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General Certificate of Education (A-level)
January 2012

Mathematics MPC2

(Specification 6360)

Pure Core 2

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC2

Q	Solution	Marks	Total	Comments
1(a)	{Area of sector =} $\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times \theta$	M1		$\frac{1}{2}r^2\theta$ seen in **(a)** or used for the area
	21.6 = 18θ so θ = 1.2	A1	2	Must be exact, not rounded to
(b)	{Arc =} $r\theta$	M1		$r\theta$ seen in **(b)** or used for the arc length
	…. = 7.2 {cm}	A1F	2	Ft on 6×c's value for θ provided 4<arc<10.
	Total		**4**	
2(a)	$h = 1$	B1		$h = 1$ stated or used. (PI by x-values 0,1,2,3,4 provided no contradiction)
	$\mathrm{f}(x) = \frac{2^x}{x+1}$ I $\approx$ h/2{…} {.}=f(0)+f(4)+2[f(1)+f(2)+f(3)]	M1		OE summing of areas of the 'trapezia'..
	$\{.\} = 1+\frac{16}{5}+2\left(\frac{2}{2}+\frac{4}{3}+\frac{8}{4}\right)$ =1+3.2+2(1+1.33…+2)	A1		OE Accept 1dp evidence. Can be implied by later correct work provided >1 term or a single term which rounds to 6.43
	(I ≈) 0.5[4.2+2×4.333..] =6.43 (to 3sf)	A1	4	CAO Must be 6.43
(b)	Increase the number of ordinates	E1	1	OE eg increase the number of strips.
	Total		**5**	
3(a)	$\sqrt[4]{x^3} = x^{\frac{3}{4}}$	B1	1	Accept k = ¾ OE
(b)	$\frac{1-x^2}{\sqrt[4]{x^3}} = \frac{1}{\sqrt[4]{x^3}} - \frac{x^2}{\sqrt[4]{x^3}}$ $= x^{-k} - \frac{x^2}{\sqrt[4]{x^3}}$ [or $\frac{1}{\sqrt[4]{x^3}} - x^{2-k}$]	M1		Split followed by at least one correct index law used to remove denominator.
	$= x^{-\frac{3}{4}} - x^{\frac{5}{4}}$	A1F	2	If incorrect, ft on c's non-integer k value answer to part (a), provided M1 has been awarded. Accept answer given in form of values for p and q.
			3	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
4(a)	Area = $\frac{1}{2}\times 10\times AC\sin 150$	M1		$\frac{1}{2}\times 10\times AC\sin 150$
	$40 = 2.5AC$ so $AC = 16$ (m)	A1	2	AG Be convinced
(b)	$\{BC^2 =\} 10^2 + 16^2 - 2\times 10\times 16\times\cos 150$	M1		RHS of cosine rule used
	$= 100 + 256 + 277.128\ldots$	m1		Correct order of evaluation
	$BC = \sqrt{633.128\ldots} = 25.162\ldots = 25.16$m	A1	3	AWRT 25.16
(c)	$\frac{10}{\sin C} = \frac{BC}{\sin 150}$ (or $\frac{BC}{\sin 150} = \frac{AC}{\sin B}$)	M1		A correct equation using sine rule or cosine rule or area formula for either B or C. Subst of BC or AC not required for this M.
	$\sin C = \frac{10\sin 150}{"25.16"}$ (=0.1987..) (or $\sin B = \frac{16\sin 150}{"25.16"}$ (=0.317… or 0.318))	m1		Correct rearrangement to either sin$C$ or cos$C$ or sin$B$ or cos$B$ equal to numerical expression ft on c's numerical value for BC. PI by correct C or (by correct B if Mscored)
	Smallest angle, (C =) 11.5º to 1dp	A1	3	Accept a value 11.4 to 11.5 inclusive.
			8	

