Version 1.0

General Certificate of Education
June 2010

Mathematics MPC1

Pure Core 1

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

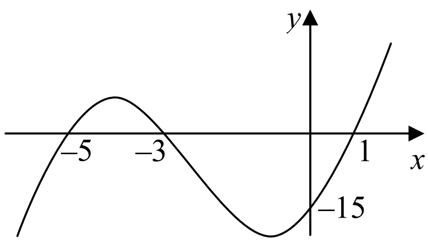
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1(a)	$y = \frac{14}{3} - \frac{2}{3}x$	M1		Attempt at $y = \ldots$
	Gradient $AB = -\frac{2}{3}$	A1	2	Condone error in rearranging equation
(b)(i)	$y - 7 = \text{"}their\ grad\ AB\text{"}(x-3)$	M1		or $2x+3y = k$ and sub $x = 3, y = 7$ or $y = mx + c$, m = *their grad AB* and attempt to find c using $x = 3, y = 7$
	$y - 7 = -\frac{2}{3}(x-3)$ OE	A1	2	$2x+3y = 27$, $y = -\frac{2}{3}x + 9$ etc
(ii)	$m_1 m_2 = -1$	M1		or *negative reciprocal* (stated or used PI)
	$\Rightarrow$ grad $AD = \frac{3}{2}$	A1$\sqrt{}$		FT their grad AB
	$y - 7 = \frac{3}{2}(x-3)$	A1		Any correct equation unsimplified
	$\Rightarrow 3x - 2y + 5 = 0$	A1	4	Integer coefficients; all terms on one side, condone different order or multiples. eg $0 = 4y - 6x - 10$
(c)	$2x+3y=14$ and $5y - x = 6$ used with x or y eliminated (generous)	M1		$2(5y-6)+3y=14$ etc
	$x = 4$,	A1		
	$y = 2$	A1	3	$B(4,2)$ full marks NMS
	Total		**11**	
2(a)	$\left(3-\sqrt{5}\right)^2 = 9 - 6\sqrt{5} + \left(\sqrt{5}\right)^2$	M1		Allow one slip in one of these terms M0 if middle term is omitted
	$= 14 - 6\sqrt{5}$	A1	2	
(b)	$\frac{\left(3-\sqrt{5}\right)^2}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$	M1		or $\ldots \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$
	$14 + 6\sqrt{5}\sqrt{5} - 6\sqrt{5} - 14\sqrt{5}$ $(= 44 - 20\sqrt{5}\ )$	m1		Expanding ***their*** numerator (condone one error or omission)
	(Denominator) = -4	B1		Must be seen as denominator
	(Answer) $= -11 + 5\sqrt{5}$	A1	4	Accept "answer $= 5\sqrt{5} - 11$"
	Total		**6**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\mathrm{p}(-3)=(-3)^3+7(-3)^2+7(-3)-15$ $=-27+63-21-15$	M1		$\mathrm{p}(-3)$ attempted; NOT long division This line alone implies M1
	$\mathrm{p}(-3)=0 \Rightarrow (x+3$ is) factor	A1	2	$\mathrm{p}(-3)$ shown = 0 plus statement
(ii)	$\mathrm{p}(x)=(x+3)(x^2+px+q)$	M1		Full long division, comparing coefficients or by inspection either p = 4 or q = –5
	(Quadratic factor) (x^2+4x-5)	A1		or M1 A1 for either x–1 or x+5 ***clearly*** found using Factor Theorem
	$(\mathrm{p}(x)=)$ $(x+3)(x-1)(x+5)$	A1	3	Must be seen as a product of 3 factors NMS full marks for correct product SC B2 for 3 correct factors listed NMS SC B1 for $(x + 3)(x - 1)(\ )$ or $(x + 3)(x + 5)(\ )$ or $(x + 3)(x + 1)(x - 5)$
(b)	$\mathrm{p}(2)=2^3+7\times2^2+7\times2-15$ or (2+3)(2–1)(2+5)	M1		NOT long division; must be $\mathrm{p}(2)$ May use "their" product of factors
	(Remainder) = 35	A1cso	2	
(c)(i)	$\mathrm{p}(-1)=-16$; $\mathrm{p}(0)=-15$ $\Rightarrow \mathrm{p}(-1)<\mathrm{p}(0)$	B1	1	Values must be evaluated correctly
(ii)		B1		y- intercept –15 marked or (0,–15) stated
		M1		Cubic graph – 1 max, 1 min
		A1		shape with –5, –3, 1 marked
	Cannot score M1A0A1 but can score B0M1A1A1	A1	4	Graph correct with minimum point to left of y-axis and going beyond both –5 and 1 Previous A1 must be scored
	Total		**12**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{x^5}{5} - \frac{8}{2}x^2 + 9x$	M1 A1 A1		One term correct Another term correct All correct (may have + c)
	$\frac{32}{5} - 16 + 18$	m1		$\mathrm{F}(2)$ attempted
	$= 8\frac{2}{5}$	A1	5	$\frac{42}{5}$, 8.4
(ii)	Shaded area $= 18$ – '*their integral*' $= 9\frac{3}{5}$	M1 A1	2	PI by 18 – (a)(i) NMS $\frac{48}{5}$, 9.6 NMS full marks
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 - 8$	M1 A1		One term correct All correct (no + c etc)
	$x = 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 4 - 8$	m1		sub $x = 1$ into *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$
	(Gradient of curve) $= -4$	A1cso	4	No ISW
(ii)	$y - 2 = -4(x - 1)$; $y = -4x + c,\ c = 6$	B1$\surd$	1	any correct form ; FT *their* answer from (b)(i) but must use $x = 1$ and $y = 2$
	Total		**12**	

