



General Certificate of Education (A-level)
January 2013

Mathematics

MM2B

(Specification 6360)

Mechanics 2B

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$KE = \frac{1}{2} \times 0.16 \times 11^2$ $= 9.68 \text{ J}$	M1 A1	2	
(b)	Change in PE: $mgh = 0.16 \times 9.8 \times 5$ $= 7.84 \text{ J}$	M1 A1	2	
(c)(i)	KE when reached point B $= 9.68 - 7.84 \text{ J}$ $= 1.84 \text{ J}$	M1 A1	2	‘(a)’ – ‘(b)’ cao
(ii)	Speed of ball is $\sqrt{\frac{1.84}{\frac{1}{2} \times 0.16}}$ $= 4.7958 \text{ m s}^{-1}$ $= 4.80 \text{ m s}^{-1}$	M1 A1	 2	If added in (c)(i) 0 marks for (c)(i) 14.8 M1A1 for c(ii) Condone 4.8, 4.79
Total			8	
2(a)	$\mathbf{a} = \frac{d\mathbf{v}}{dt}$ $= -4\pi \sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j}$	M1 A1	 2	M1 for either term correct Accept $-12 \times \frac{\pi}{3} \sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j}$ condone no i in (a)
(b)(i)	Using $\mathbf{F} = m\mathbf{a}$: $\mathbf{F} = 4 \times \left[-4\pi \sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 18t\mathbf{j} \right]$ $\mathbf{F} = -16\pi \sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 72t\mathbf{j}$	M1 A1	 2	Or either term correct
(ii)	When $t = 3$, $\mathbf{F} = 4 \times [-4\pi \sin(\pi)\mathbf{i} - 54\mathbf{j}]$ $= -216\mathbf{j}$ Magnitude is 216	B1 B1ft	2	ft finding magnitude of their F
(c)	$\mathbf{r} = \int \mathbf{v} dt$ $= \frac{36}{\pi} \sin\left(\frac{\pi}{3}t\right)\mathbf{i} - 3t^3\mathbf{j} + \mathbf{c}$ When $t = 3$, $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j}$ $\rightarrow -81\mathbf{j} + \mathbf{c} = 4\mathbf{i} - 2\mathbf{j}$ $\mathbf{c} = 4\mathbf{i} + 79\mathbf{j}$ $\mathbf{r} = \left\{ \frac{36}{\pi} \sin\left(\frac{\pi}{3}t\right) + 4 \right\} \mathbf{i} + \{79 - 3t^3\} \mathbf{j}$	M1 A1 M1 A1 A1	 5	either term correct No need for c (otherwise cao) Condone $\frac{12}{(\pi/3)}$ cao
Total			11	

Q	Solution	Marks	Total	Comments
3	<p>Force acting against gravity is $mg\sin\theta$</p> <p>Force acting against gravity and resistance is $mg\sin\theta + 8000$</p> $= 1500 \times g \times \sin\theta + 8000$ $= 8588 \text{ N or } 8590 \text{ N}$ <p>Using power = force \times velocity</p> $= 8588 \times 22$ $= 188\,936 \text{ W}$ $= 189 \text{ kW}$	<p>M1</p> <p>A1</p> <p>M1 dep</p> <p>A1</p> <p>A1</p>	5	<p>Condone $\cos\theta$ or -1 for M marks</p> <p>Accept 188.9 or 188</p>
	Total		5	
4(a)	Symmetry	E1	1	
(b)	<p>Moments about AB:</p> $300\sigma.15 + 100\sigma.5 + 300\sigma.15 = 700\sigma.x$ $x = \frac{9500}{700}$ $= \frac{95}{7} \text{ or } 13.6 \text{ cm}$	<p>M1A1</p> <p>A1</p>	3	<p>(condone lack of σ)</p> <p>M1 needs correct total marks</p>
(c)	<p>Distance from HG is 16.4 cm</p> $\tan\theta = \frac{15}{16.42857}$ $= 0.913043$ $\theta = 42.3974^\circ$ $\theta = 42^\circ$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	4	<p>Seeing both 15,16.4 and tan</p> <p>[48° probably B1, M1]</p> <p>NB $\frac{13.6}{15}$ etc $\Rightarrow 42^\circ$ no marks</p>
	Total		8	