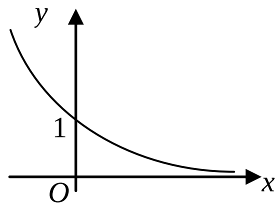
MPC2 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	Stretch(**I**) in x-direction(**II**) scale factor $\frac{1}{6}$ (**III**)	M1 A1	 2	Need (**I**) **and** either (**II**) or (**III**) Need (**I**) and (**II**) and (**III**)
(ii)	(g(x) =) = $\left(1+\frac{x-3}{3}\right)^6$ = $\left(\frac{x}{3}\right)^6$ or $\frac{x^6}{3^6}$ or $\frac{x^6}{729}$	M1 A1	 2	OE Replaces $\frac{x}{3}$ by $\frac{x-3}{3}$ Must be simplified
(b)	$\left(1+\frac{x}{3}\right)^6 = 1+\binom{6}{1}\frac{x}{3}+\binom{6}{2}\left(\frac{x}{3}\right)^2+\binom{6}{3}\left(\frac{x}{3}\right)^3$ … = (1 +) 2x $+\frac{6!}{4!2!}\left(\frac{x}{3}\right)^2+\frac{6!}{3!3!}\left(\frac{x}{3}\right)^3$ = (1 + 2x) $+\frac{15}{9}x^2+\frac{20}{27}x^3$ (a=2) $b=\frac{5}{3}$, $c=\frac{20}{27}$	B1 M1 A1 A1	 4	a=2. Condone ‘2x’ Either (1 6) 15 20 seen or $\binom{6}{2}$, $\binom{6}{3}$ written (PI) in terms of factorials (OE) b = $\frac{5}{3}$ (or $1\frac{2}{3}$). Condone …+$\frac{5}{3}x^2$ c = $\frac{20}{27}$. Condone …+$\frac{20}{27}x^3$ Accept equivalent recurring decimals Ignore terms with higher powers of x **SC** If A0A0 award A1 for either $+15\frac{x^2}{9}$, $+20\frac{x^3}{27}$ seen or $+\frac{15x^2}{9}$, $+\frac{20x^3}{27}$ seen
	Total		**8**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\{ S_{25} =\}\ \frac{25}{2}\left[2a+(25-1)d\right]$	M1		$\frac{25}{2}\left[2a+(25-1)d\right]$ OE
	$\frac{25}{2}\left[2a+24d\right]=3500$			
	$25(2a+24d)=7000$ or [$\frac{50a+600d}{2}=3500$]	m1		Forming equation and attempt to remove fraction or to expand brackets or better
	$50a+600d=7000$ (or better) so $a + 12d = 140$	A1	3	CSO AG Be convinced.
(b)	5^{th} term = $a + 4d$	M1		$a + (5 - 1)d$ used correctly
	$a + 12d = 140$, $a + 4d = 100$ $\Rightarrow 8d = 40$	M1		Solving $a + 12d = 140$ simultaneously with either $a+4d = 100$ or $a+5d = 100$ as far as eliminating either a or d.
	$\Rightarrow d = 5$	A1		
	$\Rightarrow a = 80$	A1	4	
(c)	$33\left(3500-\sum_{n=1}^{k}u_n\right)=67\sum_{n=1}^{k}u_n$	M1		Recognition that $\sum_{n=1}^{25}u_n = 3500$
	$33\times3500=67\sum_{n=1}^{k}u_n+33\sum_{n=1}^{k}u_n$	m1		Correct rearrangement PI
	$100\times\sum_{n=1}^{k}u_n = 33\times3500 \Rightarrow \sum_{n=1}^{k}u_n = 1155$	A1	3	
	Total		**10**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
7(a)		B1		Correct shaped graph in 1st two quadrants only and indication of correct behaviour of curve for large positive and negative vals. of *x*. Ignore any scaling on axes.
		B1	2	*y*-intercept indicated as 1 on diagram or stated as intercept=1 or as coords (0, 1).
(b)	$\frac{1}{2^x}=\frac{5}{4} \Rightarrow 2^{-x}=\frac{5}{4}$ (or $2^x=\frac{4}{5}$ or $2^{2-x}=5$)	M1		Correct 'rearrangement' to eg $2^x=\frac{4}{5}$ or $2^{-x}=\frac{5}{4}$ or $0.5^x=1.25$ PI or $\log 1-\log 2^x=\log(5/4)$ or better
	$\log 2^{-x}=\log 1.25 \Rightarrow -x\log 2=\log 1.25$ $[\log 2^x=\log 0.8 \Rightarrow x\log 2=\log 0.8]$ $[\log 2^{2-x}=\log 5 \Rightarrow (2-x)\log 2=\log 5]$ $[2^x=0.8,\ x=\log_2 0.8]$; $[0.5^x=1.25,\ x=\log_{0.5}1.25]$	M1		Takes logs of both sides of eqn of form either $2^x=k$ or $2^{-x}=k$ OE and uses 3rd law of logs or log to base 2 (or base ½) correctly
	$x=-0.321928\ldots$ so $x=-0.322$ (to 3sf)	A1	3	Condone >3sf [Logs must be seen to be used otherwise **max** of M1M0A0]
(c)	$\log_a b^2+3\log_a y=3+2\log_a\left(\frac{y}{a}\right)$			
	$\log_a b^2+3\log_a y=3+2[\log_a y-\log_a a]$	M1		A log law used correctly; condone missing base *a*.
	$\log_a b^2+\log_a y=3-2\log_a a$ $\log_a b^2 y=3-2\log_a a$	M1		A different log law used correctly condone missing base *a*.
	$\log_a b^2 y=3-2(1)$ [or $\log_a b^2 y+\log_a a^2=3$]	M1		Either a further different log law used correctly condone missing base *a* or $\log_a a=1$ stated/used.
	$\Rightarrow \log_a b^2 y=1 \Rightarrow b^2 y=a$	m1		$\log_a Z=k \Rightarrow Z=a^k$ used or a correct method to eliminate logs (dep on no misapplication of any log law OE in the whole solution) Rearrangements which require only two of the above Ms to eliminate logs correctly: award the remaining M with the m mark.
	$\Rightarrow y=ab^{-2}$	A1	5	ACF of RHS
	Total		**10**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$2\sin\theta = 7\cos\theta \Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{7}{2}$	M1		$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ clearly used to reach either $2\tan\theta = 7$ or $2/7 \tan\theta = 1$ or $\tan\theta = 3.5$ or even $\tan\theta = 2/7$ after seeing $\dfrac{\sin\theta}{\cos\theta} = \dfrac{2}{7}$
	$\Rightarrow \tan\theta = \dfrac{7}{2}$	A1	2	$\dfrac{7}{2}$ OE eg 3.5
(b)(i)	$6\sin^2 x = 4 + \cos x$ $6(1 - \cos^2 x) = 4 + \cos x$	M1		$\cos^2 x + \sin^2 x = 1$ used
	$6 - 6\cos^2 x = 4 + \cos x$ $\Rightarrow 6\cos^2 x + \cos x - 2 = 0$	A1	2	CSO AG Be convinced.
(ii)	$6\sin^2 x = 4 + \cos x \Rightarrow$ $6\cos^2 x + \cos x - 2 = 0$	M1		Uses (b)(i)
	$(3\cos x + 2)(2\cos x - 1) \quad (= 0)$	m1 A1		$(3c \pm 2)(2c \pm 1)$ or by formula Correct factorisation or quadratic formula with $b^2 - 4ac$ evaluated correctly. (PI by both correct values for cos x)
	$\cos x = -\dfrac{2}{3}$, $\cos x = \dfrac{1}{2}$	A1		CSO Both values for cos x correct. Accept 3sf rounded or truncated.
	$x = 132°, 228°, 60°, 300°$	B2,1,0	6	B1 for any 3 of the 4 values correct. Condone greater accuracy (131.810..; 228.189..). Ignore answers outside the given interval. Deduct 1 mark from these two B marks for each extra solution if more than 4 answers in the given interval to a min of B0 NMS: max possible is B2
	Total		**10**	