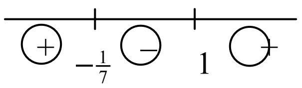
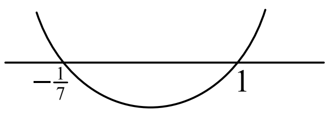
MPC1 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$(x+5)^2 + (y-6)^2 = 5^2$	M1 A1 B1	3	One term correct LHS LHS all correct RHS correct: condone = 25
(b)(i)	sub $x=-2,\ y=2$ into circle equation $3^2 + (-4)^2 = 25$ $\Rightarrow$ lies on circle	B1	1	Circle equation must be correct Must have concluding statement
(ii)	Grad $PC = -\frac{4}{3}$	B1		Condone $\frac{4}{-3}$
	Normal to circle has equation $y-6 = \text{'}\textit{their gradient PC}\text{'}(x+5)$ or $y-2 = \text{'}\textit{their gradient PC}\text{'}(x+2)$	M1		M0 if tangent attempted or incorrect coordinates used
	$y-6 = -\frac{4}{3}(x+5)$ or $y-2 = -\frac{4}{3}(x+2)$	A1cso	3	Any correct form eg $4x+3y+2=0$ $y=-\frac{4}{3}x+c,\quad c=-\frac{2}{3}$
(iii)	$PM = \frac{1}{2}\times$ radius	M1		**Alternative 1** Attempt at $M\left(-\frac{7}{2},4\right)$ with at least one correct coordinate **and** PM^2 attempted
	$= 2.5$	A1cso		$PM^2 = \frac{9}{4}+4 = \frac{25}{4}$
	$PO = \sqrt{8}$	B1		$PO^2 = 4+4 = 8$
	P is closer to the point M	E1cso	4	Statement following correct values
		(M1)		**Alternative 2** Attempt at $M\left(-\frac{7}{2},\ 4\right)$ with at least one correct coordinate **and** attempt at vectors or difference of coordinates
		(A1cso)		$\overrightarrow{PM} = \begin{pmatrix}-1.5\\2\end{pmatrix}$ OE
		(E1cso)		P is closer to the point M
		(E1)	(4)	Components of their $\overrightarrow{PM}$ and $\overrightarrow{OP}$ considered – ***totally independent*** of M1
	Total		**11**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\text{S.A.} = 4xy + 5xy + 3xy + 6x^2 + 6x^2$ OE	M1		Condone one slip or omission
	$= 12xy + 12x^2$	A1		
	$144 = 12xy + 12x^2$			Must see this line
	$\Rightarrow xy + x^2 = 12$	A1cso	3	AG
(ii)	(Volume =) $\frac{1}{2} \times 3x \times 4x \times y$ OE	M1		
	$= 6x^2 \times \frac{(12 - x^2)}{x}$			Must see $(y =)\frac{(12-x^2)}{x}$ or $xy = 12 - x^2$ for A1
	$(V =) 72x - 6x^3$	A1	2	AG must be convinced not working back from answer
(b)(i)	$\frac{\mathrm{d}V}{\mathrm{d}x} = 72 - 18x^2$	M1 A1	2	One term correct All correct (no + c etc)
(ii)	$x = 2 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}x} = 72 - 18 \times 2^2$	M1		Substitute $x = 2$ into their $\frac{\mathrm{d}V}{\mathrm{d}x}$
	$\Rightarrow \frac{\mathrm{d}V}{\mathrm{d}x} = 72 - 72 = 0$			