Q	Solution	Marks	Total	Comments
5(a)	Using $F = ma$: $-4v^{\frac{1}{3}} = 12 \frac{dv}{dt}$ $\therefore \frac{dv}{dt} = -\frac{1}{3}v^{\frac{1}{3}}$ $-3 \int \frac{dv}{v^{\frac{1}{3}}} = \int dt$ $-3 \times \frac{v^{\frac{2}{3}}}{\frac{2}{3}} = t + c$ $-\frac{9}{2}v^{\frac{2}{3}} = t + c$ When $t = 0, v = 8 \Rightarrow c = -18$ $-\frac{9}{2}v^{\frac{2}{3}} = t - 18$ $v^{\frac{2}{3}} = 4 - \frac{2}{9}t$ $v = \left(4 - \frac{2}{9}t\right)^{\frac{3}{2}}$	B1 M1 A1 M1A1 A1	 6	 condone $-$, 3 incorrect side condone lack of $+$ c
(b)	Particle is at rest when $4 - \frac{2}{9}t = 0$ The value of t is 18	B1	1	
	Total		7	
6(a)	Resolve vertically: $T \cos \theta = mg$ $34 \cos \theta = 2 \times 9.8$ $\cos \theta = \frac{19.6}{34}$ $\theta = 54.8^\circ$	M1 A1 A1	 3	M1 for $T \cos \theta$ or $T \sin \theta$ and mg
(b)	Resolve horizontally for particle: $\frac{mv^2}{r} = T \sin \theta$ $v^2 = \frac{34 \sin 54.8 \times 0.8}{2}$ $v^2 = 11.113$ Speed is 3.33 m s^{-1}	M1 A1 ft from (a) A1	 3	M1 for $T \cos \theta$ or $T \sin \theta$ Accept 3.34
(c)	Time taken is $2\pi r / v$ = 1.51 sec	M1 A1ft	 2	Or find ω and use $\frac{2\pi}{\omega}$
	Total		8	

Q	Solution	Marks	Total	Comments
7(a)	Using conservation of energy:			
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mgh$	M1		for 3 terms, 2 KE and 1 PE
	$\frac{1}{2} \times 3 \times v^2 = \frac{1}{2} \times 3 \times 4^2 - 3 \times g \times 1.2(1 - \cos 25)$	M1 A1		M1A1 for finding h [M1 for $1.2(1 - \cos 25 \text{ or } \sin 25)$]
	$v^2 = 4^2 - 2.4 \times g(1 - \cos 25)$ $v^2 = 16 - 2.2036$ $v = 3.71 \text{ ms}^{-1}$	A1	4	Accept 3.7, 3.70, 3.72
(b)	Resolving radially:			
	$T = mg \cos 25 + \frac{mv^2}{a}$	M1A1		M1 accept $\cos 25$ or $\sin 25$, + or – sign and $\neq 2$
	$= 26.645 + 34.491$ $= 61.1 \text{ N}$	A1	3	A1 fully correct and substituted Accept 61.0 or 61
	Total		7	