MPC2 (cont)

Q	Solution	Marks	Total	Comments
9(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 12 - 5x^{\frac{2}{3}}$	M1 A1	 2	$kx^{\frac{2}{3}}$ term. ACF
(b)(i)	When x=0, $\frac{\mathrm{d}y}{\mathrm{d}x} = 12$ Eqn of tangent at O is $y = 12x$	B1F B1F	 2	Ft on c's y' evaluated correctly at x=0 OE Ft on c's value for $y'(0)$ provided $y'(0)>0$.
(ii)	When $x = 8$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 12 - 5 \times (8)^{\frac{2}{3}}$ Equation of tangent at (8, 0) is $y - 0 = y'(8)[x - 8]$ $y = -8(x-8) \Rightarrow y + 8x = 64$	M1 m1 A1	 3	Attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 8$ $y = y'(8)[x - 8]$ OE CSO AG
(c)	$\int \left(12x - 3x^{\frac{5}{3}}\right) \mathrm{d}x = \frac{12x^2}{2} - \frac{3x^{\frac{8}{3}}}{\frac{8}{3}}$ $(+c)$ $= 6x^2 - \frac{9}{8}x^{\frac{8}{3}}$ $(+c)$	M1 B1 A1	 3	$kx^{\frac{5}{3}+1}$ term after integrating, condone k left unsimplified for this M mark. For $6x^2$ OE eg $(12x^2/2)$ For $-\frac{9}{8}x^{\frac{8}{3}}$ OE
(d)	Area bounded by curve and x-axis $= \int_0^8 \left(12x - 3x^{\frac{5}{3}}\right) \mathrm{d}x = 6 \times 8^2 - \frac{9}{8} \times (8)^{\frac{8}{3}}$ $= 384 - 288 = 96$ At P, $12x + 8x = 64$ $(x_P = 3.2)$ $y_P = 38.4$ Area of triangle $OPA = \frac{1}{2} \times 8 \times y_P$ Area of shaded region $=$Area Δ $OPA - \int_0^8 \left(12x - 3x^{\frac{5}{3}}\right) \mathrm{d}x$ $= 153.6 - 96 = 57.6$	 M1 A1 M1 A1 M1 M1 A1	 7	 $\pm$ F(8) {$-$ F(0)} PI following integration PI by correct final answer if evaluation not seen here Solving $y + 8x = 64$ and c's $y=kx$, $k>0$, down to an eqn in one variable… [$y+2y/3=64$] For $y_P = 38.4$ OE [If using integration to find area of triangle, award A1 if both '$x_P = 3.2$' and correct integration of correct eqns of the 2 lines] OE Need perpendicular ht to be linked to $y_P > 0$. M0 if evaluated to a value <0 OE eg 288/5
	Total		**17**	
	TOTAL		**75**	