	$\Rightarrow$ stationary (value when $x = 2$)	A1	2	Shown = 0 plus statement Statement may appear first
(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = -36x$	B1$\surd$		FT their $\frac{\mathrm{d}V}{\mathrm{d}x}$
	$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = -72$ *or* when $x = 2 \Rightarrow \frac{\mathrm{d}^2V}{\mathrm{d}x^2} < 0$			
	$\Rightarrow$ maximum	E1$\surd$	2	FT their $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$ value when $x = 2$ with appropriate conclusion
	Total		**11**	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$2(x-5)^2$ $+3$	B1 B1	 2	$p = 5$ $q = 3$
(ii)	Stating both $(x-5)^2 \geqslant 0$ and $3 > 0$ $\Rightarrow 2x^2 - 20x + 53 > 0$ *or* $2(x-5)^2 + 3 > 0$ $\Rightarrow 2x^2 - 20x + 53 = 0$ has no real roots	M1 A1cso	 2	FT their p & q, but must have $q > 0$ Must have statement and correct p & q.
(b)(i)	$b^2 - 4ac = (k+1)^2 - 4k(2k-1)$ $= -7k^2 + 6k + 1$ real roots $\Rightarrow b^2 - 4ac \geqslant 0$ $-7k^2 + 6k + 1 \geqslant 0$ $\Rightarrow 7k^2 - 6k - 1 \leqslant 0$	M1 A1 B1√ A1cso	 4	Condone one slip (including x is one slip) Condone recovery from missing brackets Their discriminant $\geqslant 0$ (in terms of k) Need not be simplified & may earn earlier AG (must see sign change)
(ii)	$(7k+1)(k-1)$ Critical values $k = 1, -\frac{1}{7}$ Use of sign diagram or sketch   $-\frac{1}{7} \leqslant k \leqslant 1$ Full marks for correct answer NMS Condone $-\frac{2}{14}$ throughout Condone $k \geqslant -\frac{1}{7}$ AND $k \leqslant 1$ for full marks ***Take their final line as their answer.***	M1 A1 M1 A1	 4	**Correct** factors or **correct** use of formula May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working. If previous A1 earned, sign diagram or sketch must be correct for M1 Otherwise M1 may be earned for an attempt at the sketch or sign diagram using ***their*** critical values. $\left(-\frac{1}{7} < k < 1\right)$, $\left(k \geqslant -\frac{1}{7} \text{ OR } k \leqslant 1\right)$, $\left(k \geqslant -\frac{1}{7},\ k \leqslant 1\right)$ score M1A1M1A0 *Answer only of* $k < -\frac{1}{7}$, $k < 1$ etc scores M1, A1, M0 since the critical values are evident. *Answer only of* $\frac{1}{7} \leqslant k \leqslant 1$ etc scores M0, M0 since the critical values are not both correct.
	Total		**12**	
	TOTAL		**75**	