4 marks)
- (d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 20^\circ)$. (2 marks)
- (e) The curve $y = \tan x$ is stretched in the x -direction with scale factor $\frac{1}{4}$ to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. (1 mark)

- 8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a (2n - 1)$$

Find the value of n . (3 marks)

- (b) Given that $\log_a x = 3$ and $\log_a y - 3 \log_a 2 = 4$:
- (i) express x in terms of a ; (1 mark)
- (ii) express xy in terms of a . (4 marks)

END OF QUESTIONS

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