Q	Solution	Marks	Total	Comments
8(a)	$\text{Work done} = \int_0^e \frac{\lambda x}{l} dx$ $= \left[\frac{\lambda x^2}{2l} \right]_0^e$ $= \frac{\lambda e^2}{2l}$	M1 A1 A1	3	SC1 $\int_0^e \frac{\lambda e}{l} de$ SC1 $\int \frac{\lambda x}{l} dx$ with no limits
(b)(i)	Using $T = \frac{\lambda x}{l}$: $5g = \frac{392x}{1.6}$ $x = \frac{5g \times 1.6}{392}$ $= 0.2$ Extension is 0.2 m	M1 A1	2	
(ii)	When extension is 0.6 m, $EPE = \frac{\lambda x^2}{2l}$ $= \frac{392 \times (0.6)^2}{2 \times 1.6}$ $= 44.1 \text{ J}$	B1 M1 A1	3	B1 for 0.6
(iii)	Let y metres be distance particle is above A . C of energy, when particle has speed 0.8 m s^{-1} , gives $5 \times g \times y + \frac{392 \times (0.6 - y)^2}{2 \times 1.6} + \frac{1}{2} \times 5 \times 0.8^2$ $= \frac{392 \times (0.6)^2}{2 \times 1.6}$ $49y + 122.5(0.6 - y)^2 + 1.6 = 122.5 \times 0.6^2$ $49y - 147y + 122.5y^2 + 1.6 = 0$ $122.5y^2 - 98y + 1.6 = 0$ $y = \frac{98 \pm \sqrt{98^2 - 4 \times 122.5 \times 1.6}}{2 \times 122.5}$ $y = \frac{98 \pm 93.9148}{245}$ $= 0.016674 \text{ and } 0.7833$ Speed first becomes 0.8 when $y = 0.0167$	M1A1 A1F A1 E1	5	M1 4 terms, 2 correct M1A1 4 terms, 3 correct M1A2 4 terms correct Ft answer to (b)(ii) if x used instead of $0.6 - y$, A1 here for $x = 0.5833...$
	Total		13	

Q	Solution	Marks	Total	Comments
9(a)	Smooth, hence reaction is perpendicular to possible movement	E1	1	
(b)				
(c)	<p>Resolving along the rod:</p> $S \cos \theta = mg \sin \theta$ <p>Moment about C: $S 2a \cos \theta \sin \theta$</p> $= mg(2a \cos \theta - \frac{1}{2}l) \cos \theta$ $4a.S \sin \theta = mg(4a \cos \theta - l)$ <p>Dividing: $4a \tan \theta = \frac{4a \cos \theta - l}{\sin \theta}$</p> $l = 4a \cos \theta - 4a \sin \theta \tan \theta$ $l = \frac{4a \cos 2\theta}{\cos \theta}$	<p>B2</p> <p>M1A1</p> <p>M1A1</p> <p>A1</p>	<p>2</p> <p>5</p>	<p>B1 for 2 forces correct</p> <p>Or geometrically:</p> <p>three forces act through a point B1</p> <p>M1 is for 2 or 3 terms; 1 term correct (could be horizontal force at C used) [forces act through point D]</p> $AD \cos 2\theta = \frac{l}{2} \cos \theta \quad \text{M1A1}$ $AD \cos \theta = 2a \cos \theta \quad \text{M1}$ $l = \frac{4a \cos 2\theta}{\cos \theta} \quad \text{A1}$

Q	Solution	Marks	Total	Comments
9 cont	<p>or</p> <p>Resolving perpendicular to S: $R \cos \theta = mg \cos 2\theta$</p> <p>Moments about A: $R 2a \cos \theta = mg \frac{1}{2} l \cos \theta$</p> <p>$4a R = mgl$ $4a mg \cos 2\theta = mgl \cos \theta$ $l = \frac{4a \cos 2\theta}{\cos \theta}$</p> <p>or</p> <p>Resolving horizontally: $R \sin \theta = S \cos 2\theta$</p> <p>Resolving vertically: $R \cos \theta + S \sin 2\theta = mg$</p> <p>Moments about A: $R 2a \cos \theta = mg \frac{1}{2} l \cos \theta$</p> <p>$4a R = mgl$ $R \cos \theta + R \frac{\sin \theta}{\cos 2\theta} \sin 2\theta = 4a \frac{R}{l}$ $l = \frac{4a \cos 2\theta}{\cos \theta}$</p>	<p>(M1A1)</p> <p>(M1A1)</p> <p>(A1)</p> <p>(M1A1)</p> <p>(M1A1)</p> <p>(A1)</p>	8	Both attempted for M1 Both correct for A1
	Total		8	
	TOTAL		75	