

General Certificate of Education
January 2006
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 2

MPC2

Tuesday 10 January 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 Given that $y = 16x + x^{-1}$, find the two values of x for which $\frac{dy}{dx} = 0$. (5 marks)

- 2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{1}{x^2 + 1} dx$$

giving your answer to four significant figures. (4 marks)

- (b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. (1 mark)

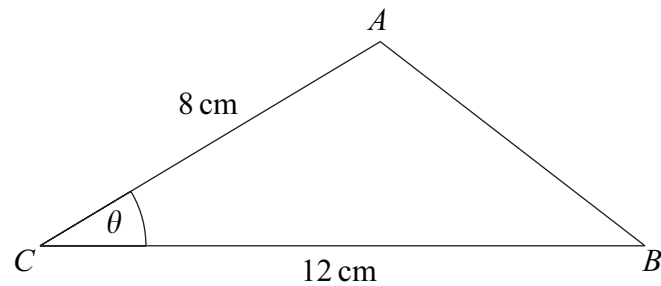
- 3 (a) Use logarithms to solve the equation $0.8^x = 0.05$, giving your answer to three decimal places. (3 marks)

- (b) An infinite geometric series has common ratio r . The sum to infinity of the series is five times the first term of the series.

(i) Show that $r = 0.8$. (3 marks)

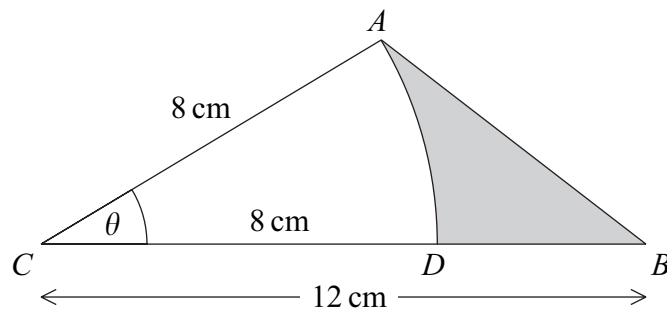
(ii) Given that the first term of the series is 20, find the least value of n such that the n th term of the series is less than 1. (3 marks)

- 4 The triangle ABC , shown in the diagram, is such that $AC = 8$ cm, $CB = 12$ cm and angle $ACB = \theta$ radians.



The area of triangle $ABC = 20$ cm².

- Show that $\theta = 0.430$ correct to three significant figures. (3 marks)
- Use the cosine rule to calculate the length of AB , giving your answer to two significant figures. (3 marks)
- The point D lies on CB such that AD is an arc of a circle centre C and radius 8 cm. The region bounded by the arc AD and the straight lines DB and AB is shaded in the diagram.



Calculate, to two significant figures:

- the length of the arc AD ; (2 marks)
- the area of the shaded region. (3 marks)

- 5 The n th term of a sequence is u_n .

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \quad u_2 = 150 \quad u_3 = 120$$

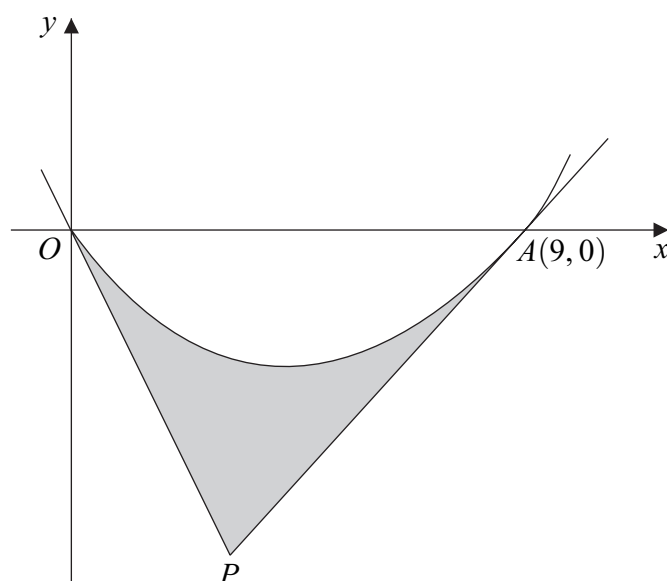
- (a) Show that $p = 0.6$ and find the value of q . (5 marks)
- (b) Find the value of u_4 . (1 mark)
- (c) The limit of u_n as n tends to infinity is L . Write down an equation for L and hence find the value of L . (3 marks)
- 6 (a) Describe the geometrical transformation that maps the curve with equation $y = \sin x$ onto the curve with equation:
- (i) $y = 2 \sin x$; (2 marks)
- (ii) $y = -\sin x$; (2 marks)
- (iii) $y = \sin(x - 30^\circ)$. (2 marks)
- (b) Solve the equation $\sin(\theta - 30^\circ) = 0.7$, giving your answers to the nearest 0.1° in the interval $0^\circ \leq \theta \leq 360^\circ$. (3 marks)
- (c) Prove that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$. (4 marks)

- 7 It is given that n satisfies the equation

$$2 \log_a n - \log_a(5n - 24) = \log_a 4$$

- (a) Show that $n^2 - 20n + 96 = 0$. (3 marks)
- (b) Hence find the possible values of n . (2 marks)

- 8 A curve, drawn from the origin O , crosses the x -axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P , as shown in the diagram.



The curve, defined for $x \geq 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

- (a) Find $\frac{dy}{dx}$. (2 marks)
- (b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O . (2 marks)
- (ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$. (3 marks)
- (iii) Hence find the coordinates of the point P where the two tangents meet. (3 marks)
- (c) Find $\int \left(x^{\frac{3}{2}} - 3x \right) dx$. (3 marks)
- (d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP . (5 marks)

END OF QUESTIONS